Accuracy of Quasi-Interpolation

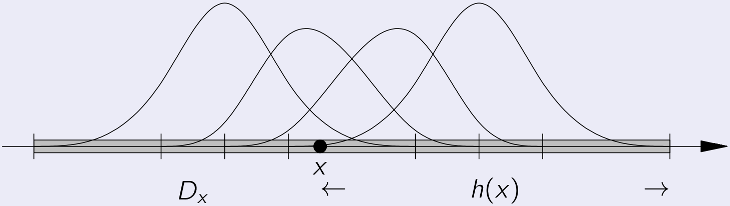
The error of a quasi-interpolant

$$f \mapsto Qf = \sum_k (Q_k f)\, b_k \in S_\xi^n$$

of maximal order satisfies

$$|f(x) - (Qf)(x)| \le \frac{\|Q\|}{(n+1)!}\, \|f^{(n+1)}\|_{\infty, D_x}\, h(x)^{n+1}, \quad x \in D_\xi^n\,,$$

where D_x is the union of the supports of all B-splines b_k, $k \sim x$, which are relevant for x, and $h(x) = \max_{y \in D_x} |y - x|$.



If the local mesh ratio is bounded, i.e., if the quotients of the lengths of adjacent knot intervals are $\le r_\xi$, then the error of the derivatives on the knot intervals $(\xi_\ell, \xi_{\ell+1})$ can be estimated by

$$|f^{(j)}(x) - (Qf)^{(j)}(x)| \le c(n, r_\xi)\, \|Q\|\, \|f^{(n+1)}\|_{\infty, D_x}\, h(x)^{n+1-j}$$

for all $j \le n$.
Choosing a standard projector for Q shows, in particular, that splines approximate smooth functions with optimal order; the norm $\|Q\|$ does depend only on n in this case.