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we read "if we assume that $u(x, y)$ and $v(x, y)$ are twice differentiable..." and later "Now, one can show that the solutions of the Cauchy–Riemann equations indeed must be twice differentiable...." No sign of the duck? Next, on page 1114, "one may ask if $f'(z)$ itself has a derivative in $\Omega, \ldots$. The plain answer is YES, which we prove below." A few lines later: "To answer the question posed, it is sufficient to notice that if $u(x, y)$ and $v(x, y)$ satisfy the Cauchy–Riemann equations, then so do all derivatives...." But the *existence* of these derivatives is still not settled. The next hope comes from Cauchy's integral formula ("We prove that if $f(z)$ is analytic..."). However, in the given proof appears a sentence "and K a bound for $|f^{(3)}(z)|$, which proves the desired result" (p. 1126). But no hypothesis on the existence of a third derivative is formulated in Theorem 82.6.

Chapters 83 and 84 compare nicely *Fourier series* with *Fourier transforms*. We read in the beginning that Fourier series arose from the solution of the heat equation and "has influenced the development of mathematical analysis profoundly.... However, as any highly specialized tool or organism, these techniques have not been able to adapt to the needs of a changing world with computational methods for nonlinear differential equations taking over as work-horse in applications." So, Fourier series are here introduced without the beautiful motivation from the heat equation or the wave equation by the idea of separating variables. Later, in section 83.14, the heat equation is nevertheless mentioned, but the solution is just written down ("We observe that ... satisfies...") and not derived. The Fourier coefficients are derived from the orthogonality by integrating an infinite series term-by-term, without justification. Many examples then illuminate the theory. A clever student may perhaps discover the fact that in all these examples the Fourier coefficients $c_m(f)$ tend to zero with one power of m^{-1} higher than stated on page 1154. The *inversion formula* (i.e., the convergence of the Fourier series to f) is stated and proved only for periodic differentiable functions "with piecewise Lipschitz continuous derivative." Without justification, it is then mentioned that the assumption on f could be relaxed to "piecewise differentiable with piecewise Lipschitz continuous derivative." It is not mentioned that even Lipschitz continuity of f is sufficient (more precisely, bounded variation) and that this has been known since 1829.

Summary. The authors present a rich amount of material, much of which belongs to the standard mathematics education, in an at times unorthodox style. The book suffers from a discrepancy between the high ambitions set out in the preface and the actual realization of the program, which lacks care and attention in many places.

Perhaps we may summarize our overall impression of the book by the sentence: the "body" is strong, but the "soul" is weak.

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A Multigrid Tutorial. Second Edition. *By Williams L. Briggs, Van Emden Henson, and Steve F. McCormick*. SIAM, Philadelphia, 2000. $43.00. xii+193 pp., softcover. ISBN 0-89871-462-1.

This is the ultimate textbook for providing a basic grounding in the subject. Students will find it very readable with just the right amount of material necessary to reflect a growing field. Completion of this book and its exercises will serve as a launching platform for more in-depth treatments of the subject found in the references.

Chapters 1–5 are essentially the same as in the first edition [1], with the following notable improvements:

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- Key concepts are nicely boxed in the exposition.
- More exercises are included for each chapter. All exercises are carefully designed to ensure the student understands and can carry out his/her own experiments to validate the concepts presented.
- Exercises are organized by concept and a related keyword to the concept appears in a title to each exercise.

I have used Chapters 1–5 to comprise half of a quarter course on iterative methods at the graduate level and as a self-paced reading course for graduate students prior to tackling more advanced treatments. In both situations, students like the book and continue to keep it on their shelves.

Chapters 6–10 have been added since the first edition to reflect the growing field of multilevel methods. The same exceptional care has been taken with this more advanced material to make it accessible to students. A nice feature is that Chapters 6–10 can be studied in any order once students have mastered Chapters 1–5. I have found that these chapters provide excellent coverage of important topics for numerical analysis seminars that can be given by either graduate students or faculty.

Chapter 6 explains the full approximation scheme (FAS) for nonlinear problems. Since the material in Chapters 1–5 deals entirely with linear problems, this addition is very important. The exposition follows naturally from that used in the first five chapters.

Chapter 7 deals with selected applications that every graduate student studying the field should encounter. Topics include the treatment of Neumann boundary conditions, anisotropic problems, semicoarsening, line relaxation, full coarsening, variable mesh problems, and variable coefficient problems.

Chapter 8 is an introduction to the algebraic multigrid method. The treatment is a beginning one that focuses mainly on symmetric positive definite M-matrices, but the book references more advanced literature.

Chapter 9 walks the reader through the tedious details of the fast adaptive composite grid (FAC) method with very simple one- and two-dimensional examples.

Chapter 10 develops a multigrid framework appropriate for self-adjoint problems that have been discretized by finite elements. A very clear development derives the multigrid interpolation and restriction operators and the coarse grid problem matrix directly from the point of view of the minimization of a functional. The first seven chapters relied on finite difference discretizations, so the inclusion of Chapter 10 gives the student a starting point to understanding the vast literature described from this significantly different point of view.

The set of topics that advanced students would not find in this book include, but are not limited to, the following:

- A detailed treatment of nonsymmetric problems, including choices for restriction and prolongation operators.
- Local mode analysis that leads to an amplification matrix rather than the amplification factor of the elementary examples.
- Issues that arise in algebraic multigrid for problems that do not have symmetric positive definite M-matrices.

The recent book by Shapira [2] gives a matrix-based treatment of these topics for the more advanced reader.

REFERENCES

[1] W. Briggs, *A Multigrid Tutorial*, SIAM, Philadelphia, 1987.
[2] Y. Shapira, *Matrix-Based Multigrid: Theory and Applications*, Kluwer Academic, London, 2003.

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Higher-Order Finite Element Methods. *By Pavel Solin, Karel Segeth, and Ivo Dolezel.* Chapman & Hall/CRC, Boca Raton, FL, 2003. $89.95. xxii+382 pp., hardcover. ISBN 1-58488-438-X.

hp-finite elements refer to an optimized finite element method where the mesh size h and the polynomial approximation degree p are selected to minimize the discretization error. Since optimal *hp*-finite element methods yield exponential convergence rates, they