

## A Golden Example

*Problem 06-003*, by JONATHAN BORWEIN<sup>1</sup> (Dalhousie University, Halifax, NS, Canada) and MARC CHAMBERLAND<sup>2</sup> (Grinnell College, Grinnell, IA).

Evaluate the sum

$$(1) \quad \Phi := \sum_{k=0}^{\infty} \left\{ \frac{G^2}{(5k+1)^2} - \frac{G}{(5k+2)^2} - \frac{G^2}{(5k+3)^2} + \frac{G^5}{(5k+4)^2} + \frac{2G^5}{(5k+5)^2} \right\} g^{5k},$$

where  $g$  is the *golden ratio*  $g := (\sqrt{5} - 1)/2$  and  $G := (\sqrt{5} + 1)/2 = g^{-1}$ . The correct result was discovered empirically by Benoit Cloitre<sup>3</sup> using integer relation methods [1]. In the irrational base  $g$ , the constant  $\Phi$  has digits independently computable since (1) gives an identity of BBP type (see [1, Chapter 4]).

*Status.* The authors have a solution. Additional solutions are welcome.

## REFERENCE

- [1] J. M. BORWEIN AND D. BAILEY, *Mathematics by Experiment*, A K Peters, Ltd., Natick, MA, 2004.

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<sup>1</sup>Email: jborwein@cs.dal.ca. This author's research was supported by NSERC and by the Canada Research Chair Programme.

<sup>2</sup>Email: chamber1@math.grinnell.edu.

<sup>3</sup>Email: abcloitre@wanadoo.fr.