Abstract

Given n non-vertical pairwise disjoint triangles in 3-space, their vertical depth (above/below) relation may contain cycles. We show that, for any $\varepsilon > 0$, the triangles can be cut into $O(n^{3/2+\varepsilon})$ pieces, where each piece is a connected semi-algebraic set whose description complexity depends only on the choice of ε, such that the depth relation among these pieces is now a proper partial order. This bound is nearly tight in the worst case. We are not aware of any previous study of this problem with a subquadratic bound on the number of pieces. This work extends the recent study by two of the authors on eliminating depth cycles among lines in 3-space. Our approach is again algebraic, and makes use of a recent variant of the polynomial partitioning technique, due to Guth, which leads to a recursive procedure for cutting the triangles. In contrast to the case of lines, our analysis here is considerably more involved, due to the two-dimensional nature of the objects being cut, so additional tools, from topology and algebra, need to be brought to bear. Our result essentially settles a 35-year-old open problem in computational geometry, motivated by hidden-surface removal in computer graphics.