Index

Anderson mixing, 18, 33
breakdown, 32
Cauchy integral, 39
Cesaro sums, 82, 97
Chebyshev parameters, 12
Chebyshev polynomials, 11, 43, 49
cluster
general, 80
outlier function, 80, 84
proper, 77
conjugate gradient method, 27, 48
alternative convergence estimates, 60
classic convergence estimates, 49
inexact, 57
preconditioned, 59
stagnation periods, 49
superlinear convergence, 53
convergence factor, 4, 12
decomposition
block ILU, 211
Cholesky, 6, 29
ILU, 182, 186
LU, 3
QR, 3, 26
SVD, 3, 41
delay of convergence, 34
domain decomposition method, 158
alternating Schwarz, 158
Neumann–Neumann preconditioner, 165, 172
nonoverlapping domain decomposition, 161
Poincare–Steklov operator, 163, 164
Schur complement, 164
ellipticity condition, 36
equation
anisotropic diffusion, 176, 185
convection-diffusion, 176, 190
diffusion, 100
Navier–Stokes, 205
of linear elasticity, 176, 201
of Love, 94
Oseen, 224
Poisson, 103, 124, 158
reaction-diffusion, 175, 179
Stokes, 206
Symm’s, 96
error functional, 13, 20
error vector, 4
estimate
Kaniel–Page, 52
fast Fourier transform, 71
field of values, see numerical range
finite element method, 101
isoP₂ – P₁ element, 208
Crouzeix–Raviart element, 138, 142, 208
inverse inequality, 130
locking phenomena, 202
nonconforming, 138
orthogonality property, 180
Scott–Vogelius element, 202, 204
stiffness matrix, 104, 106, 129
SUPG method, 193
Taylor–Hood element, 208
Wilson element, 139, 142
fixed point problem, 15
formal scalar product, 30
generalized minimal residual method, see minimal residual method
geometric convergence, 4
GMRES, see minimal residual method
Gram–Schmidt procedure, 26
H²-regularity, 128, 135
index of an algorithm, 22
inequality
Friedrichs, 100
Hölder, 19
Harding, 171
Necas, 207
strengthened Cauchy–Schwarz, 148
inner-outer iterations, 220
Jordan normal form, 4, 5
K-condition number, 60, 74
Krylov subspaces, xv, 21, 23
Krylov vectors, xv
LBB condition, 208, 210
leading submatrix, 50
lemma
Lax–Milgram, 101
Reusken’s, 133
logarithmic potentials, 44
loss of orthogonality, 35
matrix
circulant, 69
Fourier image of, 74
cyclic downward shift, 69
diagonal dominant, 20
elliptic, 36
Fourier, 70
Hermitian, 2
Hessenberg, 3
isodiagonal, 67
iteration, 5
Jacobi, 16, 17
M-matrix, 9
normal, 43
positive definite, 6
sparse, 107, 184
stiffness, see finite element method, stiffness matrix
symmetric, 2
Toeplitz, 67
algebra, 68
generating function, 75
generators, 67
multilevel, 69, 92
symbol, 75
method
Anderson, see Anderson mixing
Arnoldi, 50
biconjugate gradient, 30
CG, see conjugate gradient method
consistent, 5
fixed point, 15
Galerkin, 106
Gauss-Seidel, 8, 148
GMRES, see minimal residual method
inexact Krylov subspace, 55
inexact Richardson, 55
inexact Uzawa, 217
Jacobi, 8, 108, 131, 147
Krylov subspace, 21
Lanczos, 50
linear iterative, 5
minimal residual, see minimal residual method
Newton, 17
PCG, see conjugate gradient method, preconditioned
quasi-minimal residual, 32
relaxation, 8
Richardson, 8
SIMPLE, 215, 229
SOR, 9, 134
steepest descent, 14
symmetric Gauss-Seidel, 8, 131
Uzawa, 220
minimal residual method, 22, 25, 36
algebraic approach, 25
asymptotic convergence factor, 40, 44, 48
Chebyshev polynomials, 43
convergence for diagonalizable matrices, 39
convergence for normal matrices, 43
convergence for positive definite matrices, 37
equivalence to Anderson's mixing, 33
for nonlinear problems, 33
generalized, 22, 25
geometrical approach, 25
loss of orthogonality, 35
nongeneralized, 27
numerical range, 42
pseudospectra, 43
quasi-minimization, 32
with restarts, 37
MINRES, see minimal residual method, nongeneralized
multigrid method, 116
additive, 152
approximation property, 123, 124, 129, 188, 219
as preconditioner, 117
coarse grid operator, 110, 111
complexity, 118
coarse grid correction, 109
F-cycle, 117
iteration matrix, 119
modified approximation property, 200
modified smoothing property, 200
preconditioner, 223
norm
A-norm, 7
consistent, 2
Frobenius, 3
operator, 2
spectral, 3
numerical range, 13, 41, 48
optimal algorithm, 22
outer-inner iteration, 62
preconditioner, 4, 58, 62
additive Schwarz, 161, 171
block-diagonal, 220
BPX, 153, 155, 166
by space decomposition, 147
domain decomposition, see domain decomposition method
explicit, 58
hierarchical basis, 156, 169
implicit, 58
left, 58
modified simple circulant, 81
multigrid, see multigrid method
multiplicative Schwarz, 161, 171
optimal circulant, 73
right, 58
simple circulant, 73
two-grid, 114
projected system, 21
pseudospectra, 42
quadratic functionals, 13
quasi-minimal residual method, 32
quasi-minimization, 32
Rayleigh quotient, 13, 19
residual functional, 13, 20, 21
residual vector, 6
resolvent, 39, 41
Ritz value, 51
Ritz vector, 51
Robin constant, 44
robust iterative method, 177
robust smoother, 178
roundoff errors, 34
saddle point problem, 207, 210
Schur complement, 210, 220
preconditioner, 223
Index

singular perturbed problem, 175
singular value decomposition, 3
singular values, 3
smoothing iterations
  block Gauss–Seidel, 187
distributive, 214
  ILU, 182
  of Braess and Sarazin, 215
spectral radius, 2
  spectrum, 2
stencil, 192
subspace equation, 146
superlinear convergence, 53, 78

theorem
  Bendixon’s, 227
  interlacing of eigenvalues, 77
  Toeplitz–Hausdorff, 41
  van der Sluis–van der Vorst, 54
triangulation, 104
  minimal angle condition, 104, 106
upwind differences, 191
weak formulation of a PDE, 104, 128, 139, 180, 207
weak solution of a PDE, 101, 207
Wiener class, 76