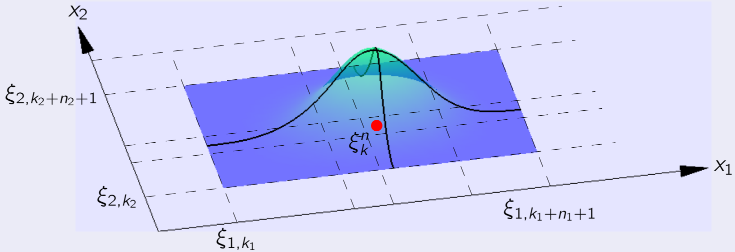
Multivariate B-Splines

The d-variate B-splines of degree $(n_1, \ldots, n_d)$ with respect to the knot sequences ξ are products of univariate B-splines:

$$b^n_{k,\xi}(x) = \prod_{\nu=1}^{d} b^{n_\nu}_{k_\nu,\xi_\nu}(x_\nu)\,.$$

Their knots in the νth coordinate direction are $\xi_{\nu,k_\nu}, \ldots, \xi_{\nu,k_\nu+n_\nu+1}$.



The multivariate knot average

$$\xi_k^n = \left(\xi_{1,k_1}^{n_1}, \ldots, \xi_{d,k_d}^{n_d}\right), \quad \xi_{\nu,k_\nu}^{n_\nu} = \left(\xi_{\nu,k_\nu+1} + \cdots + \xi_{\nu,k_\nu+n_\nu}\right)/n_\nu$$

is often used to identify multivariate B-splines on a grid and can be viewed as weighted center of their support.

Multivariate Splines

A multivariate spline p of degree $\leq n = (n_1, \ldots, n_d)$ is a linear combination of the B-splines corresponding to n-regular knot sequences $\xi = (\xi_1, \ldots, \xi_d)$:

$$p(x) = \sum_k c_k b^n_{k,\xi}(x), \quad x \in D^n_\xi .$$

The coefficients are unique; i.e., the B-splines b_k restricted to the parameter hyperrectangle D^n_ξ form a basis for the spline space denoted by S^n_ξ.

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Spline functions on a subdomain $D \subset D^n_\xi$ are obtained simply by restricting the variable x to the smaller set; the corresponding spline space is denoted by $S^n_\xi(D)$. A basis consists of the relevant B-splines b_k, $k \sim D$, which have some support in D.

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As a consequence of Marsden's identity $S^n_\xi(D)$ contains all multivariate polynomials of coordinate degre $\leq n$.