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The SIAM 100-Digit Challenge: A Study in High-Accuracy Numerical Computing. *By Folkmar Bornemann, Dirk Laurie, Stan Wagon, and Jörg Waldvogel.* SIAM, Philadelphia, 2004. $57.00. xii+306 pp., hardcover. ISBN 0-89871-561-X.

The SIAM 100-*Digit Challenge* is far too good to be confined to numerical analysts and those interested in high-precision computation. It is a tour de force of numerical problem solving that puts to work a diverse mix of analytic and computational techniques and makes excellent use of MATLAB (often with the INTLAB toolbox), *Mathematica*, and Pari/GP.

The story begins with a challenge set by Nick Trefethen in the January/February 2002 issue of *SIAM News*. He posed 10 problems, each easy to state and having a real number as answer, and asked for up to 10 correct digits for each problem. He offered a small monetary award for the best solution received by May 20, 2002. The challenge attracted entries from 94 teams in 25 countries, and 20 of the teams scored maximum points. The book begins with an interview with Trefethen about the challenge and comments from some of the contestants. I found this preliminary chapter unputdownable. However, interesting as it is, for me the story of the challenge is secondary. The significance of the book is that each chapter gives a beautiful exposition of the best ways of solving one of the problems, combining insights from the four authors—who are from four different First-Prize-winning teams—with those from other entrants. Typically, several quite different techniques are described, and the pros and cons of each explained. While each chapter is motivated by a specific problem, the techniques of solution are often generic and widely applicable. So rather than being just a catalogue of best solutions, this book is a guide and reference to numerical and analytical methods that could be of use to many readers of *SIAM Review*.

What is remarkable about the book, but not apparent from the title, is that a panoply of techniques from across scientific computing is employed. These include preconditioned iterative methods, convergence acceleration, Monte Carlo simulation, quadrature (for oscillatory integrals, and in the complex plane), Fourier analysis, special functions, symbolic computation, interval analysis, chaos and shadowing, and global optimization.

Each of the ten main chapters has been written by one of the four authors. Two highlights for me are the introduction to interval analysis in the chapter by Wagon and the survey of extrapolation methods in the appendix by Laurie. Wagon's chapter, "Think Globally, Act Locally," shows how interval analysis can be used to solve a global minimization problem in two variables; in doing so it provides the best summary of the benefits of interval analysis that I have seen. Laurie's 33-page appendix likewise is the best practical survey of extrapolation methods that I am aware of.

As discussed in the preliminary chapter, the fact that Trefethen's challenge asked for correct digits but not a proof of correctness generated some discussion in letters to *SIAM News*. The issue of correctness is fully addressed in this book, because as well as showing how to compute the digits, the authors are careful to validate their results. One approach to validation is to compute the answer by two different methods and see how many digits agree, but the authors endeavor to be more rigorous and employ suitable error bounds or interval arithmetic computations.

Although Trefethen asked for only 10 digits of the answer to each problem, the authors consider how to compute many more digits. They assess the scalability of the different methods of solution and, in an appendix, explain how they solved all but one of the problems to 10,000 digits, in computations varying by problem from a fraction of a second to a month.

An excellent website (www.siam.org/books/100digitchallenge) contains a significant amount of code and additional material relating to the book, as well as downloadable PDF versions of the foreword, preface, references, and index. At the time of writing you can get to the website simply by searching for `100 digit` in Google and clicking on the first search result returned.

This book reminds me of Acton's classic *Numerical Methods That Work* [1]. While very different and more advanced than Ac-

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ton, *100-Digit Challenge* has the same mix of contagious enthusiasm, pragmatic advice, and ingenious tricks that you mark in the margin because you feel convinced that one day you will be able to put them to good use.

The authors—who did not know each other prior to writing the book—are to be congratulated on a consistently high standard of writing and presentation that reads smoothly from one chapter to the next. Excellent reference is made to the literature, including to some items not so widely known.

For those inspired to try their hand at this kind of problem solving, an appendix provides 22 further problems in the same vein as the original 10.

In his foreword, David Bailey writes, "Savor every bite of this 10-course feast." I certainly did, and if I were to choose one math book to take to a desert island this one would be a contender—especially if I had a solar-powered laptop on which to try out the codes and ideas!

REFERENCE

[1] F. S. Acton, *Numerical Methods That Work*, Harper and Row, New York, 1970. Reprinted by the Mathematical Association of America, Washington, D.C., with new preface and additional problems, 1990.

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Functional Analysis. *By Peter D. Lax.* Wiley-Interscience, New York, 2002. $99.95. xx+580 pp., hardcover. ISBN 0-471-55604-1.

Functional analysis is one of the miracles of modern mathematics. The idea of looking at a *family* of functions, rather than single functions one at a time, is a tool of immense power and versatility. For instance, showing that norm convergence for Fourier series fails in the L^1 topology is child's play with the use of a little functional analysis. All one need show is that the Hilbert transform is unbounded on L^1, and almost any example you can think of will do the job. As a second illustration, showing that the Fourier transform acting on L^1 is not surjective onto the collection of continuous functions that vanish at infinity is a cute exercise with functional analysis. Proving the result from first principles is actually quite arduous.

Today, functional analysis is a basic tool in the theory of differential equations, in harmonic analysis, in scattering theory, in mathematical physics, and in many other parts of the mathematical sciences. Most good graduate programs require all students to learn some functional analysis. Of course the analysis students must learn it rather well; but all students should at least have a passing acquaintance.

Despite its fundamental importance, the learning of functional analysis has been rather frustrating. Peter Lax points out in his preface that, when he was a student, only Banach's book (the first book in the subject) was available. Since that time quite a number of books have appeared. Notable among these (at least from my own point of view) are Yosida's book [5], Dunford and Schwartz [2], Reed and Simon [3], and Rudin [4]. Of course, the three-volume, 2700+-page Dunford and Schwartz was a great event in modern mathematics, but it is hardly a text. Yosida's book is a modern classic, much more compact than Dunford and Schwartz and containing all the key ideas. But both these books, and in fact many of the modern functional analysis books, share one liability: it is difficult to find any result when you need it. I am no expert in functional analysis. When I need a general form of the Hahn–Banach theorem or a succinct statement of the spectral theorem, I want to just open a book and find what I am looking for. Rarely does this happen. Until now, my favorite functional analysis book has been Rudin's: he states the theorems as I would, in language that I can understand and use, and his index leads me directly to the information that I desire. With Lax's book I now have a new favorite. Lax's book has all the substance of the best functional analysis books written to date. But it also has the accessibility that makes it a useful tool, and (unlike almost any other functional analysis book around) it has a wealth of applications and examples.

Many times I have, at the request of some publisher, reviewed a book manuscript and