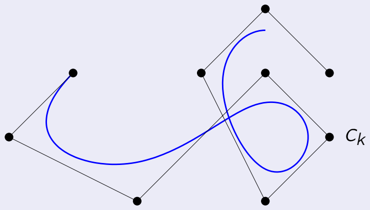
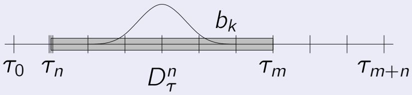
Spline Curve

A spline curve of degree $\le n$ in $\mathbb{R}^d$ has a parametrization

$$t \mapsto (p_1(t), \ldots, p_d(t)) = \sum_{k=0}^{m-1} c_k b_k(t), \quad t \in D_\tau^n,$$

with components p_ν in a spline space S_τ^n with finite knot sequence $\tau: \ \tau_0, \ldots, \tau_{m+n}$ and parameter interval $D_\tau^n = [\tau_n, \tau_m]$.





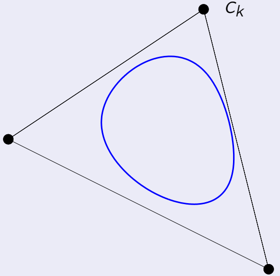
The coefficients $c_k = (c_{k,1}, \ldots, c_{k,d})$ can be combined into an $m \times d$ array C. They are called control points and form the control polygon c for p.

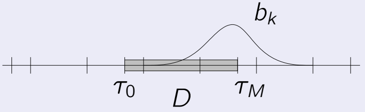
Closed Spline Curve

A closed spline curve of degree $\leq n$ in $\mathbb{R}^d$ has a parametrization

$$t \mapsto (p_1(t), \ldots, p_d(t)) = \sum_{k \in \mathbb{Z}} c_k \, b_k(t) \,, \quad t \in \mathbb{R} \,,$$

with components which are continuous T-periodic splines. This means that $p_\nu \in S^n_{\tau,T}$, $\tau = (\tau_0, \ldots, \tau_{M-1})$, and the B-splines b_k correspond to the periodically extended knot sequence $(\ldots, \tau - T, \tau, \tau + T, \ldots)$ and to M-periodic control points.





$$c_{k+M} = c_k \,, \quad \tau_{k+M} = \tau_k + T$$

According to the periodicity conditions, p is determined by M consecutive control points

$$C = \begin{pmatrix} c_0 \\ \vdots \\ c_{M-1} \end{pmatrix},$$

which form a closed control polygon for p.

Rational Parametrizations

A nonuniform rational B-spline parametrization (NURBS) $r = p/q$ is the quotient of a spline parametrization $t \mapsto p(t)$ with weighted control points

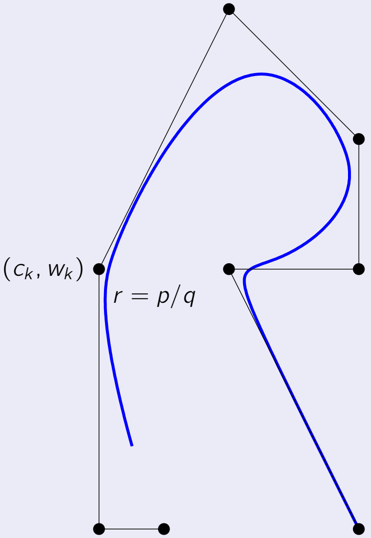
$$(c_k w_k) \in \mathbb{R}^d, \quad w_k > 0\,,$$

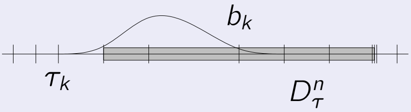
and a spline function $t \mapsto q(t)$ with coefficients w_k. The weights w_k provide additional design flexibility. Increasing a weight pulls the curve towards the corresponding control point.

We can identify r with a spline curve in homogeneous coordinates parametrized by

$$t \mapsto (p(t)\,|\,q(t)) = \sum_k (c_k w_k \,|\, w_k)\, b_k(t) \in \mathbb{R}^{d+1}\,.$$

This interpretation is convenient for implementing algorithms such as knot insertion, evaluation, and differentiation.


(c_k, w_k)
$r = p/q$


b_k
τ_k
D^n_τ