Quasi-Interpolant

A linear spline approximation scheme

$$f \mapsto Qf = \sum_k (Q_k f)\, b_k \in S_\xi^n$$

for continuous functions is called a quasi-interpolant of maximal order if the following conditions are satisfied.
(i) Q_k are locally bounded linear functionals, i.e.,

$$|Q_k f| \le \|Q\| \, \|f\|_{\infty,[\xi_k,\xi_{k+n+1}]} \,,$$

where $\|f\|_{\infty,U} = \max_{x\in U} |f(x)|$.
(ii) Q reproduces polynomials p of degree $\le n$, i.e., $Qp = p$ on the parameter interval D_ξ^n of S_ξ^n. Equivalently, for $y \in \mathbb{R}$,

$$Q_k p = \psi_k(y), \quad p(x) = (x-y)^n$$

with $\psi_k(y) = (\xi_{k+1} - y) \cdots (\xi_{k+n} - y)\,.$

Standard Projector

A quasi-interpolant

$$f \mapsto Qf = \sum_k (Q_k f)\, b_k \in S_\xi^n ,$$

for which each linear functional Q_k, $k \sim \xi$, depends only on values of f in a single knot interval in the parameter interval D_ξ^n of S_ξ^n, is a projector, i.e.,

$$Qp = p \quad \forall p \in S_\xi^n .$$

Such quasi-interpolants are called standard projectors if the norms of the linear functionals can be bounded by a constant $\|Q\|$ which depends only on the degree n. Projectors of this type exist if all B-splines have a largest knot interval of their support in D_ξ^n.