Preface

Probably, the most extended (pseudo)definition of the set of functions known as "special functions" refers to those mathematical functions which are widely used in scientific and technical applications, and of which many useful properties are known. These functions are typically used in two related contexts:

1. as a way of obtaining simple closed formulas and other analytical properties of solutions of problems from pure and applied mathematics, statistics, physics, and engineering;

2. as a way of understanding the nature of the solutions of these problems, and for obtaining numerical results from the representations of the functions.

Our book is intended to provide assistance when a researcher or a student needs to get the numbers from analytical formulas containing special functions. This book should be useful for those who need to compute a function by their own means, or for those who want to know more about the numerical methods behind the available algorithms. Our main purpose is to provide a guide of available methods for computations and when to use them. Also, because of the large variety of numerical methods that are available for computing special functions, we expect that a broader "numerical audience" will be interested in many of the topics discussed (particularly in the first part of the book). Several levels of reading are possible in this book and most of the chapters start with basic principles. Examples are given to illustrate the use of the methods, pseudoalgorithms are given to describe technical details, and published algorithms for computing a selection of functions are described as practical illustrations for the basic methods of this book.

The presentation of the topics is organized in four parts: Basic Methods, Further Tools and Methods, Related Topics and Examples, and Software. The first part (Basic Methods) describes a set of methods which, in our experience, are the most popular and important ones for computing special functions. This includes convergent and divergent series, Chebyshev expansions, linear recurrence relations, and quadrature methods. These basic chapters are mostly self-contained and start from first principles. We expect that many of the contents are appropriate for advanced numerical analysis courses (parts of the chapters are in fact based on classroom notes); however, because the main focus is on special functions, detailed examples of application are also provided.

The second part of the book (Further Tools and Methods) contains a set of important methods for computing special functions which, however, are probably not so well known as the basic methods (at least for readers who are not very familiar with special functions).

Certainly, this does not mean that these tools are less effective than the selected basic methods; for example, the performance of uniform asymptotic expansions is quite impressive in many instances. The chapters in this second part are: Continued Fractions, Computation of the Zeros of Special Functions, Uniform Asymptotic Expansions, and Other Methods (Padé approximations, sequence transformations, best rational approximations, Taylor's method for ordinary differential equations, and further quadrature methods including the Clenshaw–Curtis and Filon methods).

The third part (Related Topics and Examples) describes some methods that are specific to certain functions. A first chapter is devoted to the (asymptotic) numerical inversion of a class of distribution functions with details for gamma and beta distributions (a topic which researchers in statistics, probability, and econometrics may find useful). A second chapter (Further Examples) describes varied topics such as the Euler summation formula (and applications), the computation of symmetric elliptic integrals (Carlson's method), and the numerical inversion of Laplace transforms.

We thank NIST for the permission to quote part of a section in the DLMF project (from the chapter "Numerical Methods") on solving ordinary differential equations by using Taylor series (our §9.5), and Frank Olver for his assistance in writing this part. We thank the SIAM editorial staff, in particular Louis Primus, for their patience and splendid cooperation.

Finally, the fourth part illustrates the use of the methods by providing descriptions of specific algorithms for computing selected functions: Airy functions, Legendre functions, and parabolic cylinder functions, among others. The corresponding Fortran 90 routines can be downloaded from

http://functions.unican.es.

The web page will hold successive actualizations and extensions of the available software.

We would like to thank Dr. Van Snyder for his extensive and useful comments, and Dr. Ernst Joachim Weniger for providing us with notes, and further useful information, on Padé approximations and sequence transformations. Finally, we thank the Spanish Ministry of Education and Science for financial support (projects MTM2004-01367, MTM2006-09050).