

**IP0****Hot Topic Session: The Existence of Designs**

We prove the existence conjecture for combinatorial designs, answering a question of Steiner from 1853. More generally, we show that the natural divisibility conditions are sufficient for clique decompositions of simplicial complexes that satisfy a certain pseudorandomness condition.

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**IP1****The Graph Regularity Method**

Szemerdi's regularity lemma is one of the most powerful tools in graph theory, with many applications in combinatorics, number theory, discrete geometry, and theoretical computer science. Roughly speaking, it says that every large graph can be partitioned into a small number of parts such that the bipartite subgraph between almost all pairs of parts is random-like. Several variants of the regularity lemma have since been established with many further applications. I will discuss recent progress in understanding the quantitative aspects of these lemmas and their applications.

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**IP2****Rota's Conjecture**

In 1970, Gian-Carlo Rota posed a conjecture giving a succinct combinatorial characterization of the linear dependencies among a finite set of vectors in a vector space over any given finite field. I will discuss the conjecture as well as joint work with Bert Gerards and Geoff Whittle that led to a solution.

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**IP3****Covering Systems of Congruences**

A distinct covering system of congruences is a collection  $a_i \pmod{m_i}$ ,  $1 < m_1 < m_2 < \dots < m_k$ , whose union is the integers. Covering systems were introduced by Paul Erdos, who used the system

$$\mathbb{Z} = (0 \pmod{2}) \cup (0 \pmod{3}) \cup (1 \pmod{4}) \cup (3 \pmod{8}) \cup (7 \pmod{12}) \cup (23 \pmod{24})$$

to produce an odd arithmetic progression none of whose members is of the form a prime plus a power of two. Two well-known questions of Erdos concern covering systems. The minimum modulus problem asks if there exist distinct covering systems for which  $m_1$  is arbitrarily large. The odd modulus problem asks for a distinct covering system with all moduli odd. I will describe aspects of my negative answer to the minimum modulus problem and ongoing joint work with Pace Nielsen toward the odd modulus problem. The arguments involve techniques from graph theory and PDE.

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**IP4****Submodular Functions and Their Applications**

Submodular functions, a discrete analogue of convex functions, have played a fundamental role in combinatorial optimization since the 1970s. In the last decade, there has been renewed interest in submodular functions due to their role in algorithmic game theory, as well as numerous applications in machine learning. These developments have led to new questions as well as new algorithmic techniques. I will discuss the concept of submodularity, its unifying role in combinatorial optimization, and a few illustrative applications. I will describe some recent algorithmic advances, in particular the concept of multilinear relaxation, and the role of symmetry in proving hardness results for submodular optimization. I will conclude by discussing possible extensions of the notion of submodularity, and some future challenges.

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**IP5****Approximation Algorithms Via Matrix Covers**

I will describe a general technique for obtaining approximate solutions of hard quadratic optimization problems using economical covers of high dimensional sets by small cubes. The analysis of the method leads to intriguing algorithmic, combinatorial, geometric, extremal and probabilistic questions. Based on joint papers with Troy Lee, Adi Shraibman and Santosh Vempala.

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**IP6****Interlacing Families and Kadison–Singer**

This talk will focus on the resolution of two conjectures that have been shown to be equivalent to open problems in a number of fields, most famously a problem due to Kadison and Singer concerning the foundations of mathematical physics. This will include introducing a new technique for establishing the existence of certain combinatorial objects that we call the "method of interlacing polynomials." The technique seems to be interesting in its own right (it was also the main tool for resolving the existence of Ramanujan graphs of arbitrary degree). This represents joint work with Don Spielman of Yale University and Nikhil Srivastava of Microsoft Research, India.

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**IP7****Hamilton Decompositions of Graphs and Digraphs**

In this talk I will survey recent results on Hamilton decompositions, i.e. decompositions of the edge-set of a graph or digraph into edge-disjoint Hamilton cycles. One example is a conjecture of Kelly from 1968, which states that every

regular tournament has a Hamilton decomposition. We prove this conjecture for large tournaments. In fact, we prove a far more general result, which has further applications. Another example is the Hamilton decomposition conjecture, which was posed by Nash-Williams in 1970. It states that every regular graph on  $n$  vertices with minimum degree at least  $n/2$  has a Hamilton decomposition (joint work with Bela Csaba, Allan Lo, Deryk Osthus and Andrew Treglown).

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## IP8

### The Complexity of Graph and Hypergraph Expansion Problems

The expansion of a graph is a fundamental parameter with many connections in mathematics and applications in computer science. Two basic variants of expansion are edge-expansion (minimum ratio of the number of edges leaving a subset of vertices to the total number of edge incident to the subset) and vertex expansion (minimum ratio of number of vertices adjacent to a subset to the size of the subset). Both are NP-hard to compute exactly and the current best polynomial-time algorithms provide either an  $O(\sqrt{\log n})OPT$  solution these problems, or, for edge-expansion, an  $O(\sqrt{OPT})$  solution. The latter is via Cheeger's inequality relating edge expansion to the second smallest eigenvalue of the graph Laplacian, and provides an efficient method to check if a graph is an edge expander (i.e., has edge expansion is at least some constant). In this talk, we survey recent developments on the following lines: 1. What is the analog of expansion for partitioning a graph into multiple parts (more than 2), and how does Cheeger's inequality generalize? 2. Is there a Cheeger-type inequality for vertex expansion? How to efficiently verify whether a graph is a vertex expander? 3. Do there exist algorithms with better performance? 4. How do these parameters, inequalities and algorithms extend to hypergraphs?

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## SP1

### 2014 Dnes Knig Prize Lecture-The number of $K_{s,t}$ -free graphs

The problem of enumerating  $H$ -free graphs with a given number of vertices has been studied for almost forty years. As a result, a great deal is known about this problem. When  $H$  is not bipartite, we have not only good estimates of the number of  $H$ -free graphs, but also fairly precise structural characterisation of a typical such graph. On the contrary, not much is known in case when  $H$  is bipartite. A few years ago, we proved asymptotically optimal upper bounds on the number of graphs not containing a fixed complete bipartite graph  $K_{s,t}$ . This was the first infinite family of bipartite graphs for which such a bound was established. Since then, the method developed in our proof has found several interesting applications to a large class of related problems.

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## CP1

### Percolation in the Weighted Random Connection Model

We propose the weighted random connection model, with the goal to predict and better understand the spread of infectious diseases. Each individual of a population is characterized by two parameters: its position and risk behavior, where the probability of transmissions among individuals increases in the individual risk factors, and decays in their Euclidean distance. Our main result is on the almost sure existence of an infinite component in the weighed random connection model, which corresponds to the outbreak of an infectious disease. We use results on the random connection model and site percolation in  $\mathbf{Z}^2$ .

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## CP1

### Degeneracy in R-Mat Generated Graphs

The degeneracy of a graph  $G$  is the minimum possible value of  $d$  such that every subgraph of  $G$  has a vertex with degree at most  $d$ . It is known that the degeneracy of an Erdős-Rényi random graph is bounded with high probability. We examine degeneracy in the popular recursive matrix (R-MAT) random graph model, of which Erdős-Rényi is a special case, and determine whether its asymptotic behavior parallels that of Erdős-Rényi graphs.

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## CP1

### Identifying Codes and Searching with Balls in Graphs

We address the following search theory problem: what is the minimum number of queries that is needed to determine an unknown vertex in a graph  $G$  if queries are balls with respect to the graph distance and the answer to a query is YES if the unknown vertex lies in the ball and NO otherwise. We study adaptive or non-adaptive versions of this problem for the following classes of graphs: hypercubes, the Erdős-Rényi random graph model and graphs of bounded maximum degree.

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## CP1

### Hyperbolicity of Random Intersection Graphs

Graphs are a popular way to model a wide range of applications, including rapidly-growing data drawn from social and information sciences. There is a large body of empirical evidence suggesting that complex networks have "tree-like" properties at intermediate to large size-scales (e.g., hierarchical structures in biology, hyperbolic routing in the Internet, and core-periphery behavior in social networks). We look the hyperbolicity of a promising model for real world networks: the random intersection graph.

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## CP1

### Anti-Ramsey Number of Matchings in Hypergraphs

A  $k$ -*matching* in a hypergraph is a set of  $k$  edges such that no two edges intersect. The *anti-Ramsey number* of a  $k$ -matching in a complete  $r$ -uniform hypergraph  $\mathcal{H}$  on  $n$  vertices, denoted by  $\text{ar}(n, r, k)$ , is the smallest integer  $c$  such that in any coloring of the edges of  $\mathcal{H}$  with exactly  $c$  colors, there is a  $k$ -matching whose edges have distinct colors. The *Turán number*, denoted by  $\text{ex}(n, r, k)$ , is the maximum number of edges in an  $r$ -uniform hypergraph on  $n$  vertices with no  $k$ -matching. For  $k \geq 3$ , we conjecture

that if  $n > rk$ , then  $\text{ar}(n, r, k) = \text{ex}(n, r, k-1) + 2$ . We prove this conjecture for  $k = 2, k = 3$ , and sufficiently large  $n$ , as well as provide upper and lower bounds.

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## CP1

### De-Preferential Attachment Random Graphs

We consider a random graph sequence where a new vertex prefers attachment to a vertex with low degree. We call such a sequence a "de-preferential attachment random graph". We consider two models, "inverse de-preferential" and "linear de-preferential". If each new vertex comes with exactly one half-edge, the degree of a fixed vertex is  $\Theta(\sqrt{\log n})$  for the inverse de-preferential case and  $\Theta(\log n)$  for the linear case. We show that for the inverse de-preferential model the tail of the limiting degree distribution is faster than exponential while that for the linear de-preferential model is exactly the Geometric ( $\frac{1}{2}$ ) distribution. If each new vertex comes with  $m > 1$  half-edges, similar asymptotic results hold for fixed vertex degree in both models.

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## CP2

### A Predictive Model for Gis

Any map in an Euclidean plane may be represented by one graph, as follows. A nation is represented by a node. If two nations are adjacent; i.e., if they share a common border (other than a single point), then they are joined by an edge. We assume that nations are contiguous; i.e., that a nation may not be split up into smaller unconnected areas. If we want to predict the value for a given statistic that varies cyclically, and depends upon geographical location for a given sample, we have to determine first its index of incidence, and its index of prevalence. By performing elementary algebraic calculations, based on our graph, we will arrive at an estimate of the next indexes of incidence and prevalence, respectively. However, this calculation will not be the same for every situation; and will vary according to the sample size, and the corresponding indexes of incidence and prevalence.

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## CP2

### Statistical and Hierarchical Graph Analysis for Cy-

**ber Security**

Motivated by the need for more resilient networks, we develop methods to understand cyber data through graph analysis. We model network traffic as a graph connecting IP addresses when there is communication between them. Using entropy based measures on vertices we identify when there is a significant change in behavior. Additionally, we study these graphs hierarchically to quickly obtain estimates for measurable quantities on graphs such as the shortest path length between two IPs.

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**CP2****The Balance Optimization Subset Selection (boss) Model for Causal Inference**

Matching is used to estimate treatment effects in observational studies. One mechanism for overcoming its restrictiveness is to relax the exact matching requirement to one of balance on the covariate distributions for the treatment and control groups. The Balance Optimization Subset Selection (BOSS) model is introduced to identify a control group featuring optimal covariate balance. This presentation discusses the relationship between the matching and BOSS model, providing a comprehensive picture of their relationship.

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**CP2****Qubit Unextendible Product Bases**

Qubit unextendible product bases are a useful tool in the theory of quantum entanglement. We show that the existence of qubit unextendible product bases can be rephrased as a question of whether or not there exists an edge coloring of the complete graph with certain properties. We use this characterization to find the minimum size of qubit unextendible product bases and to find all of them on four or fewer qubits.

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**CP2****Online Streaming of Intersection Graphs**

We will present an online-streaming algorithm for intersection graph processing. The graphs are generated as a stream from VLSI layout data comprising of 10e12 congruent shapes. The intersection graph of such a large number congruent shapes has many interesting theoretical properties which can be efficiently exploited for producing efficient algorithms and techniques. Real world problems motivated from layout analysis and verifications will be presented.

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**CP2****Accumulation Behavior and Product Forecasting**

Patterns are observed from buyers buying and accumulating products. These patterns can be used in Cross Selling and Inventory Management analysis. We introduce a Markov Chain variant to model the accumulation behavior of a population and provide a forecasted confidence set of items to be accumulated in a future period which may differ in length from the derived period. In non-sequential accumulation it will be established that the eigenvalues of this model are always real.

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**CP2****Various Notions and Generalizations of Algebraically Coded Security System with Applications to Security and Privacy**

In this paper, we present new results on algebraically coded security system with their applications to security and privacy of data. Moreover, we give various notions and generalizations of algebraic coding. Lastly, we provide practical examples with applications to data security.

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**CP3****Aliens Vs Zombies: An Introduction To Bipartite Dot Product Graphs**

A *bipartite dot product representation* of a bipartite graph  $G$  with  $V(G) = X \cup Y$  is a function  $f : V(G) \rightarrow \mathbb{R}^k$  such that for vertices  $x \in X$  and  $y \in Y$ ,  $xy \in E$  if and only if  $f(x)^T f(y) \geq 1$ . We illustrate how these representations may be used to illuminate structure in social and biological networks, especially where there are specified sets within which relationships are ignored. Aliens and Zombies, for example. We will also characterize the (bipartite) graphs with a representation  $f : V(G) \rightarrow \mathbb{R}^1$ ; i.e., the graphs with *bipartite dot-product-dimension* 1, and present results for bipartite graphs with bipartite dot-product-dimension

$k > 1$ .

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### CP3

#### The Abstract Algebra of Big Data and Associative Arrays

Spreadsheets, databases, hash tables and dictionaries; these are the fundamental building blocks of big data storage, retrieval and processing. The MIT Dynamic Distributed Dimensional Data Model (D4M) is an interface to databases and spreadsheets that allow developers to write analytics in the language of linear algebra, significantly reducing the time and effort required. The core of D4M are associative arrays that allow big data to be represented as large sparse matrices. This sparse matrix D4M schema has been widely adopted in a number of domains. We explore the mathematics behind the D4M system by formulating an axiomatic definition for associative arrays in terms of semi-modules over semi-rings, then exploring the algebraic properties of the latter. We are primarily concerned with the linear systems theory and spectral theory of a three particular families of semi-rings. We present a structure theorem for the solutions of linear systems of an important family of semi-rings.

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### CP3

#### The Global Invariant of Graphic Hyperplane Arrangements

The fundamental group of the complement of a hyperplane arrangement in a complex vector space is an interesting and complicated invariant. The third rank of successive quotients in the lower central series of the fundamental group was called the global invariant of the arrangement by Falk. Falk gave a general formula to compute the global invariant, and asked for a combinatorial interpretation of the global invariant. Schenck and Suciu proved that the global invariant of a graphic arrangement is double of the number of cliques with three or four vertices in the graph with which the arrangement associated. This solved Falk's problem in the case of graphic arrangements, but their argument used fairly heavy homological algebra. In this paper, we give a direct and simple proof for this combinatorial formula.

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### CP3

#### Maps on Surfaces and Matroids

Maps on surfaces are graphs embedded onto closed orientable surfaces such that the complement of the graph is a disjoint union of topological disks. To such graphs a bond matroid can be associated and also a slightly more general structure: a symmetric or Lagrangian matroid. I will discuss some properties of Lagrangian matroids (such as duality and representability) and if time permits, introduce a more general structure: the Coxeter matroid.

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### CP3

#### On the Type(s) of Minimum Size Subspace Partitions

Let  $V = V(kt + r, q)$  be a vector space of dimension  $kt + r$  over the finite field with  $q$  elements. Let  $\sigma_q(kt + r, t)$  denote the minimum size of a subspace partition  $\mathcal{P}$  of  $V$  in which  $t$  is the largest dimension of a subspace. We denote by  $n_{d_i}$  the number of subspaces of dimension  $d_i$  that occur in  $\mathcal{P}$  and we say  $[d_1^{n_{d_1}}, \dots, d_m^{n_{d_m}}]$  is the *type* of  $\mathcal{P}$ . We show that a partition of minimum size has a unique partition type if  $t + r$  is an even integer and we give partial results for the more intricate case when  $t + r$  is an odd integer.

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### CP3

#### Necessarily Flat Polytopes

Call a  $d$ -dimensional polytope  $P$  *necessarily flat* if every embedding of its  $(d - 1)$ -skeleton is affinely  $d$ -dimensional. For example, a connected sum of polytopes cannot be necessarily flat, whereas every cone over a polytope is necessarily flat. Such polytopes are instrumental in Richter-Gebert's proof of universality for 4-dimensional convex polytopes. We demonstrate strong evidence of a conjecture concerning necessarily flat polytopes in the case  $d = 3$ .

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### CP3

#### On the Number of Bases in a Matroid

Given a triple of integers  $(n, r, b)$ ,  $0 < r \leq n$  and  $1 \leq b \leq \binom{n}{r}$ , Welsh asked if there exists an  $(n, r, b)$ -matroid, a matroid with  $n$  elements and rank  $r$  that has exactly  $b$  bases. Even stronger, we may ask whether there exists an  $r \times n$  matrix of rank  $r$  with exactly  $b$  invertible  $r \times r$  submatrices. Mayhew and Royle suggested that an  $(n, r, b)$ -matroid always exists except when  $(n, r, b) = (6, 3, 11)$ . In this talk, we shall verify their conjecture for a large family of triples.

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**CP4****On the Matrix Sequence  $\{\Gamma(A^m)\}_{m=1}^\infty$  for a Boolean Matrix  $A$  Whose Digraph Is Linearly Connected**

In this talk, we study the convergence of the matrix sequence  $\{\Gamma(A^m)\}_{m=1}^\infty$  for a matrix  $A \in \mathcal{B}_n$  the digraph of which is linearly connected. We completely characterize  $A$  for which  $\{\Gamma(A^m)\}_{m=1}^\infty$  converges and find its limit. We go further to characterize  $A$  for which the limit of  $\{\Gamma(A^m)\}_{m=1}^\infty$  is a  $J$  block diagonal matrix. All of these results are derived by studying the  $m$ -step competition graph of the digraph of  $A$ .

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**CP4****Forcing Sets in Self-Folding Origami**

We describe some of the combinatorial geometry problems based on microscale self-folding polymer gels, especially the concept of a forcing set: a subset of the creases whose mountain-valley parity force the mountain-valley parity of all the other creases. We describe an algorithm for finding minimal forcing sets for 1D origami. For 2D origami, we present results for finding forcing sets in single-vertex crease patterns and in the classic Miura-ori map fold.

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**CP4****First Eigenvectors of Nonsingular Unicyclic 3-Colored Digraphs**

In this talk we consider the class of connected 3-colored digraphs containing exactly one nonsingular cycle. Our main objective is to discuss the smallest Laplacian eigenvalue and the corresponding eigenvectors of such graphs. We show that the smallest Laplacian eigenvalue of such a graph can be realized as the algebraic connectivity, the second smallest Laplacian eigenvalue of a suitable undirected graph. The eigenvector corresponding to the smallest Laplacian eigenvalue of a nonsingular unicyclic 3-colored digraph is called the first eigenvector. We discuss the monotonicity property on the real and imaginary parts of the first eigenvectors of such graphs, which is analogous to Fiedler's monotonicity Theorem. The sign structure of the real and imaginary parts of the first eigenvectors is also described.

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**CP4****Strong Double Graphs: Spectral Properties, Energy and Laplacian Energy**

A graph  $G$  is a finite, undirected, simple graph with  $n$  vertices and  $m$  edges having vertex set  $V(G) = \{v_1, v_2, \dots, v_n\}$ . The adjacency matrix  $A = (a_{ij})$  of  $G$  is a  $(0, 1)$ -square matrix of order  $n$  whose  $(i, j)$ -entry is equal to one if  $v_i$  is adjacent to  $v_j$  and equal to zero, otherwise. The spectrum of the adjacency matrix is called the  $A$ -spectrum of  $G$ . If  $\lambda_1, \lambda_2, \dots, \lambda_n$  is the adjacency spectrum of  $G$ , the energy of  $G$  is defined as  $E(G) = \sum_{i=1}^n |\lambda_i|$ . Let  $D(G) = \text{diag}(d_1, d_2, \dots, d_n)$  be the diagonal matrix associated to  $G$ , where  $d_i$  is the degree of vertex  $v_i$ . The matrices  $L(G) = D(G) - A(G)$  and  $Q(G) = D(G) + A(G)$  are respectively called Laplacian and signless Laplacian matrices and their spectras are respectively called Laplacian spectrum ( $L$ -spectrum) and signless Laplacian spectrum ( $Q$ -spectrum) of  $G$ . If  $0 = \mu_n \leq \mu_{n-1} \leq \dots \leq \mu_1$  and  $0 \leq \mu_n^+ \leq \mu_{n-1}^+ \leq \dots \leq \mu_1^+$  are respectively the  $L$ -spectrum and  $Q$ -spectrum of  $G$ , the Laplacian energy of a graph  $G$  is defined as  $LE(G) = \sum_{i=1}^n |\mu_i - \frac{2m}{n}|$ . For a graph

$G$  with vertex set  $V(G) = \{v_1, v_2, \dots, v_n\}$ , the strong double graph  $SD(G)$  is a graph obtained by taking two copies of  $G$  and joining each vertex  $v_i$  in one copy with the closed neighbourhood  $N[v_i] = N(v_i) \cup \{v_i\}$  of corresponding vertex in another copy. We study some basic properties like the bipartiteness, Hamiltonianity, strong regularity, chromatic number and spectra of the graph  $SD(G)$  together with its energy and Laplacian energy. As an application, we obtain some new families of equienergetic and Laplacian equienergetic graphs, an infinite family of graphs  $G$  for which  $LE(G) < E(G)$ . We derive a formula for the number of spanning trees of  $SD(G)$  in terms of the number of spanning trees of  $G$ .

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**CP4****G-Ham Sandwich Theorems: Harmonic Analysis and Measure Partitions**

The Ham Sandwich Theorem – any  $n$  finite measures on  $\mathbb{R}^n$  can be bisected by single hyperplane – is the most classical result of equipartition theory, a topic central to geometric combinatorics. We provide group theoretic generalizations of this result, showing how finite measures can be “ $G$ -balanced” by unitary representations of a compact Lie group  $G$ . For abelian groups, one has an equivalent interpretation in terms of vanishing Fourier transforms, from which equipartitions and other measure partition theorems for convex fundamental domains follow.

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**CP4****A Combinatorial Characterization of Binary Positional Number Systems**

Although the representation of the real numbers in terms

of a base and a set of digits has a long history, new questions arise even in the binary case - digits 0 and 1. A binary positional number system (binary radix system) with base equal to the golden ratio  $(1 + \sqrt{5})/2$  is fairly well known. The main result of this paper is a combinatorial characterization of all binary radix systems for the real numbers.

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## CP5

### Cycle Extendability of Chordal Graphs and Tournaments

$S$  is a subset of  $\{1, 2, \dots, k\}$ . A cycle  $C$  in a graph or digraph is  $S$ -*extendable* if there is a cycle  $C^*$  which contains the vertices of  $C$  and is of length  $i$  greater than  $C$  for some  $i \in S$ . A graph or digraph is  $S$ -*cycle-extendable* if every cycle is  $S$ -extendable. We prove that Hamiltonian chordal graphs (graphs with a Hamiltonian cycle and no induced cycle of length greater than three) and strong tournaments (orientations of complete graphs in which there is a path from  $x$  to  $y$  and from  $y$  to  $x$  for every pair of distinct vertices  $x$  and  $y$ ) are  $\{1, 2\}$ -cycle-extendable. Also, in the case of tournaments, we characterize those regular and nearly regular tournaments which are not  $\{1\}$ -extendable. We present and suggest directions for future research on  $S$ -cycle-extendability.

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## CP5

### Loose Cycles in 3-Uniform Hypergraphs

Let  $H$  be a 3-uniform hypergraph. In this talk, we will discuss sufficient conditions for minimum co-degree of  $H$  which guarantee the existence of a spanning sub-hypergraph of  $H$  consisting of independent copies of a loose cycle. In particular, using the stability approach, we give a tight condition for the existence a Hamilton loose cycle.

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## CP5

### On 2-Factors with a Bounded Number of Odd Cycles

In this talk we consider conditions that guarantee that a 2-factor in a graph has a bounded number of odd cycles. In particular, we consider various conditions on claw-free graphs, extending a result from Ryjáček, Saito and Schelp. We also consider conditions that ensure the existence of a pair of disjoint 1-factors in a claw-free graph, as the union of such a pair is a 2-factor with no odd cycles.

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## CP5

### Hamiltonian Cycles in $\{1, 4\}$ -Leaper Graphs

A knight can travel to every square of a standard  $8 \times 8$  chessboard exactly once and return to the starting square, a tour which corresponds to a Hamiltonian cycle in a graph with 64 vertices and with edges defined by the knight's legal moves. Many variations of this well-known Knight's Tour problem have been studied, including boards of various dimensions, boards on tori and other surfaces, and variations on the traditional knight's move. In this talk, we present preliminary results about which  $n \times m$  rectangular boards admit a knight's tour with the variation that the knight moves 1 square either vertically or horizontally and 4 squares in the perpendicular direction, that is, a  $\{1, 4\}$ -leap.

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## CP5

### Near Perfect Matchings in $k$ -uniform Hypergraphs

Let  $H$  be a  $k$ -uniform hypergraph on  $n$  vertices where  $n$  is a sufficiently large integer not divisible by  $k$ . We prove that if the minimum  $(k - 1)$ -degree of  $H$  is at least  $\lfloor n/k \rfloor$ , then  $H$  contains a matching with  $\lfloor n/k \rfloor$  edges. This confirms a conjecture of Rödl, Ruciński and Szemerédi, who proved that the minimum  $(k - 1)$ -degree  $n/k + O(\log n)$  suffices. More generally, we show that  $H$  contains a matching of size  $d$  if its minimum codegree  $d$  is less than  $n/k$ , which is also best possible. This extends a result of Rödl, Ruciński and Szemerédi.

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## CP5

### Nonempty Intersection of Longest Paths in Series-Parallel Graphs

In 1966 Gallai asked whether all longest paths in a connected graph have nonempty intersection. Even though this is not true in general, the answer to Gallai's question is positive for several well-known classes of graphs, e.g. outerplanar graphs, split graphs, and 2-trees. We present a proof that all series-parallel graphs (i.e. connected  $K_4$ -minor free graphs) have a vertex that is common to all of its longest paths.

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## CP5

### Cycle Decompositions with a Certain Degree of Symmetry

A *cycle decomposition* of a graph  $\Gamma$  is a set  $\mathcal{C}$  of cycles whose edges partition the edge-set of  $\Gamma$ ; partitioning  $\mathcal{C}$  into factors of  $\Gamma$  results in a 2-*factorization*. Cycle decompositions having a certain *automorphism group* with (possibly) a prescribed action on the vertex-set or on the edge-set have attracted much attention. In this talk I will select some of the most recent results on this topic.

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## CP6

### New Characterizations of Proper Interval Bigraphs

In this paper, we define astral triple of edges and characterize proper interval bigraphs via absence of astral triple of edges. We also characterize proper interval bigraphs in terms of dominating pair of vertices as defined by Corneil et al. Sen and Sanyal characterized biadjacency matrix of a proper interval bigraph in terms of monotone consecutive arrangement(MCA). We have shown an interrelation between MCA and circularly compatible 1's of binary matrix as defined by Tucker.

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## CP6

### Analytical Properties of Horizontal Visibility Graphs Generated by the Feigenbaum Universal Route to Chaos in Discrete Models

This paper establishes some analytical properties of Horizontal Visibility Graphs along the lines of Feigenbaum period doubling route from regular system to chaos, time series analysis, Lyapunov exponents and fractal dimensions in discrete models like Rickers Population model :  $f(x) = xe^{r(1-x/k)}$ , where  $r$  is the control parameter and  $k$  is the carrying capacity. This also leads us to build bridges between time series analysis, nonlinear dynamics and graph theory in chaotic discrete systems.

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## CP6

### Excluded Vertex-minors for Graphs of Linear

## Rank-width at Most $k$

Linear rank-width is a graph width parameter, which is a variation of rank-width by restricting its tree to a caterpillar. It is known for each  $k$ , there is a finite obstruction set  $\mathcal{O}_k$  of graphs such that a graph  $G$  has linear rank-width at most  $k$  if and only if no vertex-minor of  $G$  is isomorphic to a graph in  $\mathcal{O}_k$ . However, no attempts have been made to bound the number of  $\mathcal{O}_k$  for  $k \geq 2$ . We show that for each  $k$ , there are at least  $2^{\Omega(3^k)}$  pairwise locally non-equivalent graphs in  $\mathcal{O}_k$ . To prove this theorem, we generalize a theorem of Bouchet, which states that if two trees are locally equivalent, then they are isomorphic, into block graphs without simplicial vertices.

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## CP6

### $N$ -Flips in 4-Connected Eulerian Triangulations on the Sphere

It is known that any two Eulerian triangulations on the sphere with the same tripartition can be transformed into each other by  $N$ -flips. In this paper, we shall prove that any two 4-connected Eulerian triangulations on the sphere with the same tripartition can be transformed into each other by  $N$ -flips, preserving the 4-connectivity, if they are not isomorphic to the exception.

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## CP6

### Characterizing Graph Classes Using Twin Vertices in Regular Induced Subgraphs

A graph is chordal and only if every two vertices are twins in every connected regular induced subgraph (deleting ‘connected’ characterizes the split graphs). Conjecture: A graph is weakly chordal if and only if twin vertices exist in every nontrivial regular induced subgraph (equivalently with ‘connected’ inserted). I describe the new class of graphs in which every vertex has a twin in every nontrivial regular induced subgraph (equivalently with ‘connected’ inserted).

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## CP6

### Weak Universality for 2-Dimensional Parallel Drawings

Let  $G$  be a  $d$ -regular graph equipped with a  $d$ -edge-coloring. A *parallel drawing* of  $G$  is a 2-dimensional drawing of  $G$  such that edges sharing a common color are mutually parallel (disregarding crossings). It is an open problem

to find a graph-theoretic characterization of graphs which have such a drawing in which the edges are represented by distinct lines. The purpose here is to demonstrate that in the case when  $d = 4$ , there is an ‘algebraic’ obstruction for the classification of such graphs.

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## CP6

### Almost Isometric Drawings of Graphs

Every connected undirected (not necessarily finite) graph  $G$  induces a metric space  $(V, d)$  on its vertex set  $V$ . For an injective mapping  $f$  from  $V$  into the Euclidean plane let the distortion of  $f$  be the smallest number  $q$  such that  $\frac{1}{q} \leq \frac{d(x,y)}{d_2(f(x),f(y))} \leq q$  for all vertices  $x \neq y$  of  $G$ , where  $d_2$  denotes the Euclidean distance. In the talk we study upper bounds on  $q$  in terms of the structure of  $G$ .

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## CP7

### Colorings with Fractional Defect

Consider a coloring of a graph such that each vertex is assigned a fraction of each color, with the total amount of colors at each vertex summing to 1. We define the fractional defect of a vertex  $v$  to be the sum of the overlaps with each neighbor of  $v$ , and the fractional defect of the graph to be the maximum of the defects over all vertices. Note that this coincides with the usual definition of defect if every vertex is monochromatic. We provide results on the minimum fractional defect of 2-colorings of some graphs.

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## CP7

### Coloring Planar Digraphs

A conjecture of Neumann-Lara (1985) states that the vertex set of any planar oriented graph  $G$  can be 2-colored such that each color class induces an acyclic subdigraph. We show that the conjecture holds when one further assumes that  $G$  is of digirth five, and discuss some related results. This is joint work with Bojan Mohar.

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## CP7

### Chromatic-Choosability of the Power of Graphs

The  $k$ th power  $G^k$  of a graph  $G$  is the graph defined on  $V(G)$  such that two vertices  $u$  and  $v$  are adjacent in  $G^k$  if the distance between  $u$  and  $v$  in  $G$  is at most  $k$ . A graph  $H$  is called *chromatic-choosable* if  $\chi_l(H) = \chi(H)$ . A natural question raised by Xuding Zhu is whether there exists a constant integer  $k$  such that  $G^k$  is chromatic-choosable for every graph  $G$ . Motivated by the List Total Coloring Conjecture, Kostochka and Woodall (2001) asked whether  $G^2$  is chromatic-choosable for every graph  $G$ . In 2013, Kim and Park answered the Kostochka and Woodall’s question in the negative by finding a graph  $G$  such that  $G^2$  is a complete multipartite graph. In this paper, we answer Zhu’s question by showing that for every integer  $k \geq 2$ , there exists a graph  $G$  such that  $G^k$  is not chromatic-choosable. Moreover, for any fixed  $k$  we show that the value  $\chi_l(G^k) - \chi(G^k)$  can be arbitrarily large.

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## CP7

### Equitable Coloring of Graphs with Intermediate Maximum Degree

A graph  $G$  is equitably  $k$ -colorable if the vertex set of  $G$  can be properly colored with  $k$  colors such that the sizes of any two color classes differ by at most one. We show that a connected  $G$  is equitably  $\Delta(G)$ -colorable when  $(|G| + 1)/3 \leq \Delta(G) < |G|/2$ , where  $|G|$  and  $\Delta(G)$  denote the order and the maximum degree of  $G$ , respectively.

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## CP7

### A Distance-Two Coloring with Application to Wireless Sensor and Actor Networks

Navarra et al. in 2010 proposed a new virtual infrastructure for the coarse-grained localization of wireless sensor and actor networks, and proposed distance-two coloring for the adjacency graph of their virtual infrastructure, named  $G_\ell$ . Navarra et al. proposed optimal coloring for  $G_4, G_5$ ,

and  $G_{3,2^x}$ ,  $x \geq 0$ . Our main contribution of this work is to propose optimal colorings for all the remaining  $G_\ell$ 's.

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### CP7

#### Steinberg's Conjecture, the Bordeaux Coloring Conjecture and Near-Coloring

An important result in the theory of graph coloring is Grotzsch's theorem, which states that every triangle-free planar graph is 3-colorable. A famous related question is due to Steinberg and states that any planar graph without 4- or 5-cycles is 3-colorable. In this talk, we will discuss some of the recent progress made towards proving Steinberg's conjecture and discuss joint work with Ken-ichi Kawarabayashi that planar graphs with no 5-cycles, 6-cycles or intersecting triangles are 3-colorable. In addition, we discuss recently completed senior thesis work based on near-coloring with Kyle Yang.

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### CP8

#### A Result on Clique Convergence for Join of Graphs

Let  $G$  be a  $(p, q)$ -graph and  $\mathcal{K}_G$  be the set of all maximal cliques of  $G$ , then the  $K$ -graph (clique graph) operator of  $G$  denoted by  $K(G)$  is the graph with vertex set  $\mathcal{K}_G$  and two elements  $Q_i, Q_j \in \mathcal{K}_G$  form an edge if and only if  $Q_i \cap Q_j \neq \emptyset$ . We define iterated clique graphs by  $K^0(G) = G$ , and  $K^n(G) = K(K^{n-1}(G))$  for  $n > 0$ . We study the dynamical behavior of some simple graphs  $G = G_1 + G_2$  under the iterated application of the clique graph operator  $K$ . We determine the number of cliques in  $K(G)$  when  $G = G_1 + G_2$ . Also, we prove a necessary and sufficient condition for clique graph  $K(G)$  when  $G = G_1 + G_2$  is complete, i.e., let  $G = G_1 + G_2$  such that  $K(G)$  is a complete graph if and only if either  $K(G_1)$  or  $K(G_2)$  is a complete graph. Further we gave the characterization for convergence of the join of graphs using clique graph operator.

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### CP8

#### On the Centralizing Number of Graphs

Hedetniemi observed that every graph  $G$  is the center of some graph. More formally, for every graph  $G$  there is a graph  $H$  such that  $C(H) \cong G$ , where  $C(H)$  is the subgraph of  $H$  induced by the vertices in the center of  $H$ . Buckley, Miller and Slater introduced the *centralizing number*  $A(G)$  of a graph  $G$ , which is the minimum number of vertices that must be added to  $G$  to form a supergraph  $H$  where  $C(H) \cong G$ . Hedetniemi's original construction showed that  $A(G) \leq 4$  for any graph  $G$ , and it is straightforward to see  $A(G) \neq 1$ . We investigate the distribution of graphs with respect to  $A(G)$  as the number  $n$  of vertices increases, showing that the number of graphs with fixed centralizing number is growing with  $n$ . Our results derive from a closer examination of the centralizing number, including the effect on this parameter from cloning and twinning vertices.

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### CP8

#### Deciding the Bell Number for Hereditary Graph Classes

The *speed* of a hereditary graph class is the number of (labelled) graphs on  $\{1, 2, \dots, n\}$  in the class, as a function of  $n$ . It is known that not every function can be obtained as the speed of some such class; e.g., if the speed grows faster than any polynomial, then it is at least exponential. Another such jump was identified by Balogh–Bollobás–Weinreich (2005): if the speed is at least  $n^{(1-o(1))n}$ , then it is bounded from below by the  $n$ th Bell number (the number of partitions of an  $n$ -element set). We study the computational problem to decide for a given hereditary class whether its speed is below or above\* the Bell number. We provide an algorithm that solves this problem for classes described by a finite list of forbidden induced subgraphs. It is based on a characterisation of minimal classes with speed above\* the Bell number. \* By "above" I mean "greater than or equal to".

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### CP8

#### Modified Regular Line Digraphs for Optimal Connectivity and Small Diameters

Regular digraphs of optimal connectivity and  $O(\log n)$  diameter are designed from a seed by recursively generating modified line graphs based on  $n$  and  $d$ . To our knowledge, this result is the best known in literature for this class of

graphs. Shortest paths are obtained in  $O(\log n)$  steps with and without the presence of faulty nodes. Applications are to fault tolerant networks, low diameter degradation, rerouting for transient faults, and modifications for permanent faults.

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## CP8

### Some Problems on Paths and Cycles

In 1976, Thomassen conjectured that every longest cycle in a 3-connected graph has a chord. In 1979, Kotzig conjectured that there exists no graph in which each pair of vertices is connected by a unique path of length  $k$  ( $k \geq 3$ ). In this talk, some results on these problems and related conjectures are summarized.

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## CP8

### Signed Edge Domination Numbers of Complete Tripartite Graphs

The closed neighborhood  $N_G[e]$  of an edge  $e$  in a graph  $G$  is the set consisting of  $e$  and of all edges having an end-vertex in common with  $e$ . Let  $f$  be a function on  $E(G)$ , the edge set of  $G$ , into the set  $\{-1, 1\}$ . If  $\sum_{x \in N[e]} f(x) \geq 1$  for each edge  $e \in E(G)$ , then  $f$  is called a signed edge domination function (SEDF) of  $G$ . The signed edge domination number  $\gamma'_{sk}(G)$  of  $G$  is defined as  $\gamma'_s(G) = \min\{\sum_{e \in E(G)} f(e) \mid f \text{ is an SEDF of } G\}$ . In this presentation, we find the signed edge domination number for the complete tripartite graph  $K_{m,n,p}$ , where  $m \leq n \leq p$  and  $p \leq m+n$ .

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## CP9

### Scheduling Problem with Uncertain Processing Times

Scheduling problem with uncertain processing times is addressed with the objective of minimizing mean completion time. Some polynomial time heuristics, utilizing the lower and upper bounds of processing times, are proposed. The proposed and existing heuristics are compared by extensive computational experiments. The computational results indicate that the proposed heuristics perform significantly better than the existing heuristics in terms of both error and computation time.

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## CP9

### Finding the Minimum Value of a Vector Function Using Lipschitz Condition

Exclusion algorithms have been used recently to find all solutions of a system of nonlinear equations or to find the global minimum of a function over a compact domain. These algorithms are based on a minimization condition that can be applied to each cell in the domain. In this paper, we consider Lipschitz functions of order  $\alpha$  and give a new minimization condition for the exclusion algorithm. Furthermore, convergence and complexity results are presented for such algorithm.

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## CP9

### On the Stable Matchings That Can Be Reached When the Agents Go Marching in One by One

The *Random Order Mechanism* (ROM) is a sequential version of Gale and Shapley's deferred-acceptance (DA) algorithm where agents are arriving one at a time, and each newly arrived agent has an opportunity to propose. Surprisingly, it is possible for ROM to output a stable matching that is *different* from the man-optimal and woman-optimal stable matchings. In this talk, we consider the following question: Given a stable matching  $\mu$  of instance  $I$ , can ROM output  $\mu$ ?

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## CP9

### Characterization of Convex Sets Which Are Semidefinite Representable at Infinity

We characterize a convex set which is semidefinite representable (SDR) at  $\infty$ . We prove: a closed convex set is SDR iff it is SDR at each point of  $K \cup \{\infty\}$ . We define new class of sets which are SDR away from a point. We prove: let  $\phi \in K$ .  $K$  is SDR iff  $K$  and  $K^0$  both are SDR away from  $\phi$ .

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## CP9

### Discrete Tropical Optimization Problems

Multidimensional optimization problems are presented, which are formulated and solved in the framework of tropical (idempotent) mathematics. The problems consist of minimizing nonlinear objective functions defined on finite-dimensional semimodules over integer and rational semifields with idempotent addition, subject to linear inequality constraints. We offer explicit solutions to the problems in a compact vector form suitable for both further analysis and practical implementation. Application examples of problems in operations research are provided to illustrate the results.

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## CP9

### Deriving Compact Extended Formulations via LP-based Separation Techniques

Some combinatorial optimization problems can be modeled via Linear Programs with an exponential number of rows and/or columns, which can be solved by generating rows and columns via separation techniques. A crucial question concerns the possibility of having alternative formulations which require additional variables but only a polynomial number of inequalities. These formulations are called compact extended. We show how to derive compact extended formulations if the separation problem can be solved by a linear program. We illustrate the technique by providing examples of several known combinatorial optimization problems.

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## CP9

### The Minimum Number of Hubs Needed in Networks

A hub refers to a non-terminal vertex of degree  $\geq 3$ . We study the minimum number of hubs needed in a network to guarantee certain flow demand constraints imposed between multiple pairs of sources and sinks. We prove that under the constraints, regardless of the size of the network, such minimum number is always upper bounded and we derive tight upper bounds for some special parameters. In particular, for two pairs of sources and sinks, we present a novel path-searching algorithm, the analysis of which is instrumental for the derivations of the tight upper bounds. Our results are of both theoretical and practical interest: in theory, they can be viewed as generalizations of the classical Menger's theorem to a class of undirected graphs with multiple sources and sinks; in practice, our results, roughly speaking, suggest that for some given flow demand constraints, not too many hubs are needed in a network. See

<http://arxiv.org/abs/1308.2365> for the full manuscript.

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## CP10

### On Minimal Partial Words with Subword Complexity $2n$

Recently, it was shown that compressed binary De Bruijn sequences can be built using minimal partial words of subword complexity  $2n$ . More precisely, a  $2n$ -complex partial word of a positive integer order  $N$  has  $2n$  distinct subwords for all  $n = 1, 2, \dots, N$ . Such a partial word is minimal if there are no shorter  $2n$ -complex partial words also of order  $N$ . In this paper, we show how to construct minimal  $2n$ -complex partial words of a given order  $N$  with a given number of holes  $h$ . We provide some insights into the lengths of such partial words, as well as how many there are, as functions of  $h$  and  $N$ . In the case when  $h = 0$ , we also provide an algorithm to generate them. Furthermore, we discuss several conjectures and open problems towards the classification of the minimal  $2n$ -complex partial words.

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## CP10

### Keeping Avoider's Graph Almost Acyclic

In this talk we consider biased  $(1 : b)$  Avoider-Enforcer games in the monotone and strict version. In particular, we show that Avoider can keep his graph acyclic for every but the last round of the game if  $b \geq 200n \log n$ . We hereby improve previous upper bounds on the threshold biases for the non-planarity game, the non- $k$ -colorability game, and the  $K_t$ -minor game. Moreover, we give a slight improvement for the lower bound in the non-planarity game.

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## CP10

### Minimal Partial Languages and Automata

Partial words are sequences of characters from an alphabet in which some positions may be marked with a “hole” symbol,  $\diamond$ . We can create a  $\diamond$ -substitution mapping this symbol to a subset of the alphabet, so that applying such a substitution to a partial word results in a set of full words (ones without holes). This setup allows us to compress regular languages into smaller partial languages. Deterministic finite automata for such partial languages, referred to as  $\diamond$ -DFAs, employ a limited non-determinism that can allow them to have lower state complexity than the minimal DFAs for the corresponding full languages. Our paper focuses on algorithms for the construction of minimal partial languages, associated with some  $\diamond$ -substitution, as well as approximation algorithms for the construction of minimal  $\diamond$ -DFAs.

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## CP10

### The Relationships Between the Bernoulli Numbers $B_{2n}$ & $A_n$ and the Euler Numbers $E_{2n}$ & $B_n$

Traditionally, the values of Riemann zeta function at the even positive integers have been formulated in terms of Bernoulli numbers  $B_{2n}$  and the sums of the alternating series of odd powers of the reciprocal of odd positive integers have been calculated in terms of Euler numbers  $E_{2n}$ . However, the present author reproduced the sum of the same series using different procedures in terms of two rational numbers  $a_n$  and  $b_n$ . In this paper, the relationships between  $B_{2n}$  &  $a_n$  and  $E_{2n}$  &  $b_n$  have been established. Consequently, some of the theorems and corollaries in [5] have been restated in terms of  $B_{2n}$  and  $E_{2n}$ .

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## CP10

### Tight Bounds on Number of Primitively-Rooted Squares in Partial Words

Partial words have been subject of much recent investigation, both combinatorial and algorithmic. They are sequences that may have undefined positions, called holes, that match, or are compatible with, any letter of the alphabet over which they are defined. These sequences allow for data that is incomplete or corrupted. The known algorithms for detecting all repetitions in strings without holes cannot be easily extended to strings with holes, one of the most important culprits being that compatibility is not transitive. This paper deals with a type of repeti-

tions in partial words over any alphabet size called squares, which consist of two adjacent occurrences of compatible substrings. We establish tight upper and lower bounds on the number of occurrences of primitively-rooted squares in a given partial word of a fixed length and number of holes. These bounds have applications to the analysis of efficient algorithms for detecting all repetitions in partial words.

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## CP10

### Pattern Occurrence Statistics and Applications to the Ramsey Theory of Unavoidable Patterns

As suggested by Currie, we apply the probabilistic method to problems regarding pattern avoidance. Using techniques from analytic combinatorics, we calculate asymptotic pattern occurrence statistics and use them in conjunction with the probabilistic method to establish new results about the Ramsey theory of unavoidable patterns in the full word case (both nonabelian sense and abelian sense) and in the partial word case. We suggest additional possible uses of these data in applications such as cryptography and musicology.

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## CP10

### A Variation of the Instant Insanity Puzzle

The Instant Insanity puzzle consists of four cubes whose faces are colored with one of four colors: red, blue, green and white. The object of the game is to stack the four cubes on top of one another so that each side of the stack has one face of each color. A variation of this puzzle will be presented in this talk.

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## MS1

### Cycle Packing

In the 1960s, Erdős and Gallai conjectured that the edge set of every graph on  $n$  vertices can be partitioned into  $O(n)$  cycles and edges. They observed that one can easily get an  $O(n \log n)$  upper bound by repeatedly removing the

edges of the longest cycle. We make the first progress on this problem, showing that  $O(n \log \log n)$  cycles and edges suffice. We also prove the Erdős-Gallai conjecture for random graphs and for graphs with linear minimum degree.

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**MS1**

**A Refinement of the Ore-type Version of the Corrádi-Hajnal Theorem**

This result is joint with H. Kierstead, T. Molla and E. Yeager. In 1963, Corrádi and Hajnal proved that for all  $k \geq 1$  and  $n \geq 3k$ , every  $n$ -vertex graph  $G$  with minimum degree  $\delta(G) \geq 2k$  contains  $k$  disjoint cycles. The degree bound is sharp. Enomoto and Wang independently proved the Ore-type refinement of the result: the same conclusion holds under the weaker assumption that for all distinct nonadjacent vertices  $x, y$ ,  $d(x) + d(y) \geq 4k - 1$ . We describe for all  $k \geq 4$  and  $n \geq 3k$  the  $n$ -vertex graphs  $G$  with  $d(x) + d(y) \geq 4k - 3$  for all distinct nonadjacent  $x, y$  that do not contain  $k$  disjoint cycles. In particular, the result describes all sharpness examples for the Corrádi-Hajnal Theorem.

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**MS1**

**Robustness of Graphs - Case Study: Dirac's Theorem**

A typical result in graph theory can be read as following: under certain conditions, a given graph  $G$  has some property  $\mathcal{P}$ . For example, a classical theorem of Dirac asserts that every  $n$ -vertex graph  $G$  of minimum degree at least  $n/2$  is Hamiltonian, where a graph is called Hamiltonian if it contains a cycle that passes through every vertex of the graph. Recently, there has been a trend in extremal graph theory where one revisits such classical results, and attempts to see how strongly  $G$  possesses the property  $\mathcal{P}$ . In other words, the goal is to measure the robustness of  $G$  with respect to  $\mathcal{P}$ . In this talk, we discuss several measures that can be used to study robustness of graphs with respect to various properties. To illustrate these measures, we present several extensions of Dirac's theorem.

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**MS1**

**Proof of Two Conjectures of Thomassen on Tournaments**

We prove the following two conjectures of Thomassen on highly connected tournaments: (i) For every  $k$ , there is an  $f(k)$  so that every strongly  $f(k)$ -connected tournament contains  $k$  edge-disjoint Hamilton cycles (joint work with Kühn, Lapinskas and Patel). (ii) For every  $k$ , there is an  $f(k)$  so that every strongly  $f(k)$ -connected tournament has a vertex partition  $A, B$  for which both  $A$  and  $B$  induce a strongly  $k$ -connected tournament (joint with Kühn and Townsend). Our proofs introduce the concept of 'robust dominating structures', which will hopefully have further applications. I will also discuss related open problems on cycle factors and linkedness in tournaments.

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**MS1**

**Packing Trees of Bounded Degree**

Motivated by a conjecture of Gyarfás and Lehel, recently Böttcher, Hladký, Piguet and Taraz showed that every collection of trees can be packed into a complete graph with at  $(1 + o(1))n$  vertices, provided each tree has at most  $n$  vertices, bounded maximum degree, and all trees have at most  $n^2/2$  edges all together. We give an alternative proof and discuss some extensions of this result.

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**MS2**

**Electroid Varieties and a Compactification of the Space of Electrical Networks**

We construct a compactification of the space of circular planar electrical networks studied by Curtis-Ingerman-Morrow and De Verdière-Gitler-Vertigan, using cactus networks. We embed this compactification as a linear slice of the totally nonnegative Grassmannian, and relate Kenyon and Wilson's grove measurements to Postnikov's boundary measurements. Intersections of the slice with the positroid stratification leads to a class of electroid varieties, indexed by matchings. The partial order on matchings arising from

electrical networks is shown to be dual to a subposet of affine Bruhat order. The analogues of matroids in this setting are certain distinguished collections of non-crossing partitions.

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## MS2

### Asymptotics of Symmetric Polynomials with Applications to Statistical Mechanics Models

We develop methods for asymptotic analysis of normalized symmetric polynomials (Schur functions and other Lie group characters). We show how these asymptotics, together with some combinatorial interpretations, give the distributions of certain observables in the scaling limit of grid models – lozenge tilings, 6-vertex model with domain wall boundary conditions (i.e. ASMs), the  $O(1)$  dense loop model. Based on joint work with Vadim Gorin (MIT, IITP).

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## MS2

### Title Not Available at Time of Publication

Abstract not available at time of publication.

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## MS2

### Arctic Curves of the Octahedron Equation

We study the octahedron relation, which solution corresponds to the partition function for dimer coverings of the Aztec Diamond graph. We find exact solutions for a particular class of periodic initial conditions. We show that the density function, that measures average dimer occupation of a face of the Aztec graph, obeys a linear system with periodic coefficients. We explore the thermodynamic limit of the dimer models and derive exact "arctic" curves separating various phases.

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## MS2

### Rowmotion, Homomesy, and the Razumov-Stroganov Ex-conjecture

The Razumov-Stroganov ex-conjecture, an important link between statistical physics and combinatorics, relates the ground state eigenvector of the  $O(1)$  dense loop model to a refined counting of fully-packed loops on a square grid, which are in bijection with alternating sign matrices. We interpret Cantini and Sportiello's proof of this conjecture

from the perspective of a toggle group action on the alternating sign matrix poset, thereby simplifying some aspects of their proof.

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## MS3

### Generating Random Graphs in Biased Maker-Breaker Games

In this talk we present a general approach connecting biased Maker-Breaker games and problems about local resilience in random graphs. Using this method, we investigate the threshold bias  $b$  for which Maker can win certain  $(1 : b)$  games. We utilize this approach to prove new results about biased games and also to derive some known results in novel ways. We also present some related and challenging open problems.

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## MS3

### Picker-Chooser Fixed Graph Games

Given a fixed graph  $H$  and a positive integer  $n$ , a Picker-Chooser  $H$ -game is a biased game played on the edge set of  $K_n$  in which Picker is trying to force many copies of  $H$  and Chooser is trying to prevent him from doing so. In this paper we conjecture that the value of the game is roughly the same as the expected number of copies of  $H$  in the random graph  $G(n, p)$  and prove our conjecture for special cases of  $H$  such as complete graphs and trees.

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## MS3

### Does the Random Graph Intuition Help when Orientations are Involved?

We study Maker-Breaker games involving orientations. Two players, say Red and Blue, claim and direct edges of the complete graph alternately. It is of interest to determine the largest  $k$  such that Red has a strategy so that (1) the Red digraph contains a copy of a predefined tournament on  $k$  vertices, (2) the Red digraph contains a copy

of every tournament on  $k$  vertices. We also discuss connections to the random graph intuition.

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### MS3

#### Maker-Breaker Games on Random Geometric Graphs

In a Maker-Breaker game on a graph  $G$ , Breaker and Maker alternately claim edges of  $G$ . Maker wins if, after all edges have been claimed, the graph induced by his edges has some desired property. We consider four Maker-Breaker games played on random geometric graphs. For each of our four games we show that if we add edges between  $n$  points chosen uniformly at random in the unit square by order of increasing edge-length then, with probability tending to one as  $n \rightarrow \infty$ , the graph becomes Maker-win the very moment it satisfies a simple necessary condition. In particular, with high probability, Maker wins the connectivity game as soon as the minimum degree is at least two; Maker wins the Hamilton cycle game as soon as the minimum degree is at least four; Maker wins the perfect matching game as soon as the minimum degree is at least two and every edge has at least three neighbouring vertices; and Maker wins the  $H$ -game as soon as there is a subgraph from a finite list of “minimal graphs”. These results also allow us to give precise expressions for the limiting probability that  $G(n, r)$  is Maker-win in each case, where  $G(n, r)$  is the graph on  $n$  points chosen uniformly at random on the unit square with an edge between two points if and only if their distance is at most  $r$ .

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### MS3

#### Walker vs Breaker

We introduce the concept of Walker-Breaker games on graphs, a variant of Maker-Breaker games where Maker’s edge selections are required to “walk” around the underlying graph in various ways. In particular, we will consider games where Walker’s goal is to span as large a subgraph as possible.

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### MS4

#### $\sigma$ -Polynomials and Their Roots

Given a graph  $G$  of order  $n$ , the  $\sigma$ -polynomial of  $G$  is the generating function  $\sigma(G, x) = \sum a_i x^i$  where  $a_i$  is the number of partitions of the vertex set of  $G$  into  $i$  nonempty independent sets. Such polynomials arise in a natural way from chromatic polynomials, and have strong connections to several other well known polynomials in combinatorics. Brenti proved that  $\sigma$ -polynomials of graphs with chromatic number at least  $n - 2$  had all real roots, and conjectured the same held for chromatic number  $n - 3$ . We affirm this conjecture.

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### MS4

#### Independent Set, Induced Matching, and Pricing: Connections and Tight (Subexponential Time) Approximation Hardnesses

We present almost settled inapproximability results for three fundamental problems. The first one is the subexponential-time inapproximability of the maximum independent set problem, a question studied in the area of parameterized complexity. The second one is the hardness of approximating the maximum induced matching problem on bounded-degree bipartite graphs. The last one is the tight hardness of approximating the  $k$ -hypergraph pricing problem, a fundamental problem in the area of algorithmic game theory.

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#### MS4

##### A Geometric Foundation to Information Networks

We analyze the geometric properties underlying information networks modeled as large-scale finite graphs  $G = (V, E)$ , where the vertex set  $V$  defines the information location and the edge set  $E$  labeled with their logical distance. We determine the required conditions for the resulting quasi-geodesic metric space to define  $k$ -local geodesics. These properties are then exploited as the foundation of a distributed information localization algorithm which enables to retrieve information from their distance.

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#### MS4

##### Unique Vector Coloring

Strict vector coloring is one formulation of an optimization program for Lovász theta function: assign vectors to vertices so that adjacent vertices obtain vectors with large angle in-between. We study when this assignment is unique. We find analog of a classical result on uniqueness of coloring of graph products and generalize it. We use recent results of Laurent and Varvitsiotis on universal rigidity, and properties of eigenspaces of Kneser graphs.

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#### MS4

##### 4-coloring Graphs with No Induced 5-cycle and 6-vertex Path

We show that the 4-coloring problem can be solved in polynomial time on graphs with no induced 5-cycle  $C_5$  and no induced 6-vertex path  $P_6$ . The more general 4-coloring on  $P_6$ -free graphs is one of the remaining cases left to complete the complexity characterization of the  $c$ -coloring problem on  $P_k$ -free graphs for fixed  $c$  and  $k$ . While this is our ultimate goal, the case with no induced 5-cycles already reveals interesting structure and requires non-trivial arguments.

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#### MS5

##### Subgraphs in Random Non-uniform Hypergraphs

For a set of positive integers  $R := \{k_1, k_2, \dots, k_r\}$  and probabilities  $\mathbf{p} = (p_1, p_2, \dots, p_r) \in [0, 1]^r$ , let  $G^R(n, \mathbf{p})$  be the random hypergraph  $G$  on  $n$  vertices so that for  $1 \leq i \leq r$  each  $k_i$ -subset of vertices appears as an edge of  $G$  with probability  $p_i$  independently. The threshold of the occurrence of a fixed graph  $H$  in the Erdős-Rényi model  $G(n, p)$  (here  $R = \{2\}$ ) is well-studied in literature. We generalize these results to non-uniform hypergraphs. This is joint work with Linyuan Lu.

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#### MS5

##### Networks with Similar Assortativity - JDMs and PAMs

The partition adjacency matrix (PAM) encodes the number of edges connecting vertices of two partition classes in a given partition of the vertices. It is a generalization of the joint degree matrix, which encodes the information necessary to compute the assortativity of the network. In this talk we will explore what is the size of edge-swaps needed to walk the space of graphs with given degree sequence and PAM, and related questions.

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#### MS5

##### Sampling Graphs Using Markov Chains

The problem of efficiently sampling from a set of graphs (bipartite graphs, directed graphs) with a given degree sequence has many applications. One approach to this problem uses Markov chains to perform the sampling. The difficulty lies in establishing rigorously that the Markov chain converges rapidly to its stationary distribution. I will review the known results in this area and discuss ongoing extensions.

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#### MS5

##### Hypergraph Problems Arising from Networks in Algebraic Statistics

Social networks and other large sparse data sets pose significant challenges for statistical inference, as many standard statistical methods for testing model/data fit are not applicable in such settings. Algebraic statistics offers a theoretically justified approach to goodness-of-fit testing that relies on the theory of Markov bases and is intimately connected with the geometry of the model as described by its fibers. Most current practices require the computation of

the entire basis, which is infeasible in many practical settings. We present a dynamic approach to explore the fiber of a model, which bypasses this issue, and is based on the combinatorics of hypergraphs arising from the toric algebra structure of log-linear models. This algebraic statistics problem is intimately tied with graph and hypergraph sampling problems, which will be explained in this setting. Joint work with Elizabeth Gross and Despina Stasi.

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**MS5**

### Constrained Graph Construction Problems for Network Modeling Purposes

Here I will discuss some of the fundamental questions related to graph ensemble based modeling of empirical networks. In particular I will focus on degree based and joint-degree based graph existence, construction, sampling and enumeration problems and to a lesser extent, on soft-constraints based modeling using exponential random graph ensembles.

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**MS6**

### Geometric Homomorphisms and the Geochromatic Number

A geometric graph  $\overline{G}$  is a simple graph  $G$  together with a fixed straightline drawing in the plane. A geometric homomorphism  $\varphi : V(\overline{G}) \rightarrow V(\overline{H})$  is a homomorphism of the underlying abstract graphs that also preserves edge crossings. That is,  $\varphi$  is a geometric homomorphism if whenever  $v, u$  are adjacent vertices in  $\overline{G}$  then  $\varphi(v), \varphi(u)$  are adjacent in  $\overline{H}$  and whenever  $e, f$  are crossing edges in  $\overline{G}$  then  $\varphi(e)$  and  $\varphi(f)$  are crossing in  $\overline{H}$ . Analogous to the chromatic number of an abstract graph, the geochromatic number of a geometric graph is defined using geometric homomorphisms. That is, the geochromatic number of a geometric graph  $\overline{G}$  is the smallest integer  $n$  so that there is a geometric homomorphism from  $\overline{G}$  to some geometric realization of  $K_n$ . This talk will consider conditions under which we can find or bound the geometric chromatic number of a geometric graph.

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**MS6**

### Graph Homomorphisms: Edge Colours, Signs, and Crossings, An Introduction

Given graphs  $G$  and  $H$ , a *homomorphism* of  $G$  to  $H$  is a function  $f : V(G) \rightarrow V(H)$  such that  $uv \in E(G)$  implies  $f(u)f(v) \in E(H)$ . We write  $G \rightarrow H$ . Graph homomorphisms generalize colourings in the sense that  $G \rightarrow K_n$  is simply an  $n$ -colouring of  $G$ . Homomorphisms can be

naturally defined in settings like digraphs, edge-coloured graphs, signed graphs, and general relational systems. One may also place restrictions on the function  $f$ , for example we may require  $f$  to be locally injective or restrict its values to particular allowed images. In this introductory talk we examine homomorphisms in settings mentioned above: edge-coloured, signed, and directed graphs. We survey some results with particular interest given to complexity and good characterizations.

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**MS6**

### Complexity of Signed Graph Homomorphisms

We present recent advances in the study of the computational complexity of the signed graph homomorphism problem, in an attempt of a full classification based on the target signed graph. In this talk, a signed graph is a graph whose edges are either positive or negative, and switching (i.e. locally changing the signs of the edges incident to a given vertex) is allowed. A homomorphism of signed graphs is a mapping between the vertex sets that preserves the edge signs up to switching.

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**MS6**

### Colourings of Edge-Coloured Graphs and the Vertex Switching Operation

The operation of *switching* at a vertex  $x$  of an oriented graph reverses the orientation of every arc incident with  $x$ . Similarly, switching at a vertex  $x$  of a 2-edge coloured graph changes the colour of each edge incident with  $x$ . We will properties of vertex colourings and homomorphisms of 2-edge coloured graphs which are analogous to properties that hold in the case of oriented graphs, and which generalize to  $m$ -edge coloured graphs for  $m > 2$ .

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**MS6****Signed Graph Homomorphisms**

A signed graph, denoted  $(G, \Sigma)$ , is a graph where each edge has a sign,  $\Sigma$  is the set of negative edges. The notion first was introduced by F. Harary to study social networks where relationships can be of two sorts: friendship or enmity. The notion of minor for signed graphs then allowed graph theorists to formulate a better language to already existing results on coloring graphs with forbidden structures. For example, Catlin's proof of odd- $K_4$  free graphs being 3-colorable can be read as: if  $(G, E(G))$  has no  $(K_4, E(K_4))$ -minor, then  $G$  is 3-colorable. The aim of this talk is to introduce a further generalization of this sort of coloring results using the language of homomorphisms.

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**MS7****Packing Posets in the Boolean Lattice**

We consider maximizing the number of pairwise unrelated copies of a poset  $P$  in the family of all subsets of  $[n] := \{1, \dots, n\}$ . When  $P$  is a chain on  $k$  elements, the answer is asymptotic to  $\frac{1}{2^{k-1}} \binom{n}{\lfloor n/2 \rfloor}$ , as  $n \rightarrow \infty$ , by a result of Griggs, Stahl, and Trotter. While it remains open to determine asymptotically the largest size  $\text{La}(n, P)$  of a family of subsets of  $[n]$  that contains no subposet  $P$ , we solve this new problem: The maximum is  $\sim \frac{1}{c(P)} \binom{n}{\lfloor n/2 \rfloor}$ , where the integer  $c(P)$  relates to embeddings of  $P$  into the Boolean lattice. This problem was independently posed and solved by Katona and Nagy.

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**MS7****Largest Union-intersecting Families**

János Körner asked the following question. Let  $[n] = \{1, 2, \dots, n\}$  and let  $\mathcal{F} \subset 2^{[n]}$  be a family of its subsets. It is called union-intersecting if  $(F_1 \cup F_2) \cap (F_3 \cup F_4)$  is non-empty whenever  $F_1, F_2, F_3, F_4 \in \mathcal{F}$  and  $F_1 \neq F_2, F_3 \neq F_4$ . What is the maximum size of a union-intersecting family? This question is answered in the present paper. The optimal construction when  $n$  is odd consists of all subsets of size at least  $\frac{n-1}{2}$  while in the case of even  $n$  it consists of all sets of size at least  $\frac{n}{2}$  and sets of size  $\frac{n}{2} - 1$  containing a fixed element, say 1. We also proved some extensions, variants and analogues of this statement. The following one is an example. Suppose that  $\mathcal{F}$  is a union-intersecting family of  $k$ -element subsets of  $[n]$ . We found that the optimal construction for this problem consists of all  $k$ -element subsets of size  $k$  containing the element 1, and one more additional set, if for  $n > n(k)$ . The results were jointly achieved with Dániel T. Nagy.

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**MS7****Estimation of the Size of Families Not Containing the Given Posets**

We are interested in how large a family of subsets of the set  $[n] := \{1, \dots, n\}$  that avoids a given partially ordered set (poset)  $P = (P, \leq)$ . Let  $\text{La}(n, P)$  be the largest size of families of subsets of  $[n]$  that do not contain the poset  $P$ . A conjecture posed by Griggs and Lu (2009), independently by Bukh (2009), is that  $\text{La}(n, P)$  is asymptotically equal to  $e \binom{n}{\lfloor \frac{n}{2} \rfloor}$ , where  $e$  is an integer determined by  $P$  when  $n$  is large enough. The best general upper bound for  $\text{La}(n, P)$  is  $(\frac{1}{2}(|P| + h(P)) - 1) \binom{n}{\lfloor \frac{n}{2} \rfloor}$ , due to Bursi and Nagy. In the talk we will see how to improve the upper bound to be

$$\frac{1}{m+1} \left( |P| + \frac{1}{2}(m^2 + 3m - 2)(h(P) - 1) - 1 \right) \binom{n}{\lfloor \frac{n}{2} \rfloor},$$

where  $m$  can be any positive integer less than  $\lceil \frac{n}{2} \rceil$ .

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**MS7****Extremal Problems on Posets and Hypergraphs**

We are interested in how large a family of subsets of the  $n$ -set  $[n] := \{1, \dots, n\}$  there is that avoids certain patterns. The foundational result of this sort, Sperner's Theorem from 1928, solves this problem for families that contain no two-element chains. Here we will present some recent results on extremal families avoiding a given (weak) subposet, such as crowns, diamonds, etc. When viewing the family of subsets as a hypergraph, we generalized several properties of Turán density, such as supersaturation, blow-up, and suspension, from uniform hypergraphs to non-uniform hypergraphs. The Lubell function plays a key role in both areas.

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**MS7****Recent Progress on Diamond-free Families**

In the Boolean lattice, a diamond is a subposet of four distinct subsets  $A, B, C, D$  such that  $A \subset B, C$  and  $D \supset B, C$ . One of the most well-studied problems in extremal poset theory is determining the size of the largest diamond-free family in the  $n$ -dimensional Boolean lattice. We will discuss some recent progress on this problem.

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**MS8****Ordered Ramsey Numbers**

Given a labeled graph  $H$  with vertex set  $\{1, 2, \dots, n\}$ , the

ordered Ramsey number  $r_{<}(H)$  is the minimum  $N$  such that every two-coloring of the edges of the complete graph on  $\{1, 2, \dots, N\}$  contains a copy of  $H$  with vertices appearing in the same order as in  $H$ . The ordered Ramsey number of a labeled graph  $H$  is at least the Ramsey number  $r(H)$  and the two coincide for complete graphs. However, we prove that even for matchings there are labelings where the ordered Ramsey number is superpolynomial in the number of vertices. We prove several additional results regarding ordered Ramsey numbers, including an ordered analogue of the Burr-Erdős conjecture. We also show a close connection between ordered Ramsey numbers of graphs and the ordinary Ramsey number for 3-uniform hypergraphs.

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**MS8**

### On the Size Ramsey Numbers and their Variations

For a given graph  $G$ , its *size-Ramsey number* is the minimum number of edges in a graph  $H$  such that any 2-coloring of the edges of  $H$  yields a monochromatic copy of  $G$ . This concept was introduced by Erdős, Faudree, Rousseau, and Schelp and considered by several other researchers. Somewhat surprisingly this idea has not yet been extended to hypergraphs, even though classical Ramsey numbers for hypergraphs have been studied extensively. In this talk, we discuss this new direction of research.

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**MS8**

### Solution to the Erdős-Gyárfás Conjecture on Generalized Ramsey Numbers

Fix positive integers  $p$  and  $q$  with  $2 \leq q \leq \binom{p}{2}$ . An edge-coloring of the complete graph  $K_n$  is said to be a  $(p, q)$ -coloring if every  $K_p$  receives at least  $q$  different colors. The function  $f(n, p, q)$  is the minimum number of colors that are needed for  $K_n$  to have a  $(p, q)$ -coloring. This function was introduced by Erdős and Shelah about 40 years ago, and Erdős and Gyárfás in 1997 initiated the systematic study of this function. In particular, they were interested how for each fixed  $p$ , the asymptotic behavior of  $f(n, p, q)$  (as  $n$  goes to infinity) changes for different values of  $q$ . They proved several interesting results, but the question of determining the maximum  $q$  for which  $f(n, p, q)$  is sub-polynomial in  $n$  remained open. In this talk, we prove that the answer is  $p - 1$ .

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**MS8**

### Sharp Thresholds for Van Der Waerden's Theorem

It was shown by Rödl and Ruciński that for  $m = C_r n^{(k-2)/(k-1)}$  almost every  $m$ -element subset  $M$  of the first  $n$  integers has the property that every  $r$ -coloring of  $M$  yields a monochromatic arithmetic progression of length  $k$ , while sets of size  $o(n^{(k-2)/(k-1)})$  fail to have this property. We discuss sharp thresholds for this and related properties. This is joint work with E. Friedgut, H. Hàn, and Y. Person.

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**MS8**

### On the Ramsey Number of the Clique and the Hypercube

Abstract not available at time of publication.

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**MS9**

### Acyclic List Coloring on Planar Graphs

Let  $G = (V, E)$  be a graph. A proper vertex coloring of  $G$  is acyclic if  $G$  contains no bicolored cycle. Namely, every cycle of  $G$  must be colored with at least three colors.  $G$  is acyclically  $L$ -list colorable if for a given list assignment  $L = \{L(v) : v \in V\}$ , there exists a proper acyclic coloring  $\pi$  of  $G$  such that  $\pi(v) \in L(v)$  for all  $v \in V$ . If  $G$  is acyclically  $L$ -list colorable for any list assignment with  $|L(v)| \geq k$  for all  $v \in V$ , then  $G$  is acyclically  $k$ -choosable. In 1976, Steinberg conjectured that every planar graph without 4- and 5-cycles is 3-colorable. This conjecture cannot be improved to 3-choosable basing on the examples given by Voigt and independently, by Montassier. In this talk, We will show that planar graphs without 4- and 5-cycles are acyclically 4-choosable. This result is also a new approach to the conjecture proposed by Montassier, Raspaud and Wang, which

says that every planar graph without 4-cycles is acyclically 4-choosable.

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### MS9

#### Colorings and Independent Sets of Triangle-Free Planar Graphs

We use coloring arguments to give a sufficient condition for the size of the largest independent set in an  $n$ -vertex triangle-free planar graph to be at least  $n/3 + k$  for a fixed constant  $k$ , and use it to show the fixed parameter tractability of the corresponding decision problem.

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### MS9

#### On Thue Colourings of Graphs

Here we deal with different versions of nonrepetitive total colourings of graphs and define the minimum number of colours required in every strong (weak) total Thue colouring. We prove some general upper bounds for the normal as well as the list version of the problem and show some better bounds for special classes of graphs.

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### MS9

#### Subcubic Triangle-free Graphs have Fractional Chromatic Number at most $14/5$

We prove that every subcubic triangle-free graph has fractional chromatic number at most  $14/5$ , thus confirming a conjecture of Heckman and Thomas [A new proof of the independence ratio of triangle-free cubic graphs. *Discrete Math.* 233 (2001), 233–237].

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### MS9

#### Algorithms to Color Sparse Graphs

Abstract not available at time of publication.

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### MS10

#### Erdős–Szekeres-type Statements in Dimension 1

Consider a geometric property of  $k$ -tuples of points expressible by a system of polynomial equations and inequalities. For some properties, every large set contains a subset each  $k$ -tuple of which satisfies the property. For example, every very large set contains a large subset in convex position (Erdős–Szekeres theorem). We treat the general problem in dimension 1, and show partial results in higher dimensions.

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### MS10

#### Simplifying Inclusion - Exclusion Formulas

The classical inclusion-exclusion formula expresses the measure of a union of sets from the measures of their intersections. For  $n$  sets this formula has  $2^n - 1$  terms, and a significant amount of research, originating in applied areas, has been devoted to constructing simpler formulas for particular families. We show that every family of  $n$  sets with Venn diagram of size  $m$  admits an inclusion-exclusion formula with  $m^{O(\log^2 n)}$  terms, with  $\pm 1$  coefficients, computable in  $m^{O(\log^2 n)}$  expected time. Joint work with J. Matousek, Z. Safernova, M. Tancer and P. Patak

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### MS10

#### Splitting Points and Hyperplanes

In this talk, we will discuss some of the discrete structures that arise while studying finite families of points and hyperplanes in  $\mathbb{R}^d$ . Given a set of points and hyperplanes in  $\mathbb{R}^d$ , we aim to find a large subset of them such that no selected hyperplane splits two selected points. This will be related to classic convex partition results of Yao and Yao which were originally motivated by geometric range queries.

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### MS10

#### The Cylindrical Crossing Number of the Complete Bipartite Graph

A cylindrical drawing of the complete bipartite graph on  $(n, m)$  vertices consists of  $n$  vertices placed on the upper rim and  $m$  vertices on the lower rim of a straight circular cylinder where every two vertices from different rims are joined by an edge on the surface. We present a formula for the minimum number of edge-crossings among all such drawings for each  $(n, m)$  and an algorithm for creating drawings that achieve this crossing number.

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### MS11

#### Gaps in Eigenfunctions of Graphs

We will discuss some problems and results on various properties of eigenfunctions of graphs. For, we will describe methods to bound the ‘stretches’ of edges which depend on the eigenfunctions.

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### MS11

#### Construction of Matrices with a Given Graph and Prescribed Interlaced Spectral Data

A result of Duarte [Linear Algebra Appl. 113 (1989) 173182] asserts that for real  $\lambda_1, \dots, \lambda_n$  and  $\mu_1, \dots, \mu_{n-1}$  with  $\lambda_1 \leq \mu_1 \leq \lambda_2 \leq \dots \leq \mu_{n-1} \leq \lambda_n$  and each tree  $T$  on  $n$  vertices there exists an  $n \times n$ , real symmetric matrix  $A$  whose graph is  $T$  such that  $A$  has eigenvalues  $\lambda_1, \dots, \lambda_n$  and the principal submatrix obtained from  $A$  by deleting its last row and column has eigenvalues  $\mu_1, \dots, \mu_{n-1}$ . This result is extended to connected graphs through the use of the implicit function theorem.

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### MS11

#### The Spectrum of Erdos-Renyi Random Graphs Near the Connectivity Threshold

We give sharp estimates for the spectral gap of an Erdos-Renyi random graph near the connectivity threshold. We show applications of these results to random topological

spaces.

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### MS11

#### Interlacing Families and Bipartite Ramanujan Graphs

Abstract not available at time of publication.

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### MS11

#### An Alon-Boppana Result for the Normalized Laplacian

We prove a Alon-Boppana style bound for the spectral gap of the normalized Laplacian for general graphs via a bound on the weighted spectral radius of the universal cover graph

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### MS12

#### Generalized Chromatic Polynomials

We show that many counting functions of graph properties give rise to graph polynomials similar to the chromatic polynomial of graphs. We survey what is known so far and formulate problems and conjectures.

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### MS12

#### Algebraic Properties of the Chromatic Polynomial

There is increasing interest in algebraic properties of chromatic polynomials including which algebraic numbers are roots, *chromatic factorisation* (where a chromatic polynomial is the product of chromatic polynomials) and relationships between chromatic polynomials with the same splitting field. Morgan and Farr introduced *certificates* for chromatic factorisation. Certificates are a powerful tool in proving algebraic properties of graph polynomials without the cost of computing the polynomial. We give an intro-

duction to using certificates in these proofs.

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## MS12

### Counting Cuts, Matchings, Bipartite Subgraphs, and Dominating Sets: The Bipartition Polynomial of a Graph

We introduce a new graph polynomial, the bipartition polynomial, as a common generalization of the domination polynomial and the Ising model, which is also the generating function for the number of matchings, bipartite subgraphs, and (for regular graphs) independent sets. The smallest known pair of non-isomorphic graphs with the same bipartition polynomial consists of two graphs of order 22. The bipartition polynomial can be used to prove properties of other graph polynomials and graph invariants.

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## MS12

### Caterpillars and the U-polynomial

The  $U$ -polynomial (Noble and Welsh) and the chromatic symmetric function (Stanley) are known to be equivalent for trees. A long-standing open question is whether there exist non-isomorphic trees with the same chromatic symmetric function. Caterpillars are trees such that the graph induced by the non-leaf vertices is a path. We establish a relation between caterpillars and integer compositions and thereby prove that proper caterpillars (caterpillars without vertices of degree two) are distinguished by the  $U$ -polynomial.

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## MS13

### New Metrics for Protein Interaction Networks

In protein-protein interaction (PPI) networks, functional similarity is often inferred based on the function of directly interacting proteins, or more generally, some notion of interaction network proximity among proteins in a local neighborhood. Prior methods typically measure proximity as the shortest-path distance in the network, but this has only a limited ability to capture fine-grained neighborhood distinctions, because most proteins are close to each other, and there are many ties in proximity. We introduce diffusion state distance (DSD), a new metric based on a graph diffusion property, designed to capture finer-grained distinctions in proximity for transfer of functional annotation in PPI networks. We present a tool that, when input a PPI network, will output the DSD distances between every pair of proteins. We show that replacing the shortest-path metric by DSD improves the performance of classical function prediction methods across the board. If time permits, we will also mention our most recent work that incorporates information about confidence scores and explicit pathways

into the model.

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## MS13

### Large Highly Connected Clusters in Protein-Protein Interaction Networks

The edge (vertex) connectivity of a graph is the minimum number of edges (vertices) that must be removed to disconnect the graph. Connectivity is an important property of a graph but has seldom been used in the study of protein-protein interaction (PPI) and other biological networks. Connectivity differs from edge density in that it is based on the number of paths between all vertices in the graph, while edge density is based solely on the number of edges. Connectivity may be a better indicator of large clusters than edge density: as the number of vertices in a cluster increases, the edge density tends to decrease rapidly, but the connectivity remains constant. We developed algorithms to search for subgraphs with high connectivities and applied these algorithms to *Saccharomyces cerevisiae* and human PPI networks. We discovered that PPI networks have large subgraphs (20-130 vertices) with high connectivity that were previously unrecognized. The function of these subgraphs remains unknown, but they are far more highly connected than subgraphs in random subgraphs and have significant numbers of proteins with shared biological functions, suggesting that these are biologically significant.

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## MS13

### Efficiently Enumerating All Connected Induced Subgraphs of a Large Molecular Network

In systems biology, the solution space for a broad range of problems is composed of sets of functionally associated biomolecules, which can be identified from connected induced subgraphs of molecular interaction networks. Applications typically quantify the relevance (e.g., modularity, conservation, disease association) of connected subnetworks using an objective function and use a search algo-

rithm to identify sets of subnetworks that maximize this objective function. Here, we propose and comprehensively evaluate an algorithm that significantly reduces the computations necessary to enumerate all maximal subgraphs that satisfy a hereditary property.

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### MS13

#### Network-Based Rnaseq Quantification and Survival Analysis for Cancer Genomics

High-throughput mRNA sequencing (RNA-Seq) is a promising technology for gene transcript quantification. We introduce a Network-based method for RNA-Seq-based Transcript Quantification (Net-RSTQ) to integrate protein domain-domain interaction information with short read alignment for transcript abundance estimation. Net-RSTQ models the expression of interacting transcripts as Dirichlet priors on the likelihood of the observed read alignments against the transcripts in one gene. Laboratory validation with qPCR confirmed the performance of Net-RSTQ for transcript quantification. The transcript abundances estimated by Net-RSTQ are also more informative for cancer sample classification.

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### MS13

#### Computational Approaches for Analyzing Large-scale Genetic Interaction Networks

Many phenotypes are the result of interactions between multiple genetic variants. Our lab has been involved in an on-going effort to measure quantitative phenotypes for millions of double mutants in yeast in an attempt to understand the fundamental principles of genetic interactions. I will describe what we can learn from these large-scale experiments, highlight insights that contribute to our understanding and treatment of human disease, and discuss where innovations in machine learning and data mining are particularly relevant in addressing these questions.

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### MS14

#### Delaunay Complexes, Noneuclidean Geometry, and Complexity

I will discuss Delaunay tessellations in the plane, connections to hyperbolic geometry and other kinds of geometry, and some surprising complexity results, some stemming from old and some from very new work.

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### MS14

#### Improvements on the Elekes-Rónyai Method

Elekes and Rónyai considered the following problem. Let  $A, B$  be two sets of  $n$  real numbers, and let  $f$  be a real bivariate polynomial. They showed that if  $|f(A \times B)| \leq cn$ , for some constant  $c > 0$  and for  $n \geq n_0$ , where  $n_0$  depends on  $c$  and  $deg f$  only, then  $f$  must be of one of the special forms  $f(u, v) = h(g(u) + k(v))$ , or  $f(u, v) = h(g(u)k(v))$ , for some univariate polynomials  $h, k, h$  over  $R$ . Let us denote the number of triples  $(a, b, c) \in A \times B \times C$  such that  $c = f(a, b)$  by  $M$ . We show that either  $M = O(n^{11/6})$  where the constant of proportionality depends on  $deg f$ , or else  $f$  has one of the above special forms. This improves bounds on various Erdős type problems in discrete geometry.

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### MS14

#### Embeddability in the 3-sphere is Decidable

During the talk we sketch a proof that the following algorithmic problem is decidable: given a 2-dimensional simplicial complex, can it be embedded (topologically, or equivalently, piecewise linearly) in  $\mathbf{R}^3$ ? This is a natural higher-dimensional analogue of the graph-planarity question. By a known reduction, it suffices to decide the embeddability of a given triangulated 3-manifold  $X$  into the 3-sphere  $S^3$ . The main step relies on a search for a short meridian, via (marked) normal surface theory.

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#### MS14

##### On the Number of Plane Graphs with Polyline Edges

It is shown that on every  $n$ -element point set in the plane, at most  $\exp(O(kn))$  labeled planar graphs can be embedded using polyline edges with  $k$  bends per edge. This is the first exponential upper bound for the number of labeled plane graphs where the edges are polylines of constant size. Several standard tools developed for the enumeration of straight-line graphs, such as triangulations and crossing numbers, are unavailable in this scenario. Furthermore, the exponential upper bound does not carry over to other popular relaxations of straight-line edges: for example, the number of plane graphs realizable with  $x$ -monotone edges on  $n$  points is already super-exponential. (Joint work with Andrea Francke.)

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#### MS14

##### Space Curve Arrangements with Many Incidences

If a collection of lines in  $\mathbb{R}^3$  has too many pairwise intersections, then many of these lines must lie on planes or reguli. In this talk I will discuss a generalization of this result to algebraic space curves; if a collection of curves has too many pairwise intersections, then these curves must congregate on lower dimensional varieties.

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#### MS15

##### Fighting Fires on Layered Grids

In the Firefighter Problem, a fire starts at a vertex of a graph at time 0, and in discrete time steps, it spreads from burning vertices to their neighbors unless they are protected by one of the firefighters that are deployed between each time step. We assume that the number of firefighters per turn is a fixed positive integer. We also assume that once a vertex is burning or protected, it remains in that state. The process terminates when fire can no longer spread. In the case of locally-finite infinite graphs, the key question is to determine the minimum number of firefighters needed to stop a fire in finite time. In this talk, we will consider graphs that are the Cartesian product of a path and an infinite 2-dimensional grid, and will give an upper bound on the number of firefighters needed in such a graph given that the number of firefighters that can be deployed in each layer per time unit does not exceed a fixed number.

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#### MS15

##### The Robber Strikes Back

We consider the new game of Cops and Attacking Robbers, which is identical to the usual Cops and Robbers game except that if the robber moves to a vertex containing a

single cop, then that cop is removed from the game. We study the minimum number of cops needed to capture a robber on a graph  $G$ , written  $cc(G)$ . We give bounds on  $cc(G)$  in terms of the cop number of  $G$  for bipartite graphs.

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#### MS15

##### Ambush Cops and Robbers on Graphs with Small Girth

In this variation of the game with two robbers, the cops win by moving onto the same vertex as *one* of the robbers after a finite number of moves. The robbers win by avoiding capture indefinitely or by both moving onto the same vertex as the cop. (Otherwise, the robbers are on distinct vertices.) We present results on graphs with small girth.

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#### MS15

##### Cops and Invisible Robbers

In this talk we will consider the game of cops and robbers, as in the classic model, with the added restriction that the robber is invisible to the cops. This alteration brings the game more in line with the pursuit-evasion model known as graph searching. We will consider some classic investigations along this line, including relations to monotonicity, pathwidth, and complexity.

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#### MS15

##### Edge Contraction and Cop-Win Critical Graphs

In the game of Cop and Robber, a single cop tries to apprehend a robber as they move alternately along edges of a reflexive graph. Graphs in which one cop can always win are called cop-win graphs. A graph is Cop-win Edge Critical with respect to Contraction (CECC) if it is not itself cop-win, but with the contraction of any edge, the resulting graph is cop-win. In this talk, I will present characterizations of some particular classes of CECC graphs such as bipartite, minimum degree three, and 4-regular. I will also compare these results with known results for graphs that are Cop-win Edge Critical with respect to Edge Addition.

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**MS16****Long Cycles in Graphs with Bounded Degrees**

In 1993 Jackson and Wormald conjectured that for any positive integer  $d \geq 4$ , there exists a positive real number  $\alpha$  depending only on  $d$  such that if  $G$  is a 3-connected  $n$ -vertex graph with maximum degree  $d \geq 4$ , then  $G$  has a cycle of length  $\geq \alpha n^{\log_{d-1} 2}$ . They showed that the exponent in the bound is best possible if the conjecture is true. In this paper, we report the recent progress toward this conjecture.

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**MS16****Minimum Degree and Disjoint Cycles in Generalized Claw-free Graphs**

For  $s \geq 3$  a graph is  $K_{1,s}$ -free, if it does not contain an induced subgraph isomorphic to  $K_{1,s}$ . For  $s = 3$ , such graphs are called claw-free graphs. Results on disjoint cycles in claw-free graphs satisfying certain minimum degree conditions will be discussed, such as if  $G$  is claw-free of sufficiently large order  $n = 3k$  with  $\delta(G) \geq n/2$ , then  $G$  contains  $k$  disjoint triangles. Also, the extension of results on disjoint cycles in claw-free graphs satisfying certain minimum degree conditions to  $K_{1,s}$ -free graphs for  $s > 3$  will be presented. These results will be used to prove the existence of minimum degree conditions that imply the existence of powers of Hamiltonian cycles in generalized claw-free graphs.

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**MS16****On Chorded Cycles**

I will introduce results that have been shown over the last

few years dealing with chorded cycles. I will then concentrate on some new results concerned with placing elements on chorded cycles, when a set of  $k$  independent edges can be made the chords of  $k$  vertex disjoint chorded cycles, and finally, how many chords we should be hoping to find.

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**MS16****Vertex-Disjoint Theta Subgraphs**

A theta graph is a graph consisting of three internally disjoint paths joining the same two vertices. In 2009, Chiba et al. proved that every graph with minimum degree at least  $2k + 1$  and order  $\gg k$  contains  $k$  vertex-disjoint theta subgraphs. In this talk, we consider a minimum degree condition for a graph to contain  $k$  vertex-disjoint *isomorphic* theta subgraphs. The result implies the existence of  $k$  vertex-disjoint cycles of the same even length.

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**MS16****Halin Graphs and Generalized Halin Graphs**

A *Halin graph* is a plane graph  $H = T \cup C$  such that  $T$  is a spanning tree of  $H$  with no vertices of degree 2 where  $|T| \geq 4$  and  $C$  is a cycle whose vertex set is the set of leaves of  $T$ . It is known that Halin graphs satisfy a lot of nice properties. So, it is a natural question whether relaxations of Halin graphs preserve such properties or not. Now we define a *generalized Halin graph* which is a graph  $H = T \cup C$  such that  $T$  is a spanning tree of  $H$  with no vertices of degree 2 where  $|T| \geq 4$  and  $C$  is a cycle whose vertex set is the set of leaves of  $T$  (note that some generalized Halin graphs may not be plane graphs). In this talk, we introduce some known results on Halin graphs and generalized Halin graphs. After that, we investigate difference between Halin graphs and generalized Halin graphs.

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**MS17****Large Forbidden Configurations**

Define a matrix  $A$  to be simple if it is a  $(0,1)$ -matrix with no repeated columns. Given a matrix  $F$ , we say  $A$  has no *configuration*  $F$  if there is no submatrix of  $A$  which is a row and column permutation of  $F$ . Our extremal problem, given  $m$  and  $F$ , is to determine an upper bound  $\text{forb}(m, F)$  on the

number of columns in an  $m$ -rowed simple matrix which has no configuration  $F$ . For this talk we consider forbidding a configuration  $F(m)$ , a configuration that grows with  $m$ . Some design theory questions arise.

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#### MS17

##### The Manickam-Miklós-Singhi Conjectures for Sets and Vector Spaces

More than twenty-five years ago, Manickam, Miklós, and Singhi conjectured that for positive integers  $n, k$  with  $n \geq 4k$ , every set of  $n$  real numbers with nonnegative sum has at least  $\binom{n-1}{k-1}$   $k$ -element subsets whose sum is also nonnegative. We verify this conjecture when  $n \geq 8k^2$ , which simultaneously improves and simplifies a bound of Alon, Huang, and Sudakov and also a bound of Pokrovskiy when  $k < 10^{45}$ . Moreover, our arguments resolve the vector space analogue of this conjecture.

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#### MS17

##### Maximizing the Number of Nonnegative Subsets

Given a set of  $n$  real numbers, if the sum of elements of every subset of size larger than  $k$  is negative, what is the maximum number of subsets of nonnegative sum? In this talk we will show that the answer is  $\binom{n-1}{k-1} + \dots + \binom{n-1}{0} + 1$  by establishing and applying a weighted version of Hall's Theorem. This settles a problem of Tsukerman. Joint work with Noga Alon.

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#### MS17

##### Turan Problem for Hypergraph Forests

The Turan number  $ex_r(n, H)$  of an  $r$ -uniform hypergraph  $H$  is the largest size of an  $r$ -uniform hypergraph on  $n$  vertices that does not contain  $H$  as a subgraph. A hypergraph is a hypergraph forest if its edges can be ordered as  $E_1, \dots, E_m$  such that for any  $i > 1$  there exists  $a(i) < i$  such that  $E_i \cap (\bigcup_{j < i} E_j) \subseteq E_{a(i)}$ . A subgraph of a hypergraph forest is called a partial forest. Many results in extremal set theory are Turan results on very special hypergraph forests or partial forests. Here, we establish a general result: For all  $r \geq q + 2 \geq 4$ , if  $F$  is a partial forest of edge sizes at most  $q$  and  $H$  is obtained from  $F$  by enlarging its edges to  $r$ -sets using disjoint sets of new vertices then  $ex_r(n, H) = (\sigma(H) - 1) \binom{n}{r-1} + O(n^{r-2})$ , where

$\sigma(H)$  is the smallest size of a set of vertices that meets each edge of  $H$  in exactly one vertex. We also determine the exact values of  $ex_r(n, H)$  for all large  $n$  for certain so-called  $\sigma$ -critical  $H$ . Our results generalize many recent asymptotic or exact results on the topic. The work is joint with Z. Füredi. We also survey related results and pose some questions.

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#### MS17

##### Turán Numbers for Bipartite Graphs Plus Odd Cycles

In this talk, I will discuss a general approach to a class of extremal problems for bipartite graphs. Specifically, Erdős and Simonovits made three important conjectures concerning the asymptotic behavior of the extremal number for a bipartite graph containing a cycle. One of the conjectures states that if  $\mathcal{C}_k$  is the set of odd cycles of length less than  $k$ , then for every bipartite graph  $F$  containing a cycle, the extremal  $\mathcal{C}_k$ -free  $F$ -free graphs should have asymptotically as many edges as an extremal  $F$ -free bipartite graph. We prove this conjecture for a wide class of graphs  $F$ , in addition exhibiting a “near-bipartite” structure of extremal graphs. The proof depends strongly on the existence of an exponent for a bipartite graph  $F$ , which constitutes another of the three conjectures of Erdős and Simonovits.

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#### MS18

##### Sudoku, Latin Squares, and Defining Sets in Graph Coloring

Over the last decade, Sudoku, a combinatorial number-placement puzzle, has become a favorite pastime of many all around the world. In this puzzle, the task is to complete a partially filled 9 by 9 square with numbers 1 through 9, subject to the constraint that each number must appear once in each row, each column, and each of the nine 3 by 3 blocks. As it turns out, this is very similar to the notion of critical sets for Latin squares, or more generally, defining sets for graph colorings. In this talk, we discuss this connection and present a number of new results and open problems for Sudoku squares.

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#### MS18

##### Defining Sets in Graph Coloring

In a given graph  $G$ , a set of vertices  $S$  with an assignment of colors is called a defining set (of a  $k$ -coloring), if there exists a unique extension of the colors of  $S$  to a proper  $k$ -coloring of the vertices of  $G$ . The minimum cardinality between all defining sets is denoted by  $d(G, k)$ . Defining sets are defined and discussed for many concepts and parameters in graph theory and combinatorics. For example in Latin squares a critical set is a partial Latin square that has a unique completion to a Latin square of order  $n$ , the interest is to find the size of the smallest critical

set. The following conjecture (1995), is still open: For any  $n$ ,  $d(K_n \times K_n, n) = \lfloor n^2/4 \rfloor$ . Defining sets in graph coloring are closely related with the idea of “uniquely  $k$ -list colorable graphs”. In this talk we mention these concepts in different areas and introduce some more open problems.

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**MS18**

### Coloring Maps on Surfaces

Albertson conjectured every surface  $F^2$  admits an integer  $N(F^2)$  such that if  $G \subset F^2$ , then  $G$  admits some  $S \subset V(G)$  with  $|S| \leq N(F^2)$  such that  $G - S$  is 4-colorable. In my talk, as a variation of it, we prove that every locally planar graph  $G$  on an orientable surface has a 5-coloring such that the cardinality of the smallest color class is bounded by  $\varepsilon|V(G)|$ . Moreover, we prove similar results for some classes of maps on orientable surfaces.

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**MS18**

### Distinguishing Colorings of 3-regular Maps on Closed Surfaces

A  $k$ -coloring of a map on a closed surface is said to be *distinguishing* if there is no automorphism of the map other than the identity map which preserves the colors. In particular, if a map has a  $k$ -coloring which uses color  $k$  at most once, then it is said to be *nearly distinguishing*  $(k-1)$ -colorable. We shall show that any 3-regular map on a closed surface is nearly distinguishing 3-colorable unless it is one of three exceptions.

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**MS18**

### Small Snarks and 6-chromatic Triangulations on the Klein Bottle

We take a dual approach to the embedding of snarks on the Klein bottle, and investigate edge-colorings of 6-chromatic triangulations of the Klein bottle. In the process, we discover the smallest snarks that embed polyhedrally on the Klein bottle. Additionally, we find that every triangulation containing certain 6-critical graphs on the Klein bottle must have a Grünbaum coloring and thus cannot admit a dual embedded snark.

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**MS19**

### Properly Coloured Hamilton Cycles in Edge-coloured Complete Graphs

Let  $K_n^c$  be an edge-coloured complete graph on  $n$  vertices.

Let  $\Delta_{\text{mon}}(K_n^c)$  denote the maximum number of edges of the same colour incident with a vertex of  $K_n^c$ . In 1976, Bollobás and Erdős conjectured that every  $K_n^c$  with  $\Delta_{\text{mon}}(K_n^c) < \lfloor n/2 \rfloor$  contains a properly coloured Hamilton cycle, that is, a spanning cycle in which adjacent edges have distinct colours. In this talk, we show that for any  $\varepsilon > 0$  and for all  $n \geq n_0(\varepsilon)$ ,  $\Delta_{\text{mon}}(K_n^c) < (1/2 - \varepsilon)n$  is sufficient. Hence the conjecture of Bollobás and Erdős is true asymptotically.

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**MS19**

### Decomposition of Random Graphs into Complete Bipartite Graphs

For a graph  $G$ , the bipartition number  $\tau(G)$  is the minimum number of complete bipartite subgraphs whose edge sets partition the edge set of  $G$ . In 1971, Graham and Pollak proved that  $\tau(K_n) = n - 1$ . For a graph  $G$  with  $n$  vertices, one can show  $\tau(G) \leq n - \alpha(G)$  easily, where  $\alpha(G)$  is the independence number of  $G$ . Erdős conjectured that almost all graphs  $G$  with  $n$  vertices satisfy  $\tau(G) = n - \alpha(G)$ . In this talk, we present upper and lower bounds for  $\tau(G(n, p))$  which gives support to Erdős' conjecture.

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**MS19**

### A Blow-up Lemma for Sparse Pseudorandom Graphs

We present a new blow-up lemma for spanning graphs with bounded maximum degree in sparse pseudorandom graphs and discuss its possible applications.

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**MS19**

### The Typical Structure of Sparse $H$ -free Graphs

Two central topics of study in combinatorics are the so-called evolution of random graphs, introduced by the seminal work of Erdős and Rényi, and the properties of  $H$ -free

graphs, that is, graphs which do not contain a subgraph isomorphic to a given (usually small) graph  $H$ . A widely studied problem that lies at the interface of these two areas is that of determining how the structure of a typical (random)  $H$ -free graph with  $n$  vertices and  $m$  edges changes as  $m$  grows from 0 to  $\text{ex}(n, H)$ . We first show how a recent result of Balogh, Morris, and the speaker, proved independently by Saxton and Thomason, allows one to derive an approximate structural description of a typical  $H$ -free graph for each non-bipartite  $H$ . Then, extending classical results of Kolaitis, Osthus, Prömel, Rothschild, Steger, and Taraz, we show that in several interesting cases, such as when  $H$  is a clique, these approximate structural descriptions may be made precise.

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## MS19

### The Structure of Digraphs not Containing a Fixed Subgraph

We count the number of digraphs and oriented graphs not containing a fixed digraph  $H$ . The corresponding question has been studied extensively for graphs and hypergraphs. Our proof uses the hypergraph container method developed by Saxton and Thomason and independently by Balogh, Morris, and Samotij. Moreover, we describe the structure of typical  $H$ -free digraphs and oriented graphs for certain  $H$ . This answers two questions of Cherlin.

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## MS20

### Size Conditions for Rainbow Matchings

Let  $\mathcal{E} = (E_i \mid 1 \leq i \leq k)$  be a collection of graphs (or hypergraphs). A choice of disjoint edges, one from each  $E_i$ , is called a *rainbow matching* for  $\mathcal{E}$ . In this talk I will discuss some recent results pertaining to rainbow matchings. In particular, I will focus on size conditions for the  $E_i$ 's such that there exists a rainbow matching for  $\mathcal{E}$ .

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## MS20

### A Refinement of the Grid Theorem

A  $k$ -connected set of vertices in a graph  $G$  is a set  $X$  of vertices in  $G$  such that if  $X_1, X_2 \subseteq X$  then  $G$  contains  $\min\{|X_1|, |X_2|, k\}$  vertex-disjoint paths between  $X_1$  and  $X_2$ . The unavoidable minors for graphs containing a large  $k$ -connected set will be presented. In particular, for graphs containing a large, highly connected set, all of the unavoidable minors contain a large grid-minor, which implies the grid theorem of Robertson and Seymour. This is joint work with Jim Geelen.

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## MS20

### Packing Edge-Disjoint Odd S-Cycles in 4-Edge-Connected Graphs

It has been known that for a 4-edge-connected graph  $G$ , the Erdős-Pósa property holds for edge-disjoint odd cycles, and there is a simple polynomial-time algorithm to test whether or not  $G$  has  $k$  edge-disjoint odd cycles for fixed  $k$ . We generalize these results to the problem with parity constraints.

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## MS20

### Unavoidable Vertex-minors in Large Prime Graphs

A graph is *prime* (with respect to the split decomposition) if its vertex set does not admit a partition  $(A, B)$  (called a *split*) with  $|A|, |B| \geq 2$  such that the set of edges joining  $A$  and  $B$  induces a complete bipartite graph. We prove that for each  $n$ , there exists  $N$  such that every prime graph on at least  $N$  vertices contains a vertex-minor isomorphic to either a cycle of length  $n$  or a graph consisting of two disjoint cliques of size  $n$  joined by a matching.

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## MS20

### Packing $A$ -paths with Specified Endpoints

Consider a graph  $G$  and a specified subset  $A$  of vertices. An  $A$ -path is a path with both ends in  $A$  and no internal vertex in  $A$ . Given a list of  $\{(s_i, t_i) : 1 \leq i \leq k\}$  of pairs of vertices in  $A$ , consider the question of whether there exist many pairwise disjoint  $A$ -paths  $P_1, \dots, P_t$  such that for all

$j$ , the ends of  $P_j$  are equal to  $s_i$  and  $t_i$  for some value  $i$ . This generalizes the disjoint paths problem and is  $NP$ -hard if  $k$  is not fixed. We further restrict the question, and ask if there either exist  $t$  pairwise disjoint such  $A$ -paths or alternatively, a bounded set of  $f(t)$  vertices intersecting all such paths. In general, there exist examples where no such function  $f(t)$  exists; we present an exact characterization of when such a function exists.

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**MS21**

### Resolvable Or Near Resolvable Designs in Multiple Rooms

I will discuss the existence of RBIBDs or NRBs which are  $L$ -colorable in the following sense. Let  $D = (V, \mathcal{B}, \mathcal{R})$  be a  $(v, k, \lambda)$ -RBIBD (or NRB), let  $t$  be a divisor of  $\frac{v}{k}$  (or  $\frac{v-1}{k}$ ) and let  $L$  be a list of  $t$  non-negative integers summing up to  $\lambda$ . I say that  $D$  is  $L$ -colorable if it is possible to color its blocks with  $t$  colors  $1, \dots, t$  in such a way that:

- every parallel (or near parallel) class has exactly  $\frac{v}{kt}$  (or  $\frac{v-1}{kt}$ ) blocks colored  $i$  for  $1 \leq i \leq t$ ;
- if  $c_i(x, y)$  is the number of blocks colored  $i$  through  $x$  and  $y$ , then the list  $\{c_1(x, y), \dots, c_t(x, y)\}$  is equal to  $L$  for every pair of distinct points  $x$  and  $y$ .

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**MS21**

### A Three-Factor Product Construction for Mutually Orthogonal Latin Squares

It is well known that mutually orthogonal latin squares, or MOLS, admit a (Kronecker) product construction. I will discuss how, under mild conditions, 'triple products' of MOLS can result in a gain of one square. This observation leads to a few improvements to the MOLS table and a slight strengthening of the famous theorem of MacNeish. This is joint work with Alan C.H. Ling and hinges on an important construction of Rolf Rees.

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**MS21**

### Vertex-transitive Graphs of Prime-squared Order are Hamilton-decomposable

We prove that all connected vertex-transitive graphs of order  $p^2$ ,  $p$  a prime, can be decomposed into Hamilton cycles. (Joint work with Brian Alspach and Darryn Bryant.)

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**MS21**

### How to Squash a 6-Cycle System into a Steiner Triple System

The spectra for Steiner triple systems and 6-cycle systems agree when  $n = 1$  or  $9 \pmod{12}$ . Let  $(X, C)$  be a 6-cycle system of order  $n = 1$  or  $9 \pmod{12}$ . Let  $T$  be a collection of bowties obtained by squashing each 6-cycle of  $C$  into a bowtie (i.e. identifying two 'opposite' vertices of the 6-cycle). If  $(X, T)$  is a Steiner triple system we say that the 6-cycle system  $(X, C)$  is squashed into the Steiner triple system  $(X, T)$ . In this talk we construct, for every  $n = 1$  or  $9 \pmod{12}$  a 6-cycle system that can be squashed into a Steiner triple system. (This is joint work with Mariusz Meszka and Alex Rosa.)

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**MS21**

### On Large Sets of Combinatorial Objects

Resolutions of  $t$ -designs were studied as early as 1847 by Reverend T. P. Kirkman who proposed the famous 15 schoolgirls problem. In this talk we discuss *large sets*  $LS[N](t, k, v)$  of simple  $t - (v, k, \lambda)$  designs, large sets  $LS_q[N](t, k, n)$  of geometric  $t$ -designs in linear spaces of dimension  $n$  over  $GF(q)$ , *orthogonal large sets* and *large sets of large sets*. We present some recent results which use groups to extend previous results, and pose open problems.

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**MS22**

### Finitely Forcible Graphons

In extremal graph theory, we often consider large graphs that are in the limit uniquely determined by finitely many densities of their subgraphs. The corresponding limits (so-called graphons) are called finitely forcible. Motivated by classical results in extremal combinatorics as well as by recent developments in the study of finitely forcible graphons, Lovasz and Szegedy made some conjectures about the structure of such graphons. In particular, they conjectured that the topological space of typical points of every finitely forcible graphon is compact and finitely dimensional. In this talk we give a brief overview about the corresponding results, and in particular show that these two conjectures are false.

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**MS22**

### Estimating the Distance from Testable Properties

Fischer and Newman showed that it is possible to test the distance of a graph from every testable graph property. In this talk we simplify their test, and furthermore generalize it the algebraic setting of affine-invariant properties on

finite fields. This is based on a joint work with Shachar Lovett.

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## MS22

### The Inducibility of Directed Graphs

In modern extremal combinatorics, a substantial number of problems study the asymptotic relations between densities of subgraphs. One particularly interesting topic called inducibility is to find the maximum possible induced density of a given subgraph. We study this problem in the digraph setting, and solve the inducibilities for all the complete bipartite digraphs. Our result confirms a conjecture by Falgas-Ravry and Vaughan, and provides the first known explicit instance of density problems for which one can prove the extremality of an iterated blow-up construction.

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## MS22

### First Order Limits

The notion of first order convergence and the corresponding limit object, modelling, were proposed to bridge the realms of dense and sparse graph limits. In this talk, we recall the notions and we will present positive and negative results we have obtained on the existence of modelings for some classes of graphs and matroids.

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## MS22

### Rainbow Triangles in Three-colored Graphs

Erdős and Sós proposed a problem of maximizing the number  $F(n)$  of rainbow triangles in 3-edge-colored complete graphs on  $n$  vertices. They conjectured that  $F(n) = F(a) + F(b) + F(c) + F(d) + abc + abd + acd + bcd$ , where  $a + b + c + d = n$  and  $a, b, c, d$  are as equal as possible and  $F(0) = 0$ . We prove that the conjectured recurrence

holds for sufficiently large  $n$ . We also prove the conjecture if  $n = 4^k$  for all  $k \geq 0$ . These results imply that  $\lim \frac{F(n)}{\binom{n}{3}} = 0.4$ , and determine the unique limit object. In the proof we use flag algebras combined with stability arguments.

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## MS23

### Extending Partial Representations of Circle Graphs

A circle graph  $G$  is an intersection graph of chords of a circle. We study the algorithmic question of whether one can extend a given representation  $R'$  of an induced subgraph of  $G$  to a representation  $R$  of the entire  $G$ . We give a polynomial-time algorithm for this problem. To show this, we describe the structure of all representations of a circle graph based on split decomposition.

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## MS23

### Contact Representations of Planar Graph: Rebuilding Is Hard

Planar graphs have geometric representations of various types, e.g. as contacts of disks, triangles or - in the bipartite case - vertical and horizontal segments. We show that in each of these cases, the problem of completing a

partial representation becomes NP-hard. We also give two polynomial time algorithms for the grid contact case. The first one when all vertical segments are pre-represented and the second is when the vertical segments have only their x-coordinates specified.

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**MS23**

**The Recognition of Simple-Triangle Graphs and of Linear-Interval Orders is Polynomial**

Intersection graphs of geometric objects have been extensively studied, both due to their interesting structure and their numerous applications; prominent examples include interval graphs and permutation graphs. In this paper we study a natural graph class that generalizes both interval and permutation graphs, namely *simple-triangle* graphs. Simple-triangle graphs - also known as *PI* graphs (for Point-Interval) - are the intersection graphs of triangles that are defined by a point on a line  $L_1$  and an interval on a parallel line  $L_2$ . They lie naturally between permutation and trapezoid graphs, which are the intersection graphs of line segments between  $L_1$  and  $L_2$  and of trapezoids between  $L_1$  and  $L_2$ , respectively. Although various efficient recognition algorithms for permutation and trapezoid graphs are well known to exist, the recognition of simple-triangle graphs has remained an open problem since their introduction by Corneil and Kamula three decades ago. In this paper we resolve this problem by proving that simple-triangle graphs can be recognized in polynomial time. As a consequence, our algorithm also solves a longstanding open problem in the area of partial orders, namely the recognition of *linear-interval orders*, i.e. of partial orders  $P = P_1 \cap P_2$ , where  $P_1$  is a linear order and  $P_2$  is an interval order. This is one of the first results on recognizing partial orders  $P$  that are the intersection of orders from two different classes  $\mathcal{P}_1$  and  $\mathcal{P}_2$ . In complete contrast to this, partial orders  $P$  which are the intersection of orders from the same class  $\mathcal{P}$  have been extensively investigated, and in most cases the complexity status of these recognition problems has been already established.

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**MS23**

**Picking Planar Edges; Or, Drawing a Graph with a Planar Subgraph**

Given a graph  $G$  and a subset  $F \subseteq E(G)$  of its edges, is there a drawing of  $G$  in which all edges of  $F$  are free of crossings? We show that this question can be solved in polynomial time using a Hanani-Tutte style approach. If we require the drawing of  $G$  to be straight-line, and allow at most one crossing along each edge in  $F$ , the problem turns out to be as hard as the existential theory of the real numbers.

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**MS23**

**Coloring Geometric Intersection Graphs via on-line Games**

Game graphs are graphs encoding presentation strategies in on-line graph coloring games. The chromatic number of a game graph is equal to the number of colors that any on-line coloring algorithm is forced to use playing against the corresponding strategy. I will present how game graphs of appropriately defined games can be used in proving lower and upper bounds on the maximum chromatic number in classes of intersection graphs of geometric objects.

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**MS24**

**Pulling Self-avoiding Walks from an Adsorbing Plane**

In this talk a self-avoiding walk model of a polymer pulled from an adsorbing surface by a vertically applied force is considered. The self-avoiding walk is fixed in the surface at its one endpoint, and pulled at the other endpoint. I will show that the appropriate thermodynamic limit in this model exists and that the free energy is convex. An expression for the free energy in terms of the free energies of pulled and adsorbing self-avoiding walks will be given, and it will be shown that there is a phase boundary between an adsorbed phase and a ballistic phase in the model. Some qualitative properties of this phase boundary will be given, and some bounds on the location of the boundary will be shown. This work was done in collaboration with SG Whittington.

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**MS24**

**The Systematic Application of Analytic Combinatorics of Several Variables to Lattice Path Enumer-**

**ation Problems**

This work combines recent work on lattice path enumeration using the orbit sum method and the analysis of multivariate generating functions and determines new asymptotic formulas for d-dimensional lattice models with particular symmetries. We determine the dominant asymptotics for the number of walks restricted to the positive orthant taking unit steps which move positively in each coordinate and are symmetric with respect to each axis. These expressions are derived by analyzing the singular variety of a multivariate rational function whose diagonal counts the lattice path models in question. (Collaboration with Stephen Melczer (SFU))

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**MS24****On the Number of Walks in a Triangular Domain**

We consider walks on a triangular domain that is a subset of the triangular lattice. We then specialize this by dividing the lattice into two directed sub-lattices with different weights. Our central result is an explicit formula for the generating function of walks starting at a fixed point in this domain and ending anywhere within the domain. Intriguingly, the vspecialization of this formula to walks starting in a fixed corner of the triangle shows that these are equinumerous to two-colored Motzkin paths, and two-coloured three-candidate Ballot paths, in a strip of finite height.

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**MS24****Random Knots and Polymer Models**

This talk will survey some of the results on properties of random knots in 3-space and in confined volumes, with applications to enzyme action on duplex DNA and the structure and dynamics of duplex DNA confined to viral capsids.

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**MS24****Numerical Approach of Two Friendly Walks in a Sticky Slab**

In previous work we derived an exact solution of two friendly directed walks above a sticky wall with single and double interactions. This models the behaviour of a pair of attracting polymers above an adsorbing surface. In this talk, we extend this to two friendly directed walks in a sticky slab. I'll discuss some difficulties encountered with the analytic approach, the results from numerical analysis, and then compare these results with similar models anal-

ysed previously.

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**MS25****Symmetric Chain Decompositions of Quotients of Products**

Given a subgroup  $G$  of the symmetric group  $S_n$  on  $[n]$  and the Boolean lattice of all subsets of  $[n]$ , the quotient  $B_n/G$  has the ordering induced on the orbits under  $G$ . It is conjectured that  $B_n/G$  has a symmetric chain decomposition [SCD]. Earlier results are extended to  $G$  constructed from wreath products and applied to powers  $P^n$  of any poset with an SCD. Several questions involving abelian subgroups of  $G$  are raised.

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**MS25****On the Dimension of Posets with Cover Graphs of Treewidth 2**

Trotter and Moore proved that a poset has dimension at most 3 whenever its cover graph is a forest, or equivalently, has treewidth at most 1. On the other hand, a well-known construction of Kelly shows that there are posets of arbitrarily large dimension whose cover graphs have treewidth 3. The focus of this talk is the boundary case of treewidth 2. It was recently shown that the dimension is bounded if the cover graph is outerplanar (Felsner, Trotter, and Wiechert) or if it has pathwidth 2 (Biró, Keller, and Young). This can be interpreted as evidence that the dimension should be bounded more generally when the cover graph has treewidth 2. We show that it is indeed the case: Every such poset has dimension at most 2554.

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## MS25

### Forbidden Structures in the Boolean Lattice

The Lubell function measures a set of points  $\mathcal{F}$  in the  $n$ -dimensional Boolean lattice by the expected number of times that a random, full chain meets  $\mathcal{F}$ . We use the Lubell function to prove Turán-type results on the maximum size of a set  $\mathcal{F}$  that avoids certain structures, such as Boolean algebras and induced posets of height 2. Joint with J. T. Johnston and Linyuan Lu.

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## MS25

### Some Recent Results about Cross Intersecting Families

A family of subsets is called intersecting if any two members of the family have non-empty intersection. Suppose that each subset has a given weight, and we want to find an intersecting family with the maximum total weight. The Erdős–Ko–Rado theorem is a typical example. I will present some new results such as the exact bound for the product sizes of cross  $t$ -intersecting  $k$ -uniform families. I will also present two very different tools: one uses a random walk and the other uses graph spectra.

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## MS25

### Planar Posets and Minimal Elements

Streib and Trotter showed that the dimension of a planar poset is bounded in terms of its height. Answering a question posed by R. Stanley, we show that the dimension of a planar poset with  $t$  minimal elements is at most  $2t + 1$ . This is tight for  $t = 1$  and  $t = 2$ . For larger  $t$ , we have a lower bound of  $t + 3$ .

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## MS26

### The Random Greedy Algorithm for Independent Sets in Hypergraphs

Let  $r$  be a fixed constant and let  $\mathcal{H}$  be an  $r$ -uniform,  $D$ -regular hypergraph on  $N$  vertices. Consider an algorithm which forms an independent set  $I$  by iteratively inserting

random vertices. At each step we insert a vertex chosen uniformly at random from the collection of vertices  $v$  for which  $I \cup \{v\}$  is independent. This process terminates at a maximal independent set. We give a nontrivial lower bound on the size of the final independent set, assuming  $\mathcal{H}$  satisfies certain degree and codegree conditions. We show that the algorithm produces objects of interest in additive combinatorics, and gives new lower bounds on the Turán numbers of certain hypergraphs.

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## MS26

### Juggling Card Sequences

One of the many ways to represent juggling patterns is through the use of so-called juggling cards. These are templates which indicate at each time step which balls are to be thrown to which heights. In this talk, we describe a number of new combinatorial and probabilistic results (as well as some unsolved problems) arising in the study of these objects.

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## MS26

### Extremal Graphs for Connectedness

It is known that the topological connectedness of the independence complex of a line graph  $L(G)$  is bounded below by  $\nu(G)/2 - 2$ , where  $\nu(G)$  denotes the matching number of  $G$ . We study graphs  $G$  for which this parameter is close to  $\nu(G)/2 - 2$ , and describe the consequences of our work for some long-standing conjectures about hypergraphs.

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## MS26

### Hamiltonian Increasing Paths in Random Edge Orderings

If the edges of  $K_n$  are totally ordered, a simple path whose edges are in ascending order is called *increasing*. The worst-case length of the longest increasing path has remained an open problem for several decades, with asymptotic bounds between  $\sqrt{n}$  (Graham and Kleitman, 1973) and  $\frac{1}{2}n$  (Calderbank, Chung, and Sturtevant, 1984). We consider the average case. Surprisingly, if a uniformly random ordering is chosen, a Hamiltonian increasing path exists with probability at least  $\frac{1}{e}$ .

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### MS26

#### Extremal Problems on Diameter 2-critical Graphs

A graph is called diameter 2-critical if its diameter is 2 and the deletion of any edge would strictly increase the diameter. In 1979, Murty and Simon conjectured that if  $G$  is a diameter 2-critical graph with  $n$  vertices and  $m$  edges, then  $m$  is no more than  $n^2/4$ , with equality if and only if  $G$  is the complete bipartite graph  $K_{n/2, n/2}$ . In the same year, Caccetta and Haggkvist made the following conjecture for diameter 2-critical graphs, which, if true, would imply the Murty-Simon conjecture: the sum of squares of all vertex degrees is at most  $nm$ . We present some results on these two conjectures, as well as their related problems. Joint work with Po-Shen Loh.

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### MS27

#### Treewidth-destroying Partitions of Graphs

Planar graphs (and more generally, graphs avoiding a fixed minor) have the following property, useful in design of approximation algorithms: For every  $k > 0$ , there exists  $d > 0$  such that each such graph can be partitioned to  $k$  parts so that removal of any of the parts reduces the tree-width to at most  $d$ . We study the properties of graph classes with this property, as well as its variations.

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### MS27

#### Inequivalent Representations of Highly Connected Matroids

We prove that for each fixed finite field  $\mathbb{F}$ , every sufficiently connected matroid has only a bounded number of inequivalent  $\mathbb{F}$ -representations. We obtain this result via a chain theorem for highly connected matroids, which as far as we know is new even when restricted to graphs. This theorem will be used in the recently announced proof of Rota's conjecture by Geelen, Gerards and Whittle.

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### MS27

#### Well-quasi-ordering Graphs by the Topological Minor Relation

Robertson and Seymour proved that graphs are well-quasi-ordered by the minor relation and the weak immersion relation in the prominent Graph Minors series. That is, given infinitely many graphs, one graph contains another as a minor (or a weak immersion, respectively). However, the topological minor relation does not well-quasi-order graphs. An old conjecture of Robertson states that for every integer  $k$ , graphs with no topological minor isomorphic to the graph obtained from a path of length  $k$  by doubling every edge are well-quasi-ordered by the topological minor relation. We will sketch our recent proof of this conjecture in this talk. This work is joint with Robin Thomas.

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### MS27

#### Non-planar Extensions of Subdivisions of Planar Graphs

*Almost 4-connectivity* is a weakening of 4-connectivity which allows for vertices of degree three. In this talk I will describe generalizations and applications of the following theorem. Let  $G$  be an almost 4-connected triangle-free planar graph, and let  $H$  be an almost 4-connected non-planar graph such that  $H$  has a subgraph isomorphic to a subdivision of  $G$ . Then there exists a graph  $G'$  such that  $G'$  is isomorphic to a minor of  $H$ , and either

- (i)  $G' = G + uv$  for some vertices  $u, v \in V(G)$  such that no facial cycle of  $G$  contains both  $u$  and  $v$ , or
- (ii)  $G' = G + u_1v_1 + u_2v_2$  for some distinct vertices  $u_1, u_2, v_1, v_2 \in V(G)$  such that  $u_1, u_2, v_1, v_2$  appear on some facial cycle of  $G$  in the order listed.

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### MS27

#### Spanning Trees with Vertices having Large Degrees

Let  $G$  be a connected graph, and let  $f$  be a mapping from  $V(G)$  to  $Z_{>0}$ , where  $Z_{>0}$  is the set of positive integers. The purpose of this talk is to consider the existence of a spanning tree  $T$  such that for each vertex  $v$ ,

$$f(v) \deg_T(v),$$

where  $\deg_T(v)$  is the degree of  $v$  in  $T$ . Note that when  $f(v) = 2$  for all but two vertices  $v$ , a spanning tree satisfying the above condition corresponds to a Hamiltonian path with prescribed ends in a given graph. In this talk, we will concentrate on necessary conditions that are almost necessary conditions.

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## MS28

### Introduction to Hardness of Approximation

Many computational problems of interest are NP-hard. An extensively studied approach to cope with NP-hardness is to design efficient algorithms that compute approximately optimal solutions, with a guarantee on the quality of approximation so achieved. Surprisingly, different problems behave very differently in terms of how well they can be approximated. The discovery of the PCP Theorem in early nineties showed that there are inherent reasons for this varied behavior and the apparent hardness of approximation for some problems. Subsequent developments have led to a much better understanding of this topic, with several challenges ahead. This talk will give an overview of these developments, as a preparation towards the follow-up talks in the mini-symposium.

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## MS28

### Toward PCPs with Minimal Error

The Sliding Scale Conjecture in PCP states that there are PCP verifiers with a constant number of queries and soundness error that is exponentially small in the randomness of the verifier and the length of the prover's answers. The Sliding Scale Conjecture is one of the oldest open problems in PCP, and it implies hardness of approximation up to polynomial factors for problems like Max-CSP (with polynomial-sized alphabet), Directed-Sparsest-Cut and Directed-Multi-Cut. A strengthening of the conjecture to projection games has further applications, e.g., to hardness of approximation up to polynomial factors for the Closest-Vector-Problem in lattices and to exploring the threshold behavior of CSPs. In this talk we discuss the conjecture and approaches to proving it.

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## MS28

### Unique Games Conjecture and Small Set Expansion

A small set expander is a graph where every set of sufficiently small size has near perfect edge expansion. This talk concerns the computational problem of distinguishing a small set-expander, from a graph containing a small non-expanding set of vertices. This problem henceforth re-

ferred to as the Small-Set Expansion problem has proven to be intimately connected to the Unique Games Conjecture conjecture. In this talk, we survey the following recent developments in connection to the small set expansion problem:

- Higher order Cheeger inequalities and their use in approximating small-set expansion.
- Efficacy of SDP hierarchies for combinatorial optimization problems on small-set expanders.
- Complexity connections between small set expansion and unique games conjecture.

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## MS28

### Analytical Approach to Parallel Repetition

We propose an analytical framework for studying parallel repetition, a basic product operation for one-round two-player games. In this framework, we consider a relaxation of the value of projection games. We show that this relaxation is multiplicative with respect to parallel repetition and that it provides a good approximation to the game value. Based on this relaxation, we prove the following parallel repetition bound: For every projection game  $G$  with value at most  $\rho$ , the  $k$ -fold parallel repetition  $G^{\otimes k}$  has value at most

$$\text{val}(G^{\otimes k}) \leq \left( \frac{2\sqrt{\rho}}{1+\rho} \right)^{k/2}.$$

This statement implies a parallel repetition bound for projection games with low value  $\rho$ . Previously, it was not known whether parallel repetition decreases the value of such games. This result implies stronger inapproximability bounds for SET COVER and LABEL COVER. In this framework, we also show improved bounds for few parallel repetitions of projection games, showing that Raz's counterexample is tight even for a small number of repetitions. Finally, we also give a short proof for the NP-hardness of LABEL COVER(1,  $\delta$ ) for all  $\delta > 0$ , starting from the basic PCP theorem.

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## MS28

### A Characterization of Strong Approximation Resistance

For a predicate  $f : -1, 1^k \mapsto 0, 1$  with  $\rho(f) = \frac{|f^{-1}(1)|}{2^k}$ , we call the predicate strongly approximation resistant if given a near-satisfiable instance of CSP( $f$ ), it is computationally hard to find an assignment such that the fraction of constraints satisfied is outside the range  $[\rho(f) - \Omega(1), \rho(f) + \Omega(1)]$ . We present a characterization of strongly approximation resistant predicates under the Unique Games Conjecture. We also present characterizations in the mixed linear and semi-definite programming hierarchy and the Sherali-Adams linear programming hierarchy. In the former case, the characterization coincides with the one based

on UGC. Each of the characterizations is in terms of existence of a probability measure on a natural convex polytope associated with the predicate. Based on joint work with Subhash Khot and Pratik Worah.

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## MS29

### Eigenvalue Stability under Hypermatrix Perturbation and Random Hypergraphs

There has been a recent flurry of interest in the spectral theory of tensors and hypergraphs as new ideas successfully analogize spectral graph theory to hypergraphs. For one natural incarnation – the homogeneous adjacency spectrum – a substantial number of seemingly basic questions about hypergraph spectra remain out of reach. One of the problems that has yet to be resolved is the (asymptotically almost sure) spectrum of a random hypergraph in the Erdős-Rényi sense; another closely related one is that of describing the spectrum of complete hypergraphs (other than a kind of implicit description for the 3-uniform case). We present the requisite background and discuss some progress in this area that involves tools from algebraic geometry, perturbation theory, and large deviations.

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## MS29

### Minimum Rank, Maximum Nullity, and Zero Forcing Number

This talk will survey recent developments in the problem of determining the minimum rank/maximum nullity of symmetric matrices described by a graph, including equality of maximum nullity and zero forcing number for complete subdivision graphs, and Nordhaus-Gaddum type problems for minimum rank and related parameters.

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## MS29

### Numerical Methods for Estimating the Minimum Rank and Zero Forcing Number of Large Graphs

Given a graph  $G$ , the minimum rank of a graph,  $mr(G)$ , is the minimum rank over all symmetric matrices with a sparsity pattern determined by the graph. Calculating the minimum rank and the related zero forcing number of a graph are computationally difficult. We offer a iterative algorithm to estimate these parameters effectively. We show this algorithm is effective for small graphs and apply the algorithm to numerically verify related conjectures for larger graphs.

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## MS29

### Connected Hypergraphs with Small Spectral Radius

In 1970 Smith classified all connected graphs with the spectral radius at most 2. Here the spectral radius of a graph is the largest eigenvalue of its adjacency matrix. Recently, the definition of spectral radius has been extended to  $r$ -uniform hypergraphs. In this paper, we generalize the Smith's theorem to  $r$ -uniform hyper graphs. We show that the smallest limit point of the spectral radii of connected  $r$ -uniform hypergraphs is  $\rho_k = (k-1)! \sqrt[k]{4}$ . We classify all connected  $k$ -uniform hypergraphs with spectral radius at most  $\rho_k$ .

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## MS29

### Nonlinear Eigenvalues in Random Regular Graphs

Given a graph  $G$  and a metric space  $(X, d)$ , one can define the *spectral gap*  $\gamma = \gamma(G, X)$  of  $G$  with respect to  $X$  via a generalization of the Rayleigh quotient under the metric  $d$ . In this way, one can call a graph family  $G_n$  with spectral gaps  $\gamma_n = \gamma(G_n, X)$  an *expander family* with respect to  $X$  if  $\gamma_n$  is bounded away from 0 as  $n \rightarrow \infty$ . We discuss the expansion properties of  $G_{n,d}$  with respect to  $G_{m,d}$  under the standard distance metric, where  $m \ll n$ , and some applications of these concepts.

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## MS30

### Hamiltonicity of 3-connected Planar Graphs with No $K_{2,5}$ Minor

Tutte showed that 4-connected planar graphs are hamiltonian. But examples, such as the Herschel graph, show that this cannot be extended to 3-connected planar graphs without extra conditions. We show that  $K_{2,5}$ -minor-free 3-connected planar graphs are hamiltonian. Results on hamilton paths in outerplanar graphs form an important tool for our arguments. Computational evidence provided by Gordon Royle suggests that it may be feasible to use our methods to characterize  $K_{2,6}$ -minor-free 3-connected nonhamiltonian planar graphs.

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**MS30****Combining Degree and Connectivity Bounds for Graph Linkages**

For  $k \geq 2$ , it is easy to construct  $(2k - 1)$ -connected graphs with minimum degree  $\delta > n/2$  which are not  $k$ -linked. On the other hand, all  $2k$ -connected graphs with minimum degree  $\delta > 10k$  are  $k$ -linked, independently of  $n$ . In this talk we will study similar threshold behaviors for other graph linkages, and discover that this threshold is not always this sharp.

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**MS30****A Spanning Tree Homeomorphic to a Small Tree**

In this talk, we discuss the existence of a spanning tree which is a subdivision of a small tree. A hamiltonian path is a spanning tree which is also a subdivision of  $K_2$ . Inspired by this observation, for a given integer  $k$ , we give a degree sum condition for a connected graph to have a spanning tree which is a subdivision of a tree of order at most  $k$ .

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**MS30****From Hists to Halin Graphs**

As counterpart notions to path, cycle, and hamiltonicity, homeorphically irreducible trees (HIT) and homeomorphically irreducible spanning trees (HIST) are trees with no vertex of degree 2. HITs and HISTs are closely related to a family of graphs called Halin graphs. A Halin graph is obtained from a plane embedding of a HIT of at least 4 vertices by connecting the leaves into a cycle following the cyclic order on the leaves determined by the embedding. In this talk, we will briefly look into some works have been done on HISTs and Halin graphs, and some other intriguing research problems related to HISTs and Halin graphs.

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**MS30****Forbidden Pairs for the Existence of a Spanning Halin Subgraph**

$\{H_1, H_2\}$  is called a forbidden pair of  $G$  if  $G$  contains no induced subgraph isomorphic to  $H_1$  or  $H_2$ . A Halin graph is a plane graph  $H = T \cup C$  consisting of a spanning tree  $T$  without vertices of degree 2 and a cycle  $C$  induced by all leaves of the tree  $T$ . In this paper, we will talk some results about the forbidden pairs for existence of a spanning Halin subgraph in  $G$ .

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**MS31****Modeling Polymer Adsorption at an Inhomogeneous Surface**

We consider several models of polymers interacting with an impenetrable surface. We use both directed and self-avoiding walks to model the polymer configurations and introduce a regular inhomogeneity in the polymer, the surface or both. In each case, we compute the phase diagram and consider the effect of the inhomogeneity on the phase boundary. We give qualitative arguments about the form of the boundary and consider how it relates to the phenomenon of pattern recognition.

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**MS31****Exact Solution of a Simple Adsorption Model of De-naturing DNA**

We consider a DNA strand in a solvent near an attractive surface modelled as two interacting (friendly) directed walks along the square lattice. We establish a functional equation for the corresponding generating function, which is further refined by means of the obstinate kernel method. Specifically, we utilise the kernel method in a novel way to express the exact-solution of the model in terms of two simpler generating functions for the same underlying combinatorial class. The Zeilberger-Gosper algorithm is then utilised to computationally determine linear homogeneous differential equations solved by these simpler generating functions, thereby allowing us to analyse the singularity structure and thus critical behaviour of the model. We deduce the phase diagram for this model, finding that the system exhibits four phases with a special point where they all meet.

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**MS31****The Shape of a Hopf Link**

We examine the geometry of random embeddings of the hopf link. The geometry is characterised by descriptors derived from the radius of gyration tensor, and is compared against similar statistics for a single self-avoiding polygon.

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**MS31****Random Knots and Confinement Considerations**

Chromosomal DNA is tightly packed in all organisms. When the chromosomes are circular, or divided into looped domains, the packing geometry has a direct impact on the topology of the chains. Problems of packing motivate our research on minimal step lattice polygons that are unconfined, or confined to slabs and tubes.

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**MS31****Counting Knotted 2-Spheres in Tubes in  $Z^4$** 

There are many rigorous and numerical results about knotting of simple closed curves in three dimensional lattices but for higher dimensional cases very little is known. If we consider 2-spheres confined to a tube in  $Z^4$ , so that the system is quasi-one dimensional, we show how transfer matrix techniques can be used to prove that all except exponentially few such 2-spheres are knotted. This is joint work with Chris Soteros and De Witt Summers.

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**MS32****The KP Hierarchy and Generating Series for Maps**

In this talk we will introduce and describe the KP hierarchy of partial differential equations and their formal power series solutions. We will then discuss how this can be applied to problems in map enumeration.

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**MS32****A Bijection for Rooted Maps on Non-orientable Surfaces**

One of the major bijective results in the area of enumeration of maps and in the area of studying uniform random maps is the approach initiated by Cori and Vauquelin and extended by Chapuy, Marcus and Shaeffer which treats rooted orientable maps of genus  $g$ . We extend their bijection for rooted maps drawn on a non-orientable surface

of type  $h$ . It allows us to prove several results concerning behaviour of large non-orientable maps as well as enumerative properties of non-orientable maps. This is a joint work with Guillaume Chapuy.

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**MS32****The Brownian Continuum Random Tree Is the Scaling Limit of Random Dissections**

We are interested in the graph structure of large random dissections of polygons sampled according to Boltzmann weights, which encompasses the case of uniform dissections or uniform  $p$ -angulations. As their number of vertices  $n$  goes to infinity, we show that these random graphs, rescaled by  $n^{-1/2}$ , converge in the Gromov–Hausdorff sense towards a multiple of Aldous’ Brownian tree when the weights decrease sufficiently fast. This gives in particular interesting combinatorial consequences concerning the geometry of these random dissections. The scaling constant depends on the Boltzmann weights in a rather amusing and intriguing way, and is computed by making use of a Markov chain which compares the length of geodesics in dissections with the length of geodesics in their dual trees. Joint work with Nicolas Curien and Bndicte Haas.

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**MS32****Exploring Some Non-constructive Map Bijections**

Maps play a sometimes hidden role in many different branches of mathematics. As a result generating series for related classes of maps can often be obtained by several different unrelated techniques. Algebraic relationships between such generating series can guarantee the existence of bijections without providing any constructive way to identify their actions. This phenomenon is particularly prominent when maps are used to study the combinatorics of Jack symmetric functions. In this setting, a shifted version of the Jack parameter can be related to a quantified departure from orientability. Generating series in this context continue to exhibit symmetries that in the restriction to orientable or non-orientable maps are explained by simple bijections, but these bijections cease to provide an explanation in the perturbed setting.

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**MS32****A Graphical Proof of a Generalization of Boccara’s Theorem Concerning the Multiplication of Long**

## Cycles

In 1980, Boccara showed that the number of ways of writing a fixed odd permutation as a product of an  $n$ -cycle and  $n - 1$ -cycle (in the symmetric group on  $n$  symbols) is independent of the permutation chosen. We give a substantial generalization of this result, and summaries of an algebraic and combinatorial proof of it. This is joint work with V. Féray.

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## MS33

### Randomly Coloring Random Graphs

We discuss some problems related to colorings of random graphs: (i) Rainbow matchings and hamilton cycles; (ii) Game chromatic number of random graphs; (iii) Rainbow connection of random graphs; (iv) zebraic colorings.

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## MS33

### A Short Proof of Gowers' Lower Bound for the Regularity Lemma

A celebrated result of Gowers states that for every  $\epsilon > 0$  there is a graph  $G$  such that every  $\epsilon$ -regular partition of  $G$  (in the sense of Szemerédi's regularity lemma) has order given by a tower of exponents of height polynomial in  $1/\epsilon$ . In this note we give a new proof of this result that uses a construction and proof of correctness that are significantly simpler and shorter. Joint work with Asaf Shapira.

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## MS33

### On the Number of Real Roots of Kac Polynomials

Real roots of Kac polynomials have been studied extensively in the literature under various conditions. With minimal assumptions on the coefficient variables and by using purely combinatorial method, we show that the number of real roots is  $(2/\pi) \log n + O(1)$  in expected value. This bound is almost best possible. (Joint with O. Nguyen and V. Vu)

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## MS33

### Turán Function for the Generalized Triangle

The *generalized triangle*  $T_r$  is an  $r$ -graph with edges  $\{1, 2, \dots, r\}$ ,  $\{1, 2, \dots, r-1, r+1\}$  and  $\{r, r+1, \dots, 2r-2\}$ . Frankl and Füredi computed the Turán density of  $T_r$  for  $r = 5, 6$ . We strengthen their result computing the Turán function  $\text{ex}(n, T_r)$  exactly for large  $n$ . The proof is based on a technique for deriving exact results for the Turán function

from "local stability" results, which has other applications.

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## MS33

### Partitioning Random Hypergraphs

I will discuss the algorithmic problem of finding a planted partition in a random  $k$ -uniform hypergraph, a generalization of both the stochastic block model ( $k = 2$ ) and planted random  $k$ -SAT. For  $k \geq 3$ , the problem exhibits an algorithmic gap: for a large range of densities unique solutions exist but no known efficient algorithms can find them. I will present a conjecture on the location of the algorithmic threshold and a proof of it for the class of statistical algorithms. Based on joint work with Vitaly Feldman and Santosh Vempala.

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## MS34

### Phase Transitions in the Laplacian Determinant

We discuss several types of combinatorial objects enumerated by the Laplacian determinant on a planar graph, and in particular indicate various phase transitions in these objects which occur for infinite graphs. As an example, we compute the growth rate of the set of  $k$ -component essential spanning forests on a bi-infinite strip of fixed width.

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## MS34

### The Structure of the Abelian Sandpile

The Abelian sandpile is a discrete diffusion process on configurations of chips on the integer lattice, introduced in 1987 by Bak, Bank and Wiesenfeld. Recent developments have provided a mathematical explanation for the fractal structures produced by this process. We will discuss these results, and some important remaining questions.

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## MS34

### Slow Mixing for the Hard-Core Model on $Z^2$

The hard-core model has attracted much attention across several disciplines, representing lattice gases in statistical physics and independent sets in discrete mathematics and

computer science. On finite graphs, we are given a parameter  $\lambda$ , and an independent set  $I$  arises with probability proportional to  $\lambda^{|I|}$ . We are interested in determining the mixing time of local Markov chains that add or remove a small number of vertices in each step. On finite regions of  $Z^2$  it is conjectured that there is a phase transition at some critical point  $\lambda_c$  that is approximately 3.79. It is known that local chains are rapidly mixing when  $\lambda < 2.3882$ . We give complementary results showing that local chains will mix slowly when  $\lambda > 5.3646$  on regions with periodic (toroidal) boundary conditions and when  $\lambda > 7.1031$  with non-periodic (free) boundary conditions. The proofs use a combinatorial characterization of configurations based on the presence or absence of fault lines and an enumeration of a new class of self-avoiding walks called taxi walks.

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### MS34

#### Maximum Independent Sets in Random $d$ -regular Graphs

Satisfaction and optimization problems subject to random constraints are a well-studied area in the theory of computation. These problems also arise naturally in combinatorics, in the study of sparse random graphs. While the values of limiting thresholds have been conjectured for many such models, few have been rigorously established. In this context we study the size of maximum independent sets in random  $d$ -regular graphs. We show that for  $d$  exceeding a constant  $d_0$ , there exist explicit constants  $a, c$  depending on  $d$  such that the maximum size has constant fluctuations around  $na - c \log n$  establishing the one-step replica symmetry breaking heuristics developed by statistical physicists. As an application of our method we also prove an explicit satisfiability threshold in random regular  $k$ -NAE-SAT. This is joint work with Jian Ding and Allan Sly.

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### MS34

#### Local Statistics of the Abelian Sandpile Model

We show how to compute local statistics of the abelian sandpile model on the square, hexagonal, and triangular lattices. On the square lattice, all local events are rational polynomials in  $1/\pi$ , while on the hexagonal and triangular lattices they are rational polynomials in  $\sqrt{3}/\pi$ . The proofs use the burning bijection between sandpiles and spanning trees, and the methods of Kenyon and Wilson for computing grove partition functions.

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### MS35

#### Recognizing and Colouring Claw-Free Graphs Without Even Holes

An even hole is an induced even cycle. Even-hole-free graphs generalize chordal graphs. We prove that claw-free even-hole-free graphs can be decomposed by clique cutsets into, essentially, proper circular-arc graphs. This provides the basis for our algorithms for recognizing and colouring these graphs. Our recognition algorithm is more efficient ( $O(n^2 m^2 \log \log n)$ ) than known algorithms for recognizing even-hole-free graphs ( $O(n^{19})$ ). Colouring claw-free graphs is NP-hard and the complexity of colouring even-hole-free graphs is unknown, but our algorithm colours claw-free even-hole-free graphs in  $O(n^4)$  time.

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### MS35

#### Digraph Analogues of Interval Graphs and of Trivially Perfect Graphs

Interval graphs are precisely the reflexive graphs for which list homomorphism problems are solvable in polynomial time. For general digraphs, this role is played by the class of digraphs free of structures we call digraph asteroidal triples (DATs). Trivially perfect graphs are precisely the reflexive graphs for which list homomorphism problems are solvable in logarithmic space. For general digraphs, this role is taken by the class of digraphs without structures we call circular  $N$ 's (cN's). Both DAT-free and cN-free digraphs exhibit interesting combinatorial properties; in particular, they can be recognized in polynomial time. This is joint work with Arash Rafiey, and partly also with Laszlo Egri, and Benoit Larose.

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**MS35****Coloring Graphs Without Induced Paths and Cycles**

Let  $P_t$  and  $C_\ell$  denote a path on  $t$  vertices and a cycle on  $\ell$  vertices, respectively. A graph  $G$  is  $(P_t, C_\ell)$ -free if it does not contain  $P_t$  or  $C_\ell$  as an induced subgraph. In this talk we study the  $k$ -COLORING problem for  $(P_t, C_\ell)$ -free graphs. We shall show that for most combinations of  $k$ ,  $t$  and  $\ell$  the problem turns out to be NP-complete. On the other hand, we give certifying polynomial time algorithms for  $(P_6, C_4)$ -free graphs for every  $k$ . For  $k = 3$  or  $k = 4$ , we also provide an explicit and complete list of minimal non- $k$ -colorable  $(P_6, C_4)$ -free graphs.

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**MS35****Non-Planar Extensions of Planar Signed Graphs**

A *signed graph* is a pair  $(G, \Sigma)$  where  $G$  is a graph, and  $\Sigma \subseteq E(G)$ . Given a planar signed graph  $(G, \Sigma)$ , we are interested in determining the minimal non-planar signed graphs  $(H, \Gamma)$  that “contain”  $(G, \Sigma)$ . We show (assuming 3-connectivity for  $(G, \Sigma)$ ) that there are a small number of these “minimal non-planar extensions” of  $(G, \Sigma)$ , and describe them explicitly. This result generalizes work done by Robertson, Seymour, and Thomas.

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**MS35****On the Contour of Chordal Bipartite Graphs**

A vertex in a graph is a contour vertex if none of its neighbors has a larger eccentricity than the vertex. We show that the set of contour vertices of a connected chordal bipartite graph, one without chordless cycles on at least six vertices, is geodetic; thus every vertex of the graph lies on a shortest path between some pair of contour vertices. A similar result does not hold for bipartite graphs with long chordless cycles.

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**MS36****Shelah’s Grid Ramsey Problem**

Shelah introduced the following problem related to Hales-Jewett numbers. The grid graph  $\Gamma_n$  has vertex set  $[n]^2$ , and any two vertices in the same row or column are adjacent. Let  $g(k)$  be the minimum  $n$  such that every  $k$ -edge-coloring of  $\Gamma_n$  has an axis-parallel rectangle with opposite edges the same color. Shelah asked: is  $g(k)$  at most polynomial in  $k$ ? We give a negative answer.

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**MS36****Monochromatic Bounded Degree Subgraph Partitions**

Let  $\mathcal{F} = \{F_1, F_2, \dots\}$  be a sequence of graphs such that  $F_n$  is a graph on  $n$  vertices with maximum degree at most  $\Delta$ . We show that there exists an absolute constant  $C$  such that the vertices of any 2-edge-colored complete graph can be partitioned into at most  $2^{C\Delta \log \Delta}$  vertex disjoint monochromatic copies of graphs from  $\mathcal{F}$ .

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**MS36****Restricted Ramsey-Type Theorems**

Abstract not available at time of publication.

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**MS36****Multipass Random Coloring of Simple Uniform Hypergraphs**

Let  $D^*(n)$  be the largest number such that every simple

$n$ -uniform hypergraph with maximum edge degree at most  $D^*(n)$  is two colorable (has Property B). We prove that  $D^*(n) = \Omega(n2^n)$ . The method generalizes to  $b$ -simple hypergraphs (every pair of distinct edges intersects in at most  $b$  vertices) and hypergraphs of arithmetic progressions. This way we obtain an improved lower bound for Van der Waerden numbers and get  $W(n) = \Omega(2^n)$ .

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### MS36

#### Hypergraph Ramsey Numbers

In this talk I will discuss an extension of cycle versus complete graph Ramsey numbers to hypergraphs. A *generalized triangle* is a  $k$ -uniform hypergraph (or simply  $k$ -graph) of three edges  $e, f, g$  such that  $|e \cap f| = |f \cap g| = |g \cap e| = 1$  and  $e \cap f \cap g = \emptyset$ . The Ramsey number  $R_k(3, t)$  is the minimum  $n$  such that every  $k$ -graph on at least  $n$  vertices contains either a generalized triangle or a clique of size  $t$ . The celebrated Ajtai-Komlos-Szemerédi Theorem together with Kim's construction show that  $R_2(3, t) = \Theta(t^2 / \log t)$ . I will outline here the proof that for every  $k \geq 3$ ,  $R_k(3, t) = \Theta^*(t^{3/2})$  where the star denotes implicit powers of logarithms in  $t$  and a constant depending only on  $k$ . Generalizations to cycle-complete graph Ramsey numbers will be discussed, together with some open problems. This is joint work with Dhruv Mubayi and Sasha Kostochka.

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### MS37

#### Online and Size anti-Ramsey Numbers

A graph is properly edge-colored if no two adjacent edges have the same color. For a graph  $H$ , we introduce the **size anti-Ramsey number**  $AR_s(H)$  to be the smallest number of edges in a graph any of whose proper edge colorings contains a totally multicolored copy of  $H$ . In this talk we shall discuss this analogue of a size-Ramsey number. We determine the bounds on  $AR_s(H)$  for general as well as for some specific graphs and provide a number of results for the on-line setting of this problem. Several open problems will be presented.

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### MS37

#### List Coloring the Square of Sparse Graphs

The square of a graph  $G$  is obtained from  $G$  by adding all edges between vertices with a common neighbor. Borodin et al. proved that squares of planar graphs of girth  $g \geq 7$  and sufficiently large maximum degree  $\Delta$  are list  $(\Delta + 1)$ -colorable, which is optimal with regard to the girth. We strengthen their result by replacing the girth and planarity constraint by a maximum average degree constraint, which yields a more refined bound.

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### MS37

#### Painting Squares in $\Delta^2 - 1$ Shades

Cranston and Kim conjectured that if  $G$  is a connected graph with maximum degree  $\Delta$  and  $G$  is not a Moore Graph, then  $\chi_\ell(G^2) \leq \Delta^2 - 1$ ; here  $\chi_\ell$  is the list chromatic number. (This saves one color over the easy upper bound of  $\Delta^2$  given by combining Brooks' Theorem (the list-coloring version) with the fact that  $G^2$  has maximum degree at most  $\Delta^2$ .) We prove their conjecture; in fact, this upper bound holds even for online list chromatic number.

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### MS37

#### Choosability of Graphs With Bounded Order

The *choice number* of a graph  $G$ , denoted  $\text{ch}(G)$ , is the minimum integer  $k$  such that, for any assignment of lists of size  $k$  to the vertices of  $G$ , there exists a proper colouring of  $G$  in which every vertex is mapped to a colour in its list. In this talk, we discuss recent results and open problems regarding the choice number of graphs for which the number of vertices is bounded in terms of  $\chi$ . In particular, this includes a solution to Ohba's Conjecture, which says that if  $|V(G)| \leq 2\chi(G) + 1$ , then  $\text{ch}(G) = \chi(G)$ . We will also talk about a recent strengthening of this result, and propose several open problems for future study. The results of this talk are joint work with Bruce Reed, Douglas West, Hehui Wu and Xuding Zhu.

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**MS37****The Potential Technique in Graph Coloring**

We discuss recent advancements and applications of the potential technique developed by Kostochka and Yancey for lower bounding the edge density of  $k$ -critical graphs. In particular, we will discuss 3-coloring graphs of girth five on surfaces.

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**MS38****Triple Systems, Gray Codes and Snarks**

The 2-block-intersection graph of any simple twofold triple system is necessarily a cubic graph. If it is Hamiltonian then a 2-intersecting cyclic Gray code for the design can be found. If it is not Hamiltonian then the graph has the potential to be a somewhat rare type of graph known as a snark.

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**MS38****On a Problem of Mariusz Meszka**

We consider a problem due to Mariusz Meszka similar to the well-known conjecture of Marco Buratti. Does there exist a near-1-factor in the complete graph on  $Z_p$ ,  $p$  an odd prime, whose multiset of edge-lengths equals a given multiset  $L$ ? This may be viewed as a generalization of the existence problem for extended Skolem sequences.

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**MS38****Approaching the Minimum Number of Clues Sudoku Problem Via the Polynomial Method**

Determining the minimum number of clues that must be present in a Sudoku puzzle in order to uniquely complete the puzzle is known as the minimum number of clues problem. For a 9-by-9 Sudoku board, it has been conjectured that one needs 17 clues, and this has been confirmed by McGuire et al. in a massive computer search. We apply the polynomial method to the analogous problem for the 4-by-4 Shidoku board to illustrate how one might approach the more general problem. This is joint work with Aden Forrow.

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**MS38****Biembeddings and Graph Decompositions**

A face 2-colourable embedding of a simple graph in which black faces have boundary lengths  $m_1, m_2, \dots, m_r$  and white faces have boundary lengths  $n_1, n_2, \dots, n_t$  is a *biembedding* of the cycle lists  $M = m_1, m_2, \dots, m_r$  and  $N = n_1, n_2, \dots, n_t$ . We use Euler's classic embedding formula to derive necessary and sufficient conditions for the existence of such an embedding in the sphere given two arbitrary lists of cycle lengths. We also discuss a surprising relationship between these biembeddings and more general cycle decomposition problems. (Joint work with Barbara Maenhaut.)

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**MS38****Switchings for 1-Factorizations**

We define four graphs where the vertices are isomorphism classes of 1-factorisations of a complete graph  $K_n$  and edges are produced by simple switchings. These graphs are studied in detail for  $n \leq 12$ . For some switchings the graph will be disconnected for all orders because 1-factorisations have a parity that is preserved by the switchings. We also study isolated vertices, including the celebrated case of perfect 1-factorisations, and connections with atomic Latin squares.

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**MS39****On the Number of  $B_h$ -Sets**

A set of positive integers is a  $B_h$ -set if all the sums of  $h$  elements from the set are distinct. We provide asymptotic bounds for the number of  $B_h$ -sets of a given cardinality contained in  $[n] = \{1, \dots, n\}$ . As a consequence of our results, better upper bounds for a problem of Cameron and Erdős (1990) in the context of  $B_h$ -sets are obtained. We use these results to estimate the maximum size of a  $B_h$ -set contained in a random subset of  $[n]$  with a given cardinality.

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### MS39

#### On Erds Conjecture on the Number of Edges in 5-Cycles

Erdős, Faudree, and Rousseau in 1992 showed that a graph on  $n$  vertices and with at least  $\lfloor n^2/4 \rfloor + 1$  edges comprise at least  $2\lfloor n/2 \rfloor + 1$  edges on triangles and this result is sharp. They also considered a conjecture of Erdős that such a graph have at most  $n^2/36$  non-pentagonal edges. This was mentioned in other paper of Erdős and also in Fan Chung's problem book. In this talk we give a graph of  $\lfloor n^2/4 \rfloor + 1$  edges with much more, namely  $n^2/8(2 + \sqrt{2}) + O(n)$  pentagonal edges, disproving the original conjecture. We also show that this coefficient is asymptotically the best possible.

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### MS39

#### Large Subgraphs Without Short Cycles

We study two extremal problems about subgraphs excluding a family  $\mathcal{F}$  of fixed graphs. i) Among all graphs with  $m$  edges, what is the smallest size  $f(m, \mathcal{F})$  of a largest  $\mathcal{F}$ -free subgraph? ii) Among all graphs with minimum degree  $\delta$  and maximum degree  $\Delta$ , what is the smallest minimum degree  $h(\delta, \Delta, \mathcal{F})$  of a spanning  $\mathcal{F}$ -free subgraph with largest minimum degree? We study the case where  $\mathcal{F}$  is composed of all cycles of length at most  $2r + 1$ ,  $r \geq 1$ . In this case, we give meaningful lower bounds on  $f(m, \mathcal{F})$  and determine the asymptotic value of  $h(\delta, \Delta, \mathcal{F})$ . This is joint work with F. Foucaud and M. Krivelevich, and with B. Reed.

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### MS39

#### Asymptotic Distribution of the Numbers of Vertices and Arcs in the Giant Strong Component in Sparse Random Digraphs

For the random digraph on  $n$  vertices,  $D(n, \text{Prob}(\text{arc}) = p)$ , Karp (90) and Łuczak (90) found that for  $p = c/n$ ,  $c > 1$ , with probability tending to 1, there is a giant strong component of size linear in  $n$ . We show that in  $D(n, p = c/n)$  and  $D(n, \text{number of arcs} = m)$ ,  $m = cn$ , the joint distribution of the numbers of vertices and arcs in the giant strong component is asymptotically Gaussian with mean and covariance matrix linear in  $n$ .

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### MS40

#### Degree Sequences and Forced Adjacency Relationships

Degree sequences of graphs usually have several labeled realizations with differing edge sets. In some cases, however, vertices with certain degrees are adjacent (or non-adjacent) in every realization. The quintessential example of this occurs with a threshold graph, where every adjacency relationship is uniquely determined by the degree sequence. Given a degree sequence, we characterize degree pairs corresponding to vertices that are forcibly (non)-adjacent, revealing connections with the Erdős–Gallai inequalities and the majorization order.

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### MS40

#### Stability of the Potential Function

The potential number of a graph  $H$ , denoted  $\sigma(H, n)$ , is the minimum integer such that any graphic sequence of length  $n$  has a realization containing  $H$  as a subgraph. A graph  $H$  has degree-sequence stability if every graphic sequence with sum close to  $\sigma(H, n)$  having no realization containing a copy of  $H$  can be transformed into an extremal sequence with  $o(n)$  additions and subtractions. We discuss the degree-sequence stability of various families of graphs.

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### MS40

#### Degree Sequences of Uniform Hypergraphs: Results and Open Problems

A sequence of nonnegative integers is  $k$ -graphic if it is the degree sequence of a  $k$ -uniform hypergraph. We present sharp sufficient conditions for  $k$ -graphicality based on a sequence's length and degree sum. Further, Kocay and Li gave a family of edge exchanges (an extension of 2-switches) that could be used to transform one realization of a 3-graphic sequence into any other realization. We extend their result to  $k$ -graphic sequences for all  $k \geq 3$  and give several applications of edge exchanges in hypergraphs.

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#### MS40

##### Minimal Forbidden Sets for Degree Sequence Characterizations

Given a set  $\mathcal{F}$  of graphs, a graph  $G$  is  $\mathcal{F}$ -free if  $G$  does not contain any member of  $\mathcal{F}$  as an induced subgraph. Barrus, Kumbhat, and Hartke (2008) called  $\mathcal{F}$  a *degree-sequence-forcing (DSF) set* if, for each graph  $G$  in the class  $\mathcal{C}$  of  $\mathcal{F}$ -free graphs, every realization of the degree sequence of  $G$  is also in  $\mathcal{C}$ . A DSF set is *minimal* if no proper subset is also DSF. In this talk, we present new properties of minimal DSF sets, including that every graph is in a minimal DSF set and that there are only finitely many DSF sets of cardinality  $k$ . Using these properties and a computer search, we characterize the minimal DSF triples.

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#### MS40

##### The Asymptotic Behavior of the Potential Function

Given a graph  $H$ , a sequence  $\pi$  of nonnegative integers is *potentially  $H$ -graphic* if there is a graph  $G$  whose vertex degrees match the entries of  $\pi$  and  $H$  is a subgraph of  $G$ . The *potential number* of  $H$  is the minimum even integer  $\sigma(n, H)$  such that every  $n$ -term sequence of nonnegative integers with sum at least  $\sigma(n, H)$  is potentially  $H$ -graphic. We present recent results on potential numbers, including precise asymptotics and stability for sequences that are not potentially  $H$ -graphic.

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#### MS41

##### Mitochondrial DNA Organization in Trypanosomatid Parasite

Trypanosomes and leishmania, two trypanosomatid parasites, are protozoa which cause fatal diseases such as sleeping sickness, Chagas disease and Leishmaniasis. A distinctive feature of trypanosomatid parasites is that their mitochondrial DNA, known as kinetoplast DNA, is organized into thousands of minicircles and a few dozen maxicircles. Minicircles in these organisms are topologically linked, forming a gigantic chainmail-like network. The topological structure of the minicircles is of great significance both because it is species-specific and because it is a promising target for the development of drugs. However, the biophysical factors that led to the formation of the network during evolution and that maintain the network remain poorly understood. In this talk I will discuss a mathematical and computational model that help us quantify possible growth mechanisms of the network. We find that minicircle networks rapidly form, following a percolation-saturation pathway, when the density of minicircles is increased, and that the valence (the number of minicircles linked to any given minicircle) grows linearly with density of minicircles. Limitations and extensions of these models and results will also be discussed.

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#### MS41

##### Assorted Topics in the Monte Carlo Simulation of Polymers

We will touch on various aspects of Monte Carlo simulations of polymers, with the particular goal of motivating the use of moves which are neither local nor global, but which occur at intermediate length scales. We will touch on applications of Monte Carlo to enumerations of walks, work in progress on possible methods to adapt the pivot algorithm to the study of confined polymers and  $\theta$  polymers, and a method to rapidly calculate the hydrodynamic radius of a polymer.

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#### MS41

##### Inhomogeneous Percolation with a Defect Plane

Let  $\mathbf{L}$  be an  $s$ -dimensional sublattice of the  $d$ -dimensional hypercubic lattice  $\mathbf{Z}^d$ , with  $2 \leq s < d$ . Each edge in  $\mathbf{L}$  is open with probability  $\sigma$ , and each edge outside  $\mathbf{L}$  is open with probability  $p$ . This percolation model has three “phases”: (1) no infinite (open) clusters; (2) infinite clusters that stay near  $\mathbf{L}$ ; and (3) infinite clusters that penetrate all of space. We present results about these phases

and their boundaries.

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**MS41**

### Subknots in Closed Chains

For a fixed knot configuration, the subknots are the knot types seen in the open subchains. For nice knot configurations (like ones minimized with respect to some knot energy), the subknots are typically simpler knot types than the host knot type. We compare and contrast the set of subknots coming from KnotPlot configurations, tight knot configurations, and random configurations. This is joint work with Ken Millett and Andrzej Stasiak.

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**MS41**

### Enumerating Knots in Lattice Tube Models of Polymers

Self-avoiding polygons on the cubic lattice are the standard statistical mechanics model for ring polymers in dilute solution. For this model, the effects of geometrical confinement are studied by constraining the polygons to an infinite rectangular lattice tube. For small tube sizes, exact enumeration and generation of polygons is accomplished using transfer-matrices. Using this approach, I will present recent results regarding the knot complexity of self-avoiding polygons in a lattice tube.

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**MS42**

### Stable Multivariate Polynomials and the Lower Bounds: (old)Results and (new)Conjectures

Let  $p(x_1, \dots, x_m)$  be homogeneous stable polynomial, i.e. it does not have zeros with positive real parts. In the last 10 years these were several spectacular applications of this class of polynomials and its natural generalizations to many problems in various areas: open problems had been solved and very complicated proofs had been drastically simplified. One of the first applications was a radical simplification, vast generalization and even some improvement of A. Schrijver's asymptotically optimal lower bound on the number of perfect matchings in regular bipartite graphs. The polynomials related to the Schrijver's lower bound are products of linear forms. It turned out that for those very special polynomials the combination of Schrijver's approach and the Bethe approximation leads to a new very precise lower bound on the mixed derivative, i.e.

the permanent. The goal of this talk is to present several conjectures generalizing to general stable polynomials new (entropic) lower bounds on the permanent. Time permitting, we will present also conjectures related to the relative approximation of 4D Pascal's determinant and the permanent of positive semidefinite matrices.

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**MS42**

### Title Not Available at Time of Publication

Abstract not available at time of publication.

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**MS42**

### Introduction to Hyperbolic and Real Stable Polynomials

We will introduce real stable and hyperbolic polynomials and discuss their basic properties in preparation for the rest of the talks. No background will be assumed.

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**MS42**

### Hyperbolic Polynomials, Interlacers, and Sums of Squares

Hyperbolic polynomials are real polynomials whose real hypersurfaces are nested ovals, the inner most of which is convex. These polynomials appear in many areas of mathematics, including optimization, combinatorics and differential equations. I'll give an introduction to this topic and discuss the special connection between hyperbolic polynomials and their interlacing polynomials (whose real ovals interlace the those of the original). This will let us related inner oval of a hyperbolic hypersurface to the cone of non-negative polynomials and, sometimes, to sums of squares. An important example will be the bases generating polynomial of a matroid.

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**MS42**

### Stable Multivariate Eulerian Polynomials

We discuss a multivariate generalization of the well-known Eulerian polynomials that arose from the solution of the Monotone Column Permanent Conjecture. We give several examples how this stable multivariate generalization simplifies and extends results in permutation statistics. Finally, we describe another surprising connection linking these polynomials to stationary distributions of certain Exclusion Processes. This talk is based on collaboration with P. Brändén, J. Haglund, M. Leander, D.G. Wagner, and

N. Williams.

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#### MS43

##### Graph Polynomials from Graph Homomorphisms

A sequence of graphs  $(H_n)$  is *strongly polynomial* if for every graph  $G$  the number of homomorphisms from  $G$  to  $H_n$  is given by a polynomial function of  $n$ . For example, the sequence  $(K_n)$  is strongly polynomial since the number of homomorphisms from  $G$  to  $K_n$  is the chromatic polynomial evaluated at  $n$ . Many important graph polynomials are determined by strongly polynomial sequences of graphs. We give a wide-ranging construction for such sequences.

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#### MS43

##### On the Complexity of Various Ising Polynomials

We study the complexity of partition functions of the Ising model with constant energies and external field. The trivariate Ising polynomial is shown to be  $\#P$ -hard to evaluate at all points, except those in set of low dimension, even on simple bipartite planar graphs. Under the Counting Exponential Time Hypothesis, we give a dichotomy theorem stating that the bivariate Ising polynomial either take exponential time to compute, or can be done in polynomial time.

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#### MS43

##### On the Largest Real Root of the Independence Polynomials of Trees

In this talk we investigate about the largest real root of independence polynomials of trees. We define an ordering on graphs. Using the mentioned ordering, we obtain some results about the largest real root. In particular we show that among all trees the paths have the minimum and the stars have the maximum real roots.

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#### MS43

##### On the Location of the Roots of Graph Polynomials

Roots of graph polynomials such as the characteristic polynomial, the chromatic polynomial, the matching polynomial, and many others are widely studied. In this paper we examine to what extent the location of these roots reflects the graph theoretic properties of the underlying graph.

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#### MS43

##### The Complexity of Counting Edge Colorings and a Dichotomy for Some Higher Domain Holant Problems

By a result of Vertigan, it is  $\#P$ -hard to count vertex  $\kappa$ -colorings over planar graphs provided  $\kappa \notin \{0, 1, 2\}$ . We consider the problem of edge coloring a graph, a special case of vertex coloring. We show that it is  $\#P$ -hard to count edge  $\kappa$ -colorings over planar  $r$ -regular graphs provided  $\kappa \geq r \geq 3$ . When this condition fails to hold, the problem is trivially tractable. We note that proving  $\#P$ -hardness for a special case of a problem gives a stronger result.

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#### MS44

##### A New Generalisation of a de Bruijn Erdos Theorem

Let  $(V, E)$  be a hypergraph. Assuming that any two edges intersect on at most  $k$  vertices and that any two vertices is contained in at least  $t$  edges, we prove that  $k \cdot |E| \geq t \cdot |V|$ . This generalizes a Theorem of De Bruijn and Erdős who proved it in the case where  $k = t = 1$ .

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#### MS44

##### Lines in Metric Spaces and Hypergraphs: A Survey

We survey the problems and results related to the points and lines in metric spaces and hypergraphs. We will discuss the definition of the lines, the (unofficially) 10-year old Chen-Chvátal conjecture, its various special cases, the efforts and results in trying to generalize the classical De Bruijn-Erdős theorem, and many recent variations on the topic.

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#### MS44

##### Points, Lines, and the Manhattan Metric

It is a well-known fact that every non-collinear set of  $n$  points in the plane determines at least  $n$  distinct lines. This is a corollary to Sylvester-Gallai theorem, as observed by Erdős in 1943. Chen and Chvátal suggested a generalization of this statement in the context of finite metric spaces with lines defined using the notion of *betweenness*. We prove this generalization in the plane with the  $L_1$  metric, assuming additionally that no two points share their  $x$ - or  $y$ -coordinate. Without this additional condition, we can prove the existence of  $n/37$  lines.

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#### MS44

##### A De Bruijn-Erdos Theorem for Distance-hereditary Graphs

A theorem of De Bruijn and Erdős implies that  $n$  non-collinear points in the plane determine at least  $n$  lines. Chen and Chvátal conjectured a generalization to finite metric spaces. An interesting case is that of metric spaces induced by graphs. In this talk I present a proof of the conjecture for metric spaces induced by distance-hereditary graphs. These are graphs where the distance between two vertices is the same in any connected induced subgraph.

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#### MS44

##### Lines in Metric Spaces Induced by General Graphs

A theorem of De Bruijn and Erdős implies that  $n$  non-collinear points in the plane determine at least  $n$  lines. Chen and Chvátal conjectured a generalization to finite metric spaces. An interesting case is that of metric spaces induced by graphs. We show that every metric space induced by a graph has either a line containing all the ver-

tices or at least  $\sqrt{n}/5$  distinct lines. We also show that a random graph induces  $\Omega(n^2)$  distinct lines.

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#### MS45

##### Lazy Cops and Robbers

We consider a variant of the game of Cops and Robbers, called Lazy Cops and Robbers, where at most one cop can move in any round. We investigate the analogue of the cop number for this game, which we call the lazy cop number. In this talk we will discuss results concerning the lazy cop number of binomial random graphs, hypercubes, and graphs with genus  $g$ .

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#### MS45

##### Line-of-Sight Pursuit in Sweepable Polygons

We consider a pursuit game in a simply polygon  $Q$  between a pursuer  $P$  and an evader  $E$ , where  $P$  only has line-of-sight visibility. We provide a winning strategy for  $P$  a *monotone polygon* in which every line orthogonal to a fixed line  $L$  intersects  $Q$  at most twice. The capture time is linear in the area of  $Q$ , and independent of the number of vertices of the polygon. Finally, we also extend our algorithm to more general *sweepable polygons*. Joint work with Volkan Isler, Jane Butterfield, Lindsay Berry, Zack Keller, Alana Shine and Junyi Wang.

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#### MS45

##### The Computational Complexity of Cops and Robbers

In the classic *Cops and Robbers* game, a team of cops attempts to capture a robber on a graph. The game has been extensively studied, with applications ranging from artificial intelligence to counterterrorism. In this talk, we discuss the computational complexity of deciding whether  $k$  cops can capture a robber on a graph  $G$ : how fast (or how slow) are the best possible computer algorithms for determining who wins? In 1995, Goldstein and Reingold conjectured that the problem is EXPTIME-complete – in other words, that Cops and Robbers is among the “hardest” problems that can be solved in time exponential in the size of the input. We sketch a recent proof of this conjecture.

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#### MS45

##### Cop Versus Gambler

We consider a variation of Cops and Robbers in which the robber is not restricted by the graph edges and instead picks a time-independent probability distribution on  $V(G)$  and moves according to this fixed distribution. The cop moves from vertex to adjacent vertex with the goal of minimizing expected capture time. Players move simultaneously. We show that when the distribution is known, the expected capture time (with best play) on any connected  $n$ -vertex graph is exactly  $n$ . Time permitting, we will also discuss what is known about the case of an unknown distribution.

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#### MS45

##### The Eternal Dominating Set Problem on Grids

In the eternal dominating set problem, guards form a dominating set on a graph and at each step, a vertex is attacked. After each attack, if the guards can ‘move’ so that they form a dominating set containing the attacked vertex, then the guards have *defended against the attack*. We wish to determine the minimum number of guards required to defend against any sequence of attacks, the *eternal domination number*. As the domination number for grid graphs

was recently determined (2011), grid graphs are a natural class of graphs to consider for the eternal dominating set problem. Though the eternal domination number has been determined for  $2 \times n$  grids and  $4 \times n$  grids, it has remained only loosely bounded for the  $3 \times n$  grid. We provide major improvements to both the upper and lower bounds for the  $3 \times n$  grid. Joint work with S. Finbow, M. van Bommel,

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#### MS46

##### Counting Split Graphs and Nordhaus-Gaddum Graphs

A graph  $G$  is an *NG-graph* if it satisfies the Nordhaus-Gaddum inequality,  $\chi(G) + \chi(\overline{G}) \leq |V(G)| + 1$  with equality. In this talk, which is a continuation of *Split Graphs and Nordhaus-Gaddum Graphs* by Ann Trenk, we continue to explore connections between NG-graphs and split graphs and count the number of NG-graphs on  $n$  vertices. This is joint work with Ann N. Trenk (Wellesley College).

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#### MS46

##### Kekulean Benzenoids

A *Kekulé structure* for a benzenoid or a fullerene  $\Gamma$  is a set of edges  $K$  such that each vertex of  $\Gamma$  is incident with exactly one edge in  $K$ , i.e. a perfect matching. All fullerenes admit a Kekulé structure; however, this is not true for benzenoids. In this paper, we develop methods for deciding whether or not a given benzenoid admits a Kekulé structure by constructing Kekulé structures that have a high density of benzene rings. Our method of constructing Kekulé structures gives good estimates for the Clar and Fries numbers for benzenoids. Exact values for certain families of benzenoids will be given.

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#### MS46

##### Split Graphs and Nordhaus-Gaddum Graphs

A graph  $G$  is an *NG-graph* if it satisfies the Nordhaus-Gaddum inequality,  $\chi(G) + \chi(\overline{G}) \leq |V(G)| + 1$  with equality. In recent work, Collins and Trenk gave a new characterization of this class which resulted in a polynomial-time recognition algorithm. In this talk, we explore connections between NG-graphs and split graphs and discuss an improved algorithm for recognizing NG-graphs based solely on degree sequences. This topic is continued in the talk *Counting Split Graphs and Nordhaus-Gaddum Graphs* by Karen Collins.

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#### MS46

##### On $r$ -dynamic Coloring of Graphs

The  *$r$ -dynamic chromatic number* of a graph  $G$ , written  $\chi_r(G)$ , is the least  $k$  such that  $G$  has a proper  $k$ -coloring in which every vertex has neighbors of at least  $\min\{d(v), r\}$  different colors. We study upper bounds on  $\chi_r(G)$ . The greedy upper bound  $\chi_r(G) \leq r\Delta(G) + 1$  holds with equality if and only if  $G$  is  $r$ -regular with diameter 2 and girth 5. For  $n$ -vertex graphs, the bound improves to  $\chi_r(G) \leq \Delta(G) + 2r$  when  $\delta(G) \geq 2r \ln n$  and to  $\chi_r(G) \leq \Delta(G) + r$  when  $\delta(G) \geq r(r+1) \ln n$ . In terms of the chromatic number,  $\chi_r(G) \leq$

$r\chi(G)$  when  $G$  is  $k$ -regular with  $k \geq (3 + o(1))r \ln r$ , but  $\chi_r(G)$  may exceed  $r^{1.377} \chi(G)$  when  $k = r$ . In contrast,  $\chi_2(G) \leq \chi(G) + 2$  when  $G$  has diameter 2, with equality only for complete bipartite graphs and  $C_5$ . Also,  $\chi_2(G) \leq 3\chi(G)$  when  $G$  has diameter 3, which is sharp. However,  $\chi_2$  is unbounded on bipartite graphs with diameter 4, and  $\chi_3$  is unbounded on bipartite graphs with diameter 3 or 3-colorable graphs with diameter 2. We also study  $\chi_r$  on

cartesian products of two paths or two cycles.

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#### MS46

#### Dichromatic Number and Fractional Chromatic Number and

Given an undirected graph  $G$ , the chromatic number  $\chi(G)$  is the minimum order of partitions of  $V(G)$  into independent sets. Given a directed graph  $D$ , a vertex set is acyclic if it does not contain a directed cycle. The chromatic number  $\chi(D)$  is the minimum order of partitions of  $V(G)$  into acyclic sets. The dichromatic number of an undirected graph  $G$ , denoted by  $\vec{\chi}(G)$ , is the maximum chromatic number over all its orientations. Erdős and Neumann-Lara proved that  $C_1 \frac{n}{\log n} \leq \vec{\chi}(K_n) \leq C_2 \frac{n}{\log n}$  for some constants  $C_1, C_2$ . They conjectured that if the dichromatic number of a graph is bounded, so does its chromatic number. The fractional chromatic number is, over all the possible weight functions on the independent sets, the minimum total weight of all the independent sets, such that for each vertex the independent sets including it has total weight at least 1. We prove that for any graph  $G$  with fractional chromatic number  $t$ , we have  $\vec{\chi}(K_n) \geq \frac{t}{18 \log t}$ .

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