## Interpolation

Points  $p_k$  and tangent vectors  $d_k$  (if provided) can be interpolated with a spline curve at parameter values  $t_k$ , using any of the interpolation methods for spline functions. The univariate schemes are applied separately in each component to determine the components of the parametrization  $p = \sum_k c_k b_k$ . Standard choices are cubic Hermite interpolation and cubic spline interpolation with not-a-knot, natural, or clamped boundary conditions.



If only points are given, knots  $\tau_j$ , parameter values  $t_k$ , and tangent vectors  $d_k$  (if required) have to be determined based on the available information. Basic choices are

• 
$$t_k - t_{k-1} = |p_k - p_{k-1}|;$$

•  $t_k = \tau_{k+\ell}$  with the shift  $\ell$  depending on the labeling of the knots;

• 
$$d_k = (p_{k+1} - p_{k-1})/(t_{k+1} - t_{k-1}).$$

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More accurate derivative approximations employ local polynomial interpolation. The resulting formulas are used in particular at the endpoints of the parameter interval, where one-sided approximations are needed.