Problem 03-003, by Jonathan Borwein (Simon Fraser University, Burnaby, BC, Canada).

Show that the complex extension of the arithmetic-geometric mean inequality

$$\left|\frac{a + b}{2}\right| \geq \sqrt{|ab|}$$

holds for complex $a, b$ “outside” a cardioid. What can be found by way of complex extension of $(a_1 + a_2 + \cdots + a_n) / n \geq (a_1a_2\cdots a_n)^{1/n}$, which holds for positive $a_1, a_2, \ldots, a_n$?

Status. This problem appears in Mathematics by Experiment (A. K. Peters Ltd., 2003), which is coauthored by the proposer. It is easily solved, but the issue of reasonable generalizations remains entirely open.