Isogeometric Elements

Assume that

\[ R \ni t \mapsto x = \varphi(t) \in D \]

is a smooth bijective parametrization of a domain \( D \) over a hyperrectangle \( R \). Then, the B-splines \( b_k \), which span an appropriate subspace of a spline space \( S_{\tau}^{(n, \ldots, n)} \) with parameter hyperrectangle \( D_{\tau}^{(n, \ldots, n)} = R \), can be composed with \( \varphi \) to form so-called isogeometric elements

\[ B_k(x) = b_k(\varphi^{-1}(x)), \quad k \in K, \]

on \( D \). The knot sequences \( \tau \) have to be chosen so that essential boundary conditions are satisfied.
Finite Element Integrals

Assume that $u(x) = v(t)$, where the variables are related by a smooth bijective parametrization $R \ni t \mapsto x = \varphi(t) \in D$. Then,

$$
\int_D f(x, \text{grad}_x u, \ldots) \, dx = \int_R f(\varphi(t), (\varphi'(t)^{-1})^t \text{grad}_t v, \ldots) |\text{det } \varphi'(t)| \, dt,
$$

where $\varphi'$ denotes the Jacobi matrix of $\varphi$. 