## Abstract

Let $HD_d(p,q)$ denote the minimal size of a transversal that can always be guaranteed for a family of compact convex sets in $\mathbb{R}^d$ which satisfy the $(p,q)$-property ($p \geq q \geq d+1$). In a celebrated proof of the Hadwiger-Debrunner conjecture, Alon and Kleitman proved that $HD_d(p,q)$ exists for all $p \geq q \geq d+1$. Specifically, they prove that $HD_d(p,d+1)$ is $\tilde{O}(p^{d^2+d})$. This paper has two parts. In the first part we present several improved bounds on $HD_d(p,q)$. In particular, we obtain the first near tight estimate of $HD_d(p,q)$ for an extended range of values of $(p,q)$ since the 1957 Hadwiger-Debrunner theorem. In the second part we prove a $(p,2)$-theorem for families in $\mathbb{R}^2$ with union complexity below a specific quadratic bound. Based on this, we introduce a polynomial time constant factor approximation algorithm for MAX-CLIQUE of intersection graphs of convex sets satisfying this property. It is not likely that our constant factor approximation can be improved to a PTAS as MAX-CLIQUE for intersection graphs of fat ellipses is known to be APX-HARD and fat ellipses have sub-quadratic union complexity.