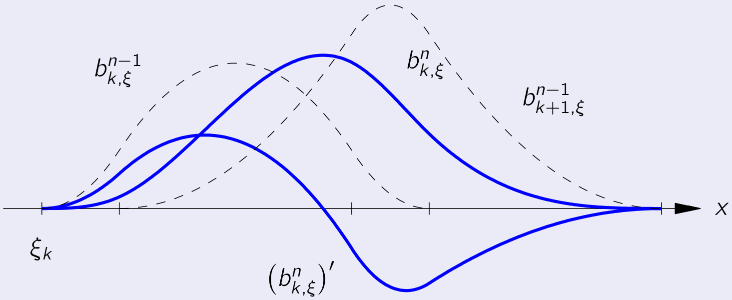
Derivative of a B-Spline

The derivative of a B-spline of degree n with knots $\xi_k, \ldots, \xi_{k+n+1}$ is a weighted difference of two B-splines of degree $n-1$. On each knot interval $[\xi_\ell, \xi_{\ell+1})$,

$$\left(b^n_{k,\xi}\right)' = \alpha^n_{k,\xi} b^{n-1}_{k,\xi} - \alpha^n_{k+1,\xi} b^{n-1}_{k+1,\xi}, \quad \alpha^n_{k,\xi} = \frac{n}{\xi_{k+n} - \xi_k},$$

where terms with denominator zero are discarded.



It follows from the recursion that $b^n_{k,\xi}$ is $(n-\mu)$-times continuously differentiable at a knot ξ_ℓ if ξ_ℓ has multiplicity $\mu \leq n$ among $\xi_k, \ldots, \xi_{k+n+1}$. In particular, $b^n_{k,\xi}$ is continuous on $\mathbb{R}$ unless one of its knots has multiplicity $n+1$.

Zeros of B-Splines

If ξ_k has multiplicity μ among the knots of $b^n_{k,\xi}$, then the left endpoint ξ_k of the B-spline support is a zero of order $n+1-\mu$:

$$b^n_{k,\xi}(x) = \beta_k (x-\xi_k)^{n+1-\mu} + \mathcal{O}\left((x-\xi_k)^{n+2-\mu}\right) , \quad x \to \xi_k^+ ,$$

with $\beta_k > 0$.
An analogous statement is valid for the right endpoint of the support of $b^n_{k,\xi}$.