Abstract

It was recently found that there are very close connections between the existence of *additive spanners* (subgraphs where all distances are preserved up to an additive stretch), *distance preservers* (subgraphs in which demand pairs have their distance preserved exactly), and *pairwise spanners* (subgraphs in which demand pairs have their distance preserved up to a multiplicative or additive stretch) [Abboud-Bodwin SODA '16, Bodwin-Williams SODA '16]. We study these problems from an optimization point of view, where rather than studying the existence of extremal instances we are given an instance and are asked to find the sparsest possible spanner/preserver. We give an $O(n^{3/5+\varepsilon})$-approximation for distance preservers and pairwise spanners (for arbitrary constant $\varepsilon > 0$). This is the first nontrivial upper bound for either problem, both of which are known to be as hard to approximate as Label Cover. We also prove Label Cover hardness for approximating additive spanners, even for the cases of additive 1 stretch (where one might expect a polylogarithmic approximation, since the related multiplicative 2-spanner problem admits an $O(\log n)$-approximation) and additive polylogarithmic stretch (where the related multiplicative spanner problem has an $O(1)$-approximation). Interestingly, the techniques we use in our approximation algorithm extend beyond distance-based problem to pure connectivity network design problems. In particular, our techniques allow us to give an $O(n^{3/5+\varepsilon})$-approximation for the Directed Steiner Forest problem (for arbitrary constant $\epsilon > 0$) when all edges have uniform costs, improving the previous best $O(n^{2/3+\varepsilon})$-approximation due to Berman et al. [ICALP '11] (which holds for general edge costs).