

A Convexity Question in Matrix Analysis

Problem 09-002, by DAVID L. RUSSELL¹ (Virginia Tech, Blacksburg, VA).

Let \mathbf{B} be an $n \times m$ real matrix of maximal rank $m \leq n$. Let $\mathbf{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n)$. Prove that the scalar-valued function

$$\mathcal{C}(\mathbf{\Lambda}) = \text{Tr}((\mathbf{B}^* \mathbf{\Lambda} \mathbf{B})^{-2})$$

is convex for $\mathbf{\Lambda} = (\lambda_1, \lambda_2, \dots, \lambda_n) \in (\mathbb{R}^+)^n$, the open positive orthant in \mathbb{R}^n .

Status. A solution is known. Other solutions are welcome.

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