Ritz–Galerkin Approximation

The finite element approximation

$$u_h = \sum_{k \in K} u_k B_k \in \mathbb{B}_h \subset H$$

for a quadratic energy functional with an elliptic bilinear form $a(\cdot, \cdot)$ and a bounded linear functional λ can be computed by solving the Ritz–Galerkin equations

$$GU = F, \quad U = (u_k)_{k \in K} \,,$$

where

$$g_{k,k'} = a(B_k, B_{k'}), \quad f_k = \lambda(B_k) \,.$$

Cea Lemma

The error of the Ritz–Galerkin approximation $u_h \in \mathbb{B}_h$ to the solution u of an elliptic problem with quadratic energy functional satisfies

$$\|u - u_h\|_H \leq (c_+/c_-) \inf_{v_h \in \mathbb{B}_h} \|u - v_h\|_H \,,$$

where $c_\pm$ are the ellipticity constants of the bilinear form.