Abstract

We introduce a novel framework of Prophet Inequalities for combinatorial valuation functions. For a (non-monotone) submodular objective function over an arbitrary matroid feasibility constraint, we give an $O(1)$-competitive algorithm. For a monotone subadditive objective function over an arbitrary downward-closed feasibility constraint, we give an $O(\log n \log^2 r)$-competitive algorithm (where r is the cardinality of the largest feasible subset). Inspired by the proof of our subadditive prophet inequality, we also obtain an $O(\log n \cdot \log^2 r)$-competitive algorithm for the Secretary Problem with a monotone subadditive objective function subject to an arbitrary downward-closed feasibility constraint. Even for the special case of a cardinality feasibility constraint, our algorithm circumvents an $\Omega(\sqrt{n})$ lower bound by Bateni, Hajiaghayi, and Zadimoghaddam in a restricted query model. En route to our submodular prophet inequality, we prove a technical result of independent interest: we show a variant of the Correlation Gap Lemma for non-monotone submodular functions.