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Discovery of Coupled Oscillation Put 17th-Century Scientist Ahead of his Time

By Erica Klarreich

Theoretical physicists may make their best discoveries while staring at the wall, but experimental physicists do so less often. Yet the great Dutch scientist Christiaan Huygens, inventor of the pendulum clock, discovered coupled oscillation in exactly this way, while confined to his bed during a brief illness in the winter of 1665. Staring absent-mindedly at two pendulum clocks suspended on a beam in his sickroom, he noticed what he called an "odd kind of sympathy" between them: No matter how the pendulums started out, within half an hour they were swinging in exactly opposite directions. Once the pendulums reached this state of anti-synchonization, they remained in it indefinitely.

Rebuffed by the newly formed Royal Society of London, the Dutch scientist Christiaan Huygens consigned his 1665 discovery of coupled oscillation to oblivion.

For more than three centuries, Huygens's observations went unexplained. Now, a team of physicists at the Georgia Institute of Technology have created a model that accounts for the strange sympathy between the pendulums. The mathematics underlying their analysis could eventually lead to ways to improve lasers, superconductors, and even the treatment of epilepsy.

The Longitude Problem

Huygens invented the pendulum clock in 1657 while working on the most important technological problem of his day: calculating longitude at sea. He knew that an accurate clock would solve the problem; mariners would be able to compare its time with the local time and calculate the time difference.

In 1665, Huygens's pendulum clocks represented the state of the art, losing only about 15 seconds in a day; the best competitors lost 15 *minutes* in a day. But even the accuracy of the pendulum clock was not good enough for the longitude problem: An error of 15 seconds each day over a six-week voyage could throw off a longitude calculation by as much as 2.5°, which at the equator spans 170 miles.

As part of an effort to get around this difficulty, Huygens built a double-clock apparatus—the idea being that if one clock ran fast, say, or needed to be cleaned during a voyage, the other clock would keep marking time. His discovery that the pendulums swung in precise anti-synchrony created an even more exciting possibility: that the two clocks would regulate each other, producing an accurate timekeeper. If, for instance, dirt accumulated in the gears of one of the clocks and made it run slightly slow, the coupling with the other clock would minimize the effect.

Reflected in the pendulum of their modern-day re-creation of Huygens's apparatus are physicists Kurt Wiesenfeld (left) and Michael Schatz.

Thrilled by the discovery, Huygens started telling everyone about the mysterious sympathy between the pendulums, writing to his father and to the fledgling Royal Society of London, which had been chartered five years earlier in large part to investigate possible solutions to the longitude problem. Unfortunately, the society didn't see the discovery in the same light as Huygens. At a meeting held to discuss his announcement, it was recorded, "Occasion was taken here by some of the members to doubt the exactness of the motion of these watches at sea, since so slight and almost insensible motion was able to cause an alteration in their going." Disappointed, Huygens consigned his discovery to oblivion.

A Mixture of Luck and Skill

Like many physicists who study coupled oscillation, Kurt Wiesenfeld of Georgia Tech made a habit of starting his talks at conferences and seminars by telling the story of Huygens's clocks. Gradually, he became intrigued by the fact that no one had explained the phenomenon. "People would ask me after a talk what's going on with the pendulum clocks," he says. "Eventually I decided I wanted to understand it." With physicists Matthew Bennett and Michael Schatz, and linguist Heidi Rockwood to help translate Huygens's Latin notes, Wiesenfeld set about recreating Huygens's observa-

Physicist Matthew Bennett was part of a Georgia Tech team that created a mathematical model to explain the surprising effect they observed in testing Huygens's observations.

tion.

Originally, he says, one of the team's hypotheses was that Huygens simply didn't see what he thought he did. "Everyone has been fooled at a traffic light, when your right-turn blinker and the blinker of the car in front of you seem to be perfectly synchronized, but then after 30 seconds they go out of phase," he says. "Huygens's experiment was done before the start of what we consider physics—the era of Newton—and so we weren't really sure whether what passes for science today is what they were doing then."

The team's translation of Huygens's notes set that hypothesis to rest. Huygens had made meticulous measurements, watching the clocks over periods of hours. He put the clocks in separate rooms, where they failed to anti-synchronize, and he placed a bureau between the clocks to make sure the cause of the coupling was not air currents. "Nowadays someone would put the clocks in a vacuum chamber, but he had a low-tech solution," Wiesenfeld says. Huygens eventually concluded that the anti-synchronization was caused somehow by "imperceptible movements" in the supporting beam.

Wiesenfeld's team recreated Huy-gens's apparatus, with a high-tech twist—lasers to track the precise motion of the pendulums. They tested the effect of varying the experiment's physical parameters—the degree to which the two clocks were matched in frequency, and the ratio of the mass of the pendulums to the total mass of the system. To their surprise, they found that although anti-synchronization occurs sometimes, it is the exception rather than the rule: It happens only when the clocks are very closely matched and the mass ratio is very low. When the mass ratio is above about .01, a phenomenon called "beating death" occurs instead: One of the pendulums simply stops swinging.

Huygens saw anti-synchronization because of a mixture of luck and skill, Wiesenfeld says. His clocks, made by the superb craftsman S. Oosterwijck, were almost perfectly matched in frequency. And to prevent the clocks from toppling over at sea, Huygens had loaded the clock boxes with 100-pound lead weights, bringing the mass ratio into exactly the right zone for anti-synchronization to occur.

A Matter of Simple Lagrangian Mechanics

To explain the effect, the team created a mathematical model with three free parameters: the angles of displacement of the two pendulums and the linear displacement of the supporting frame, which they assumed to be constrained to move only in the direction in which the pendulums are swinging. To get a sense of what was going on, the team started by ignoring the action of the clocks' escapements—the mechanisms that give the pendulums regular kicks of energy so they don't run down. With this simplification, understanding the motion of the pendulums was a matter of simple Lagrangian mechanics. After a change of variables, with the angular displacements replaced by the sum (σ) and difference (δ) of the displacements, the equations of motion become:

$$\begin{aligned} \delta'' + 2\gamma\delta' + \delta &= 0, \\ \sigma'' + 2\gamma\sigma' + \sigma &= -2Y'', \\ Y'' + 2\Gamma Y' &= -\mu\,\sigma'', \end{aligned}$$

where Y is the displacement of the supporting frame, γ and Γ are scaled friction coefficients, and μ is the ratio of the mass of the pendulums to the mass of the whole system. The equations make clear that only σ, not δ, couples to the motion of the supporting frame. So friction on the frame damps out the in-phase oscillations of the pendulums, but not the anti-phase oscillations.

Heuristically, when swinging in phase, the pendulums work together to push the supporting beam in the other direction; friction on the beam damps this kind of motion. But when the pendulums swing in opposite directions, the forces they exert on the frame cancel each other. In this case the frame doesn't move, and the system thus evolves into anti-synchronization.

This simplification has all the important ingredients, Wiesenfeld says, but doesn't explain the beating death the team observed for high mass ratios. To fill out the picture, the researchers had to factor in the action of the escapements, whose periodic kicks transform the problem into a much harder, nonlinear system. Beating death, they found, stems from the fact that the escapements function only if the pendulums are swinging above a certain critical amplitude. In simulations, they found that when the pendulums are fairly heavy compared with the supporting beam, the coupling between the pendulums is so strong that it produces wild changes in their amplitudes. When the swing of one of the pendulums dips below the critical ampli-tude, the escapement fails to engage and the pendulum gradually stops swinging.

Exploiting Coupled Oscillators

Although the "sympathy" between the pendulums was an exotic phenomenon in Huygens's day, in the last century researchers have become aware that synchronized oscillators are ubiquitous in both physical and biological settings. Engineers have exploited coupled oscillation in lasers, in which atoms oscillating in unison emit light, and in superconductors, in which synchronized coupling between pairs of electrons allows electricity to flow freely. In the natural world, crickets chirp in unison, heart pacemaker cells synchronize, and satellites go around Jupiter in synchronized orbits. "Synchronized oscillation happens on both cosmic and subatomic levels," says Steven Strogatz of Cornell University. "What's amazing is not just the different size scales but the different frequencies, which range from billions of oscillations per second to one oscillation in a million years."

Wiesenfeld's team is now trying to generalize its analysis into a broad mathematical law that will predict when coupled oscillators will become synchronized or anti-synchronized. Such a law could help engineers in designing more powerful lasers, for instance; if they knew when oscillators would synchronize, they could figure out in what circumstances a collection of weak lasers could be combined to create a single, stronger laser. Understanding why certain oscillators synchronize could even help doctors trying to treat epilepsy, Wiesenfeld suggests; epileptic seizures are thought to occur when neurons in the brain are over-synchronized, firing together in massive bursts of activity.

Why was Huygens's observation neglected for so long? Part of the cause, Strogatz suggests, was Huygens's own disappointment at the Royal Society's reaction to his discovery. "In a way the synchrony of the clocks became a nuisance for Huygens," Strogatz says. "Maybe that's why it languished for almost 350 years." Equally important, he says, is the fact that mathematicians have only recently learned how to deal with nonlinear problems like Huygens's. "People may have been interested in it before but didn't have the means to make progress on it," Strogatz says. "Huygens was ahead of his time."

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