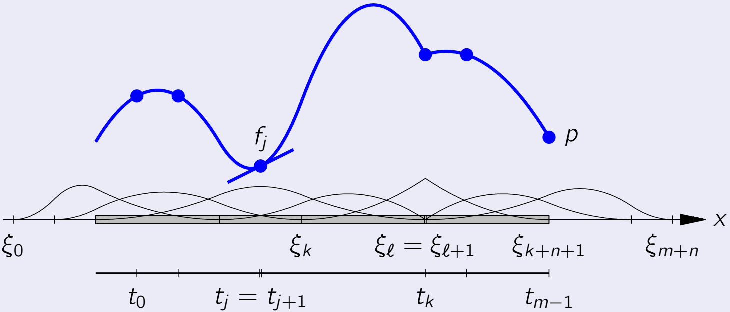
Schoenberg–Whitney Conditions

For a spline space S_ξ^n with finite knot sequence $\xi_0, \ldots, \xi_{m+n}$ and an admissible nondecreasing sequence $t: t_0 \leq \cdots \leq t_{m-1}$ of interpolation points in the parameter interval D_ξ^n there exists a unique interpolating spline $p = \sum_{k=0}^{m-1} c_k b_k \in S_\xi^n$ for arbitrary data iff

$$\xi_k < t_k < \xi_{k+n+1}$$

for $k = 0, \ldots, m-1$.



By linearity, the B-spline coefficients are determined by the linear system

$$Ac = f, \quad a_{j,k} = b_k^{(\mu_j)}(t_j),\, f_j = f^{(\mu_j)}(t_j)\,.$$

Since on any knot interval in D^n_ξ only $n+1$ B-splines are nonzero, the interpolation matrix A is banded; each row can have at most $n+1$ nonzero entries.

Error of Spline Interpolation

The error of a spline interpolant $p = \sum_{k=0}^{m-1} c_k\, b_k \in S^n_\xi$ with $\xi:\ \xi_0, \ldots, \xi_{n+m}$ to a smooth function f can be estimated by

$$|f(x) - p(x)| \leq c\left(n, \|A^{-1}\|_\infty\right)\, \|f^{(n+1)}\|_{\infty,R}\, h^{n+1}\,, \quad x \in D^n_\xi = R\,,$$

where the constant depends on the degree and the maximum norm of the inverse of the interpolation matrix

$$A:\ a_{i,k} = b_k(x_i)\,,$$

h is the maximal length of the knot intervals, and it is assumed that the parameter interval $D^n_\xi = [\xi_n, \xi_m]$ contains all interpolation points as well as for each B-spline a largest interval in its support.