Evaluation

A d-variate spline $p = \sum_{k \sim D} c_k \, b_k \in S_\xi^n(D)$ can be evaluated at a point $x \in D$ with $\xi_{\nu,\ell_\nu} \le x_\nu < \xi_{\nu,\ell_\nu+1}$ by applying the de Boor algorithm in each variable. For $\nu = 1, \ldots, d$, we modify the coefficients

$$a_{k_\nu} = c_{(\ell_1,\ldots,\ell_{\nu-1},k_\nu,k_{\nu+1},\ldots,k_d)}, \quad \ell_\nu - n_\nu \le k_\nu \le \ell_\nu \,,$$

simultaneously for all $k_\mu = \ell_\mu - n_\mu, \ldots, \ell_\mu$, $\mu > \nu$. We compute, for $i = n_\nu, \ldots, 1$,

$$a_{\ell_\nu - j} \leftarrow \gamma \, a_{\ell_\nu - j} + (1-\gamma) \, a_{\ell_\nu - j - 1}, \; \gamma = \frac{x_\nu - \xi_{\nu,\ell_\nu - j}}{\xi_{\nu,\ell_\nu - j + i} - \xi_{\nu,\ell_\nu - j}} \,, \; j = 0, \ldots, i-1 \,,$$

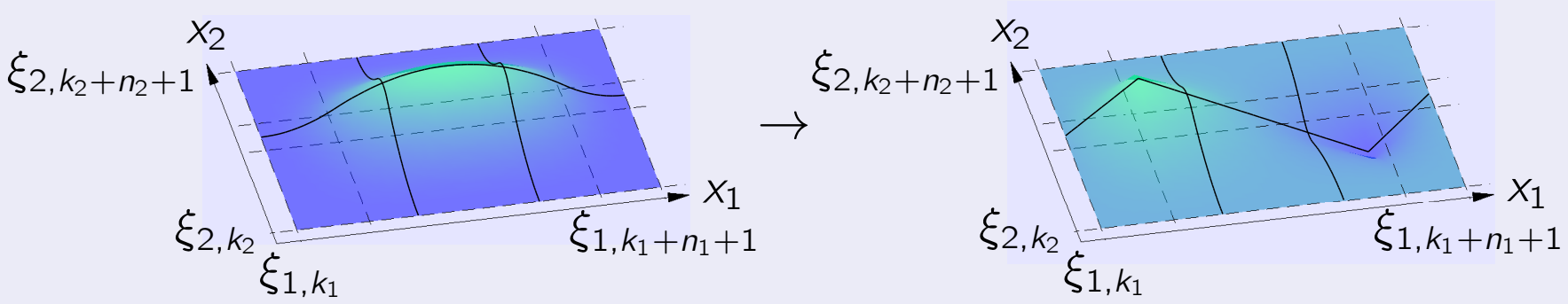
according to the triangular univariate evaluation scheme. This leads to the modified coefficients $c_{(\ell_1,\ldots,\ell_\nu,k_{\nu+1},\ldots,k_d)}$. The final computed coefficient c_ℓ then equals $p(x)$.

Differentiation

The partial derivatives ∂_ν, $\nu = 1, \ldots, d$, of a d-variate B-spline of degree $n = (n_1, \ldots, n_d)$ are differences of B-splines of lower degree:

$$\partial_\nu b^n_{k,\xi} = \alpha^{n_\nu}_{k_\nu,\xi_\nu}\, b^{n-e_\nu}_{k,\xi} - \alpha^{n_\nu}_{k_\nu+1,\xi_\nu}\, b^{n-e_\nu}_{k+e_\nu,\xi}\,, \qquad \alpha^{n_\nu}_{k_\nu,\xi_\nu} = \frac{n_\nu}{\xi_{\nu,k_\nu+n_\nu} - \xi_{\nu,k_\nu}}\,,$$

with e_ν the νth unit vector.



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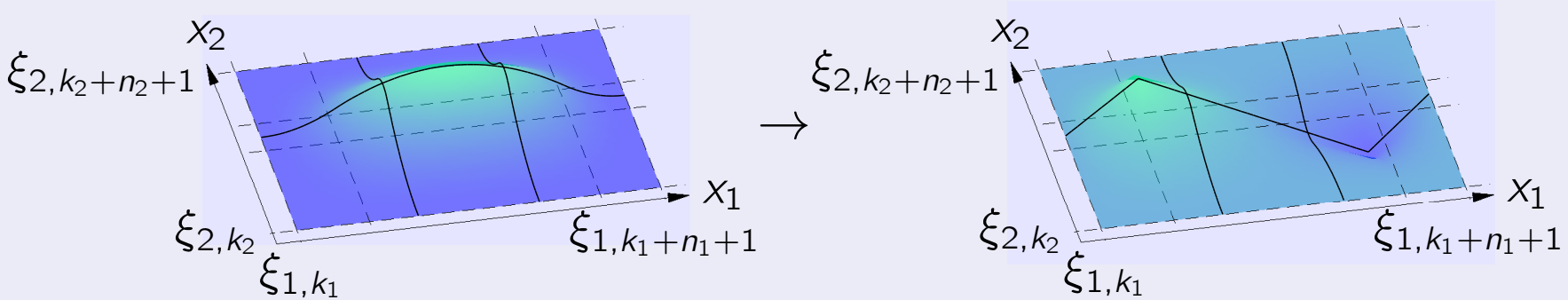

Accordingly, the partial derivative of a multivariate spline in $S^n_\xi(D)$ is given by

$$\partial_\nu \sum_{k\sim D} c_k\, b^n_{k,\xi} = \sum_{k\sim D} \alpha^{n_\nu}_{k_\nu,\xi_\nu}\,(c_k - c_{k-e_\nu})\, b^{n-e_\nu}_{k,\xi}\,,$$

where on both sides the sum is taken over the relevant B-splines for the domain D.