Isogeometric Elements

Assume that

$$R \ni t \mapsto x = \varphi(t) \in D$$

is a smooth bijective parametrization of a domain D over a hyperrectangle R. Then, the B-splines b_k, which span an appropriate subspace of a spline space $S_\tau^{(n,\dots,n)}$ with parameter hyperrectangle $D_\tau^{(n,\dots,n)} = R$, can be composed with φ to form so-called isogeometric elements

$$B_k(x) = b_k(\underbrace{\varphi^{-1}(x)}_{t}), \quad k \in K\,,$$

on D. The knot sequences τ have to be chosen so that essential boundary conditions are satisfied.

Finite Element Integrals

Assume that $u(x) = v(t)$, where the variables are related by a smooth bijective parametrization $R \ni t \mapsto x = \varphi(t) \in D$. Then,

$$\int_D f(x, \operatorname{grad}_x u, \ldots)\, dx = \int_R f(\varphi(t), (\varphi'(t)^{-1})^{\mathrm{t}} \operatorname{grad}_t v, \ldots)\, |\det \varphi'(t)|\, dt\,,$$

where φ' denotes the Jacobi matrix of φ.