Abstract

Initially developed for the min-knapsack problem, the knapsack cover inequalities are used in the current best relaxations for numerous combinatorial optimization problems of covering type. In spite of their widespread use, these inequalities yield linear programming (LP) relaxations of exponential size, over which it is not known how to optimize exactly in polynomial time. In this paper we address this issue and obtain LP relaxations of quasi-polynomial size that are at least as strong as that given by the knapsack cover inequalities. For the min-knapsack cover problem, our main result can be stated formally as follows: for any $\varepsilon > 0$, there is a $(1/\varepsilon)^{O(1)} n^{O(\log n)}$-size LP relaxation with an integrality gap of at most $2 + \varepsilon$, where n is the number of items. Prior to this work, there was no known relaxation of subexponential size with a constant upper bound on the integrality gap. Our construction is inspired by a connection between extended formulations and monotone circuit complexity via Karchmer-Wigderson games. In particular, our LP is based on $O(\log^2 n)$-depth monotone circuits with fan-in 2 for evaluating weighted threshold functions with n inputs, as constructed by Beimel and Weinreb. We believe that a further understanding of this connection may lead to more positive results complementing the numerous lower bounds recently proved for extended formulations.