Linear Elasticity

The displacement (u_1, u_2, u_3) caused by an elastic deformation minimizes the quadratic energy functional

$$\mathcal{Q}(u) = \frac{1}{2}\int_D \sum_{\nu,\nu'=1}^{3} \varepsilon_{\nu,\nu'}(u)\,\sigma_{\nu,\nu'}(u) - \int_D \sum_{\nu=1}^{3} f_\nu u_\nu, \quad u_\nu \in H^1_\Gamma(D)\,,$$

where ε is the strain and σ is the stress tensor, defined by

$$2\varepsilon_{k,\ell} = \partial_k u_\ell + \partial_\ell u_k, \quad \sigma_{k,\ell} = \lambda\,\mathrm{trace}\,\varepsilon\,\delta_{k,\ell} + 2\mu\varepsilon_{k,\ell}\,,$$

respectively.

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Via the calculus of variations, the displacement can also be characterized by the Lamé–Navier boundary value problem

$$-\operatorname{div}\sigma(u) = f \text{ in } D, \quad u = 0 \text{ on } \Gamma, \quad \sigma(u)\eta = 0 \text{ on } \partial D \backslash \Gamma\,,$$

with η the outward unit normal of ∂D.