Contents

Preface vii

1 Introduction 1

2 Linear Elliptic Partial Differential Equations 7
  2.1 Function spaces ................................................. 7
  2.2 Setting of the problem: Classical and weak formulation .......... 9
  2.3 Norms equivalent to \( \| \cdot \|_{H^1} \): Friedrichs and Poincaré inequalities . . . 15
  2.4 On \( (u, q) \) formulation and other possible generalizations . . . . 19

3 Elements of Functional Analysis 23
  3.1 Riesz representation theorem .................................. 23
  3.2 Functional equations and bilinear forms .......................... 25
  3.3 Lax-Milgram lemma ............................................. 26

4 Riesz Map and Operator Preconditioning 29

5 Conjugate Gradient Method in Hilbert Spaces 35
  5.1 Algorithmic construction of CG ................................. 36
  5.2 Analytic moment matching model reduction description of CG . . . . 42

6 Finite-Dimensional Hilbert Spaces and the Matrix Formulation of the Conjugate Gradient Method 47

7 Comments on the Galerkin Discretization 53

8 Preconditioning of the Algebraic System as Transformation of the Discretization Basis 55

9 Fundamental Theorem on Discretization 63

10 Local and Global Information in Discretization and in Computation 69

11 Limits of the Condition Number-Based Descriptions 73

12 Inexact Computations, A Posteriori Error Analysis, and Stopping Criteria 77

13 Summary and Outlook 81
<table>
<thead>
<tr>
<th>Contents</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bibliography</td>
<td>85</td>
</tr>
<tr>
<td>Index</td>
<td>101</td>
</tr>
</tbody>
</table>