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Exploring ODEs. By Lloyd N. Trefethen, Ásgeir Birkisson, and Tobin A. Driscoll. SIAM, Philadelphia, 2018. $64.00. viii+335 pp., hardcover. ISBN 978-1-611975-15-4.

Exploring ODEs is the latest excellent book by Nick Trefethen and his rotating cast of coauthors. It has the least advanced topic yet and thus the largest potential audience. As to who that audience is, the authors suggest that their book could be used as a secondary text in an introductory ODE course, or as independent reading for those with some exposure to ODEs. But the authors might be selling themselves short. I suspect that their book, if suitably supplemented, would work very well as the main text for a first course in ODEs, and I plan to use it this way myself.

The most important difference between *Exploring ODEs* and some popular alternatives is its central use of numerics. Most material is structured around concrete examples and their numerical solutions. Readers are expected to reproduce and experiment with these solutions. This is made possible by Chebfun, a MATLAB package developed by Trefethen's group that can be used to solve initial and boundary value problems with impressively concise and intuitive syntax. The essential code producing each example is provided and explained, with full code available online, and it is easy to learn the basics of Chebfun by following along. The underlying numerical methods, which are based on Chebyshev interpolation, are mostly not explained and would be above the level of the typical reader. This is all right since Chebfun comes as close as I have seen to a black box for solving ODEs numerically. This helps justify the reliance on MATLAB as opposed to an open source alternative. And, unlike MATLAB, the digital version of *Exploring ODEs* is free.

The use of Chebfun has appealing consequences. Numerical examples motivate the analytical material, and it seems realistic that some students will explore these examples further, given the ease of doing so. Nonlinear ODEs appear early and often. Some of these introduce subsequent analysis, but others are studied only numerically; this book does not overemphasize analytically tractable cases.

The book is concise, considering that figures and code take up a sizable fraction of space. It covers the core topics of a first ODE course, including first- and second-order equations, first-order systems, boundary value problems, eigenproblems, linear systems, and phase portraits. Some of this familiar material contains refreshingly unfamiliar examples involving nonlinearity, time dependence, or nonsmooth functions. Later chapters offer brief exposure to some more advanced topics: nonlinear boundary value problems, bifurcations, parameter continuation, boundary layers, random forcing, chaos, complex ODEs, and dynamical PDEs in one spatial dimension. Treatments of several topics stood out as original and better than usual: nonlinear boundary value problems, random forcing, Picard iteration, and linearization. Topics that are omitted include infinite series, integral transforms, and any explicit mention of conserved quantities or Hamiltonian structure.

Each chapter is a short linear narrative with no subsections. This style works well for the book's inquiry-driven approach but makes it hard to read less than a whole chapter at once. Subsections are not missed in most chapters, which focus on one or two ideas and end with useful summaries of key points. Some narratives are harder to follow than others, as with the introduction of linear eigenproblems in Chapter 6 or complex ODEs in Chapter 21.

Every chapter closes with a scientific application. These strike an excellent balance between science and math and do not feel superfluous. The scientific content is just enough to explain the equations, and the mathematical content is not redundant with the main text. In the chapter on bifurcations, for instance, pitchfork and Hopf bifurcations appear only in the main text and in the application, respectively. I particularly enjoyed the applications involving tides, orbital mission design, band gaps, and the KdV equation.

Analytical methods are well explained in most cases, but a few topics may be treated too briefly. One such topic is the connection between linear stability and matrix eigenvalues. Another is the integration of first-order separable equations, which is presented as symbol manipulation of differentials with no mention of the chain rule. An addition I would have found valuable is an explanation of the basic idea behind numerically integrating ODEs, even just Euler's method for initial value problems. This would give students something to picture despite not understanding Chebfun's more sophisticated workings. Some mathematics is referenced that may be unknown to students, such as the equivalence of norms on $\mathbb{R}^n$, or the regarding of functions as elements in a vector space, and a few terms are used without being explained (advection-diffusion, flow field, Jordan block, dissipative). Fortunately most of these gaps would not be hard to fill in during lectures.

The exercises that close each chapter are interesting and original, and they touch on important ideas beyond the scope of the text. Many are multipart questions with an exploratory flavor. Most chapters have enough exercises, although a few have only three or four. There are no exercises for rote practice of analytical methods like separation of variables or integrating factors. A few such exercises would be useful but can be obtained from numerous sources, unlike the questions the authors have crafted.

In short, *Exploring ODEs* is certainly good for independent study, and I expect it will be very good for an introductory ODE course as well. The Chebfun software is a testament to the maturity of methods for computing particular solutions, and its potential for numerically driven pedagogy is not limited to the scope of *Exploring ODEs*. One can imagine similar use of Chebfun in courses on dynamical systems or PDEs. Perhaps books on these topics will follow.

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