Preface

We live in an era in which ever more complex phenomena (e.g., climate change dynamics, stock markets, complex logistics, and the Internet) are being described with the help of mathematical models, frequently referred to as systems. These systems typically depend on one or more parameters that are assigned nominal values based on the current understanding of the phenomena. Since, usually, these nominal values are only estimates, it is important to know how deviations from these values affect the solutions of the system and, in particular, whether for some of these parameters even small deviations from nominal values can have a big impact.

Naturally, it is crucially important to understand the underlying causes and nature of these big impacts and to do so for neighborhoods of multiparameter configurations. Unfortunately, in their most general settings, multiparameter deviations are still too complex to analyze fully, and even single-parameter deviations pose significant technical challenges. Nonetheless, the latter constitute a natural starting point, especially since in recent years much progress has been made in analyzing the asymptotic behavior of these single-parameter deviations in many special settings arising in the sciences, engineering, and economics.

Consequently, in this book we consider systems that can be disturbed, to a varying degree, by changing the value of a single perturbation parameter loosely referred to as the "perturbation." Since in most applications such a perturbation would be small but unknown, a fundamental issue that needs to be understood is the behavior of the solutions as the perturbation tends to zero. This issue is important because for many of the most interesting applications there is, roughly speaking, a discontinuity at the limit, which complicates the analysis. These are the so-called singularly perturbed problems.

Put a little more precisely, the book analyzes—in a unified way—the general linear and nonlinear systems of algebraic equations that depend on a small perturbation parameter. The perturbation is analytic; that is, left-hand sides of the perturbed equations can be expanded as a power series of the perturbation parameter. However, the solutions may have more complicated expansions such as Laurent or even Puiseux series. These series expansions form a basis for the asymptotic analysis (as the perturbation tends to zero). The analysis is then applied to a wide range of problems including Markov processes, constrained optimization, and linear operators on Hilbert and Banach spaces. The recurrent common themes in the analyses presented is the use of fundamental equations, series expansions, and the appropriate partitioning of the domain and range spaces.

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