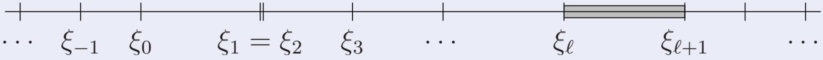
Knot Sequence

A knot sequence

$$\xi: \ \cdots \leq \xi_0 \leq \xi_1 \leq \xi_2 \leq \cdots$$

is a finite or bi-infinite nondecreasing sequence of real numbers without accumulation points. It induces a partition of a subset $R \subseteq \mathbb{R}$ into knot intervals $[\xi_\ell, \xi_{\ell+1})$.



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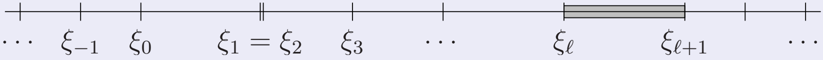


The multiplicity of a knot, $\#\xi_k$, is the maximal number of repetitions of ξ_k in the sequence ξ. In analogy with zeros of functions, we use the terms simple and double knots, etc.

Recurrence Relation

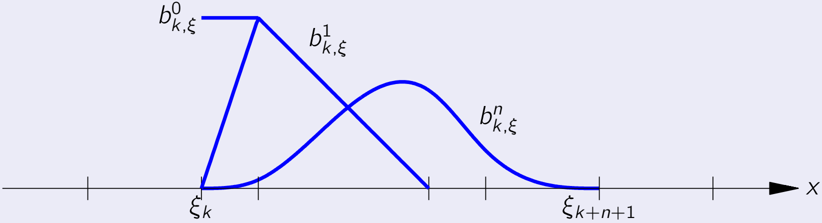
For a knot sequence ξ, the B-splines $b_{k,\xi}^n$ of degree n are defined by the recursion

$$b_{k,\xi}^n = \gamma_{k,\xi}^n b_{k,\xi}^{n-1} + (1 - \gamma_{k+1,\xi}^n) b_{k+1,\xi}^{n-1}, \quad \gamma_{k,\xi}^n(x) = \frac{x - \xi_k}{\xi_{k+n} - \xi_k},$$

starting from the characteristic functions

$$x \mapsto b_{k,\xi}^0(x) = \begin{cases} 1 & \text{for } \xi_k \le x < \xi_{k+1}, \\ 0 & \text{otherwise}, \end{cases}$$

of the knot intervals $[\xi_k, \xi_{k+1})$ and discarding terms for which the denominator vanishes.



Each B-spline $b^n_{k,\xi}$ is uniquely determined by its knots $\xi_k, \ldots, \xi_{k+n+1}$ and vanishes outside of the interval $[\xi_k, \xi_{k+n+1})$. Moreover, on each of the nonempty knot intervals $[\xi_\ell, \xi_{\ell+1})$, $k \le \ell \le k+n$, it is a nonnegative polynomial of degree $\le n$.

If the degree and the knot sequence are fixed throughout the discussion of a particular topic, we write $b_k = b^n_{k,\xi}$ to avoid the excessive use of sub- and superscripts. Similarly, parameters are omitted in other B-spline related notations.

Continuous Dependence on the Knots

If x lies in the interior of one of the knot intervals of the B-spline $b^n_{k,\xi}$ and

$$\eta_\ell \to \xi_\ell, \quad \ell = k, \ldots, k+n+1\,,$$

then, as illustrated in the figure,

$$\lim_{\eta\to\xi} b^n_{k,\eta}(x) = b^n_{k,\xi}(x)\,.$$