Preface

Inverse problems arise in many cutting-edge applications, where desired information can only be recovered from indirect measurements. Such applications make inverse problems a fascinating subject in applied mathematics; corresponding research questions include (i) identifiability of the desired quantities, and (ii) their stable reconstruction from the available data.

The applications are as diverse as the background expertise that is necessary to understand them properly. This is one of the challenges that mathematicians have to face when working in this area. Another challenge comes with the fact that inverse problems are generally ill-posed in a mathematical sense, meaning that quantities of interest do not depend continuously on the data. This requires the application of so-called regularization methods and use of the corresponding beautiful theory which has been developed in the past 50 years.

A number of excellent books on this subject have already appeared (see [6, 22, 25, 28, 29, 31, 39, 42, 44, 57, 59, 60, 85] and the German texts [40, 52, 74]) differing quite a bit in their individual emphasis and/or in their particular viewpoint. The present book deals with linear inverse problems only, but it aims to be different from the aforementioned ones in that comparable weight is given to the general theory of ill-posed problems and to a detailed treatment of a variety of applications. The presentation of the theory therefore focuses only on those results that are most relevant for a thorough understanding of the profound subtleties of ill-posed problems and the various algorithms used in practice. The individual applications, on the other hand, have been carefully selected to demonstrate the implementation of the theory.

This book is intended to be accessible both to math students and to engineers who are faced with solving inverse problems. The necessary prerequisite is undergraduate level mathematics, enriched by some basic knowledge of elementary Hilbert space theory (weak convergence, operators in Hilbert spaces, the uniform boundedness principle, basic properties of compact operators, etc.). In contrast to almost all aforementioned books the development of the theory completely avoids any version of the spectral theorem; the singular value decomposition is used for purposes of illustration only. The corresponding analysis is more elementary, yet the results are sharp and are not restricted to compact operator equations alone, as is the case in many other books.

Admittedly, some of the applications (Sections 6, 11, 14, and 17) require some facts about partial differential equations and Sobolev spaces (namely the basic H^1 theory for the Poisson equation including the appropriate trace spaces), while others (Sections 10 and 16) make use of the Fourier transform. This is a fair price to pay for dealing with interesting and nontrivial applications. However, in an effort to keep the book self-contained, relevant theoretical background is provided in a set of appendices. Readers who experience a need to refresh their knowledge of functional analysis are referred to the commendable textbooks [14, 16, 26, 27, 75].

Parts of the material in this book have been tested in class, both for lecture courses and for German "Seminars," which are somewhat similar to reading seminars at American universities. I would like to take this opportunity to thank the students in a seminar this winter term who "volunteered" to present preliminary versions of some of the chapters of this book—and stumbled across a few persistent mistakes and typos on their way. Still I fear that further mistakes may have survived and remain to be found; these will be documented on the web page `www.siam.org/books/ot153`.

Let me finish with some further acknowledgments. Above all, I am indebted to Charles W. Groetsch, who was one of my math heroes during my early career, and who became a coauthor and friend over the years. Chuck read the whole manuscript, provided a lot of encouragement, and tried his best to polish my admittedly rusty English. I am also grateful for input and comments from my student Benjamin Müller and colleagues Olaf Dössel, Frank Fiedler, Roland Griesmaier, Bastian Harrach, Jim Nagy, Erhard Scholz, Laura Schreiber, and John Sylvester. Further thanks go to Fabrice Delbary, my postdoc at the time of writing this book, for creating Figure 5.1. Finally, it is my pleasure to acknowledge the help of Fabio Frommer, a very gifted student, who has just finished his bachelor's studies and who also read very carefully a close-to-final draft of the text from beginning to end. Fabio's questions and comments made me rewrite quite a few paragraphs to clarify and improve the exposition; I have appreciated our discussions very much.

Mainz, February 2017 *Martin Hanke*