Abstract

We introduce and study the Doubly Balanced Connected graph Partitioning (DBCP) problem: Let $G = (V, E)$ be a connected graph with a weight (supply/demand) function $p : V \to \{-1, +1\}$ satisfying $p(V) = \sum_{j \in V} p(j) = 0$. The objective is to partition G into (V_1, V_2) such that $G[V_1]$ and $G[V_2]$ are connected, $|p(V_1)|, |p(V_2)| \leq c_p$, and $\max\{\frac{|V_1|}{|V_2|}, \frac{|V_2|}{|V_1|}\} \leq c_s$, for some constants c_p and c_s. When G is 2-connected, we show that a solution with $c_p = 1$ and $c_s = 3$ always exists and can be found in polynomial time. Moreover, when G is 3-connected, we show that there is always a 'perfect' solution (a partition with $p(V_1) = p(V_2) = 0$ and $|V_1| = |V_2|$, if $|V| \equiv 0 (\text{mod } 4)$), and it can be found in polynomial time. Our techniques can be extended, with similar results, to the case in which the weights are arbitrary (not necessarily ± 1), and to the case that $p(V) \neq 0$ and the excess supply/demand should be split evenly. They also apply to the problem of partitioning a graph with two types of nodes into two large connected subgraphs that preserve approximately the proportion of the two types.