An Electric Proof

Problem 02-002, *by* CECIL C. ROUSSEAU (University of Memphis).

In [4] David Singmaster proposed and Basil Rennie partially solved the following problem: find the driving-point resistance between two vertices a distance i apart in the n-dimensional cube Q_n if every edge of the cube has unit resistance. The n-dimensional cube is the graph in which the vertices are the binary sequences of length n, and two vertices are adjacent whenever they differ in precisely one coordinate. Compute two separate ways the driving-point resistance between two vertices a distance $n+1$ apart in Q_{n+1} and thereby give an "electric proof" of the well-known combinatorial identity

$$\sum_{k=0}^{n} \binom{n}{k}^{-1} = (n+1) \sum_{k=0}^{n} \frac{1}{2^k(n+1-k)}.$$

Status. The proposer has a solution. Other solutions are welcome, as are additional examples of mathematical results obtained from electrical circuit models.

Note. This is but one of several cases in which a mathematical result of interest arises from the analysis of an electrical network. In a justly famous problem from the print version of *Problems and Solutions* [3], Minkowski's inequality is obtained by considering a simple series-parallel network. Other examples, involving topics such as tiling and random walks, can be found in [1, chapters 2 and 9] and [2].

REFERENCES

[1] B. BOLLOBÁS, *Modern Graph Theory*, Springer-Verlag, New York, 1998.

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[3] A. LEHMAN, *Problem* 60-5, *A resistor network inequality*, solution by F. Reza, SIAM Rev., 4 (1962), pp. 151–155.

[4] D. SINGMASTER, *Problem* 79-16*, *Resistances in an n-dimensional cube*, partial solution by B. Rennie, SIAM Rev., 22 (1980), pp. 504–508.