Some Efficiency Measures in the Operation of Flexible Manufacturing Systems: A Stochastic Approach

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Abstract

In this document some efficiency measures of the operation of a serial line manufacturing are discussed from a stochastic approach. In this work we use the concept of operations decoupling in manufacturing systems to place buffers between each operation in order to reduce sequential interdependence between downstream and upstream operations. This will also maintain the output of the production line. Some formulas are developed to compute steady-state performance measures and expected production, including the reliability of the system.

1 Introduction.

Adoption of any technology or innovation is typically a strategic decision that will affect more than one organizational function. Advanced manufacturing technologies, especially Flexible Manufacturing Systems (FMS), clearly fall into this category.

An effective justification process for most strategic technologies requires the consideration of traditional (e.g. cost) and nontraditional investment measures (e.g. flexibility, and other performance measures). To help address this issue of effective evaluation and justification of complex strategic technologies, numerous researchers and practitioners have developed and applied several models.

The FMS can be defined as the integration of manufacturing or assembly processes, material flow and computer communications and control. The objective is to let the production floor respond rapidly and economically to changes in system operations. The importance of FMS has been increasing because of their competitive advantages. The concept of flexibility in FMS has attained significant transcendence in meeting the challenges for a variety of products at shorter lead-times, together with higher productivity and quality.

Evaluation of FMS and other strategic manufacturing technologies has included a spectrum of mathematical and systems modeling approaches. Some of the most important methodologies that have been used for justification of advanced technologies include decision theory [1], dynamic programming [2], [3], game theory [4], [5], linear/goal programming [6], [7], [8], multi-attribute utility theory [9], [10], [11], outranking approaches [12], and risk and simulation analysis [13].

For production control in a FMS system, the introduction of buffers between the manufacturing operations is a classic strategy for management of interdependence.

Two types of interdependence, sequential and reciprocal, are relevant to the production floor (See [14] and [15]). In sequential interdependence each operation's output is dependent on input from one (or more) preceding operations; i.e., operation 2 is dependent on material flowing from operation 1, and operation 3 is dependent on operation 2 and so on. Shutting down operation 1 affects all downstream operations. However, if the last operation is shut down, none of the upstream operations will be affected; they will continue processing the material, creating in-process inventory until the buffers are full. In reciprocal interdependence, mutual adjustment between operations is required for their coordination. Hence, a shutdown of an upstream operation can affect downstream operations and vice versa [14].

In this project we are interested in the sequential interdependence approach. It was initially inspired by the machine interference problem developed by Nahor [16], and widely discussed in [17]. An example is when several repairmen have charge of a set of machines and an explicit probability distribution function describes the various states of this system.

In this paper, we also considered a serial production system with n machines, where a job is divided into individual tasks which are interdependent and separated by buffers. The effect of buffers is to allow machines to operate nearly independently of each other. Buffers can avoid loss of throughput, wasted capacity, long cycle times, larger inventory levels, long lead times and poor customer service, shielding a production system against variability. This reduces idle time due to starving (no input available) and blocking (no space to dispose

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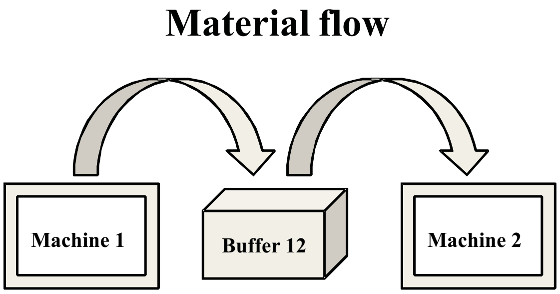

Figure 1: A single system with two machines

of output) [18]. Buffers also regulate operations and eliminate the interdependence until they are emptied when a shutdown occurs upstream. Hence, if a failure occurs in operation j, operation i, $i < j$ is insensitive to that and will keep processing and increasing inventory in the buffer following it [14].

Our aim is to obtain some measures of efficiency of the FMS with a probabilistic approach. The model first follows the basic assumptions mentioned in Nahor [16] and later we extend that method. The assumptions invoked are

1. All machines are similar in the average number of breakdowns which each experiences in its unit working time.
2. All repairmen have similar skill and aptitude in servicing the machines; all machines need similar skill to restore them into working condition.
3. Uninterrupted working time of a machine is an exponentially distributed random variable.
4. All random variables are independently distributed.
5. The system of machines and repairmen is in a state of statistical equilibrium.

2 The single case with two machines

For the development of this analysis we first study the buffered flows of matter for only two machines. Two failure prone machines M_1 and M_2 are partially decoupled by the introduction of a buffer B_{12} which may contain a maximum of q pieces, where q is an integer (Figure 1). The mean time to failure and the mean time to repair are denoted, respectively, by λ^{-1}, and μ^{-1}. The production rate of M_j is ϑ_j [parts/time-unit], $j = 1, 2$, and the time-dependent content of the buffer B_{ij}, $\mathcal{Q}(t)$ $t \geq 0$, is viewed as a Markov chain with a space stateΩ $_2$, given by [See [19]]

$$\Omega_2 = \{\vartheta_1 - \vartheta_2, -\vartheta_2, \vartheta_1, 0\},$$

where, Max $[\mathcal{Q}(t)] = \vartheta_1$, and Min $[\mathcal{Q}(t)] = -\vartheta_2$. Thus, $\mathcal{Q}(t)$ is a random variable in the interval $[-\vartheta_2, \vartheta_1]$, where $\mathcal{Q}(t)$ is represented in this proposal by the stochastic process

$$\mathcal{Q}(t) = [\vartheta_1 \pi_1(t) - \vartheta_2 \pi_2(t)]\,. \tag{2.1}$$

Also, for $j = 1, 2$ we define

$$\pi_j(t) = \begin{cases} 1, & \text{if } M_j \text{ produces in } t \\ 0, & \text{in other case} \end{cases} \tag{2.2}$$

The waiting time intervals between transitions from states $\{0\}$ to $\{1\}$ and vice versa are characterized by the exponential probability distributions. From (2.2) we state and prove the following theorems and corollaries.

Theorem 2.1. *Let* $\mathbf{P}\,[\pi(t) = 1]$ *and* $\mathbf{P}\,[\pi(t) = 0]$ *be the probabilities that the machine jth state is operating or malfunctioning respectively at time* $t \geq 0$, *then*

$$\mathbf{P}\,[\pi(t) = 1] = \frac{\mu}{\mu + \lambda} + \frac{\lambda}{\mu + \lambda} e^{-(\lambda+\mu)t}, \tag{2.3}$$

$$\mathbf{P}\,[\pi(t) = 0] = \frac{\lambda}{\mu + \lambda} - \frac{\lambda}{\mu + \lambda} e^{-(\lambda+\mu)t}, \tag{2.4}$$

Proof. It is sufficient to prove (2.3). By the definition of $\pi(t)$ we have

$$\mathbf{P}\,[\pi(t) = 0] + \mathbf{P}\,[\pi(t) = 1] = 1. \tag{2.5}$$

Assuming that $\mathbf{P}\,[\pi(0) = 1] = 1$ and $\mathbf{P}\,[\pi(0) = 0] = 0$, the change in $\mathbf{P}\,[\pi(t) = 1]$ during the interval $[t, t + \Delta t]$ has two contributions: First $\lambda(t)\Delta t$ represents the loss of availability duringΔ t, i.e., $\lambda(t)\Delta t\;\mathbf{P}\,[\pi(t) = 1]$, and the gain of availability during the same interval is given by $\mu(t)\Delta t\;\mathbf{P}\,[\pi(t) = 0]$. If $\mu(t)$ and $\lambda(t)$ are defined by the constants μ and λ respectively, it follows that

$$\mathbf{P}\,[\pi(t + \Delta t) = 1] = \tag{2.6}$$

$$\mathbf{P}\,[\pi(t) = 1] - \lambda \Delta t\,\mathbf{P}\,[\pi(t) = 1] \;+\; \mu \Delta t\;\mathbf{P}\,[\pi(t) = 0].$$

For a smallΔ t, and rearranging terms, (2.6) can be written as

$$\frac{d}{dt}\mathbf{P}\,[\pi(t) = 1] = -(\lambda + \mu)\,\mathbf{P}\,[\pi(t) = 1] + \mu,$$

whose solution (using the initial condition $\mathbf{P}\,[\pi(0) = 1]$) is

$$\frac{\mu}{\mu + \lambda} + \frac{\lambda}{\mu + \lambda} e^{-(\lambda+\mu)t}.$$

THEOREM 2.2. *Let the stochastic process defined in (2.2), then for any s, $t \geq 0$, the transition probability functions, $p_{jk}(t) = \mathbf{P}\{\pi(t+s) = k \mid \pi(s) = j\}$ are given by*

$$
\begin{aligned}
p_{00}(t) &= \frac{\lambda}{\mu+\lambda} + \frac{\mu}{\mu+\lambda}\, e^{-(\mu+\lambda)t}, \\
p_{10}(t) &= \frac{\lambda}{\mu+\lambda}\left[1 - e^{-(\mu+\lambda)t}\right], \\
p_{01}(t) &= \frac{\mu}{\mu+\lambda}\left[1 - e^{-(\mu+\lambda)t}\right], \\
p_{11}(t) &= \frac{\mu}{\mu+\lambda} + \frac{\lambda}{\mu+\lambda}\, e^{-(\mu+\lambda)t}.
\end{aligned} \tag{2.7}
$$

Proof. Let the intensities of passage from 0 to 1 be given respectively by $q_0 = \mu$ and $q_1 = \lambda$. If the transition intensities are given by $q_{01} = \mu$ and $q_{10} = \lambda$, the proof follows from the general solution of a first order linear inhomogeneus differential equation of the two-state Markov Chain [See [20], [21]].

COROLLARY 2.1. *The expected value of the process $\pi(t)$ is given by*

$$\mathbf{E}\left[\pi(t)\right] = \frac{\mu}{\mu+\lambda} - \left(p_0 - \frac{\lambda}{\mu+\lambda}\right) e^{-(\mu+\lambda)t}, \tag{2.8}$$

where $p_0 = \mathbf{P}[\pi(0) = 0]$, and $\mathbf{E}$ is the expectation operator.

Proof. The proof follows from (2.7).

COROLLARY 2.2. *The expected value of the process (2.2) is given by*

$$\mathbf{E}\left[\mathcal{Q}(t)\right] = \mathbf{E}\left[\pi(t)\right]\,(\vartheta_1 - \vartheta_2)\,. \tag{2.9}$$

Proof. The proof is trivial.

Another variable of interest in this analysis is the fraction of time $\kappa(t)$ during the interval 0 to t with $t \geq 0$, on which the stochastic process $\pi_j(t)$ takes the value 1. We define

$$\kappa(t) = \frac{1}{t}\int_0^t \pi_j(u)du, \tag{2.10}$$

and from (2.8) it is possible to demonstrate [See [20]] that, as $t \to \infty$

$$\mathbf{E}[\kappa(t)] = \frac{1}{t}\int_0^t \mathbf{E}\left[\pi_j(u)\right]du \to \frac{\mu}{\mu+\lambda}, \tag{2.11}$$

which agrees with (2.8) when $t \to \infty$ (see next section).

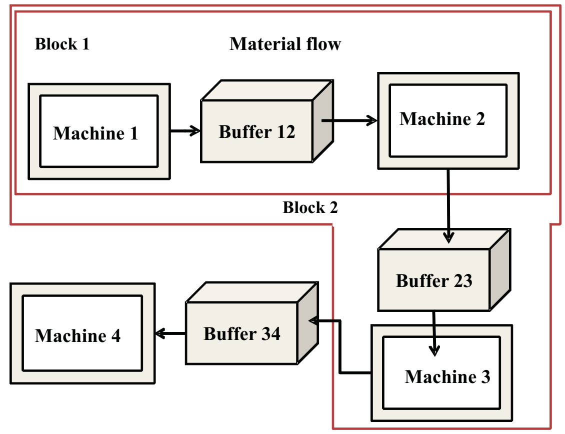

Figure 2: Generalization to more than two machines

3 Consideration of n machines

From the results obtained in the previous section, note that, as $t \to \infty$, (2.7) and (2.8) are simplified as follows

$$
\begin{aligned}
p_{00} &= \frac{\lambda}{\mu+\lambda}, \quad p_{10} = \frac{\lambda}{\mu+\lambda}, \\
p_{01} &= \frac{\mu}{\mu+\lambda}, \quad p_{11} = \frac{\mu}{\mu+\lambda},
\end{aligned} \tag{3.12}
$$

and

$$\mathbf{E}\left[\pi\right] = \frac{\mu}{\mu+\lambda}. \tag{3.13}$$

In order to generalize the above results for more complex systems we assume now that the two original machines form a single system that connects with the buffer 23 located between machines 2 and 3. Thus, we have again a system formed by two elements (Figure 2), the block 1 and machine 3 whose parameters can be generalized as follows.

The total amount of production, W_{n-1}, generated from the system with n machines (or equivalently, the production of block $(n-1)$) is, for $n = 2$

$$W_1 = \min\{\vartheta_1, \vartheta_2\}. \tag{3.14}$$

The mathematical expectation of W_1 can be obtained by

$$
\begin{aligned}
\mathbf{E}\left[W_1\right] &= \min\{\vartheta_1, \vartheta_2\}\,\left(\mathbf{P}[\pi = 1]\right)^2 \\
&= \min\{\vartheta_1, \vartheta_2\}\,\left(\tfrac{\mu}{\mu+\lambda}\right)^2,
\end{aligned} \tag{3.15}
$$

and the expectation of the random variable $\mathcal{Q}$ is

$$\mathbf{E}(\mathcal{Q}) = \frac{\mu}{\mu+\lambda}\,(\vartheta_1 - \vartheta_2).$$

The output produced in block 1 constitutes the input of the buffer 23, and then, the product enters the machine 3, which produces at a rate ϑ_3. Now, the space state for a system consisting of 3 machines is

$$\Omega_3 = \{\min\{\vartheta_1, \vartheta_2\} - \vartheta_3, -\vartheta_3, \min\{\vartheta_1, \vartheta_2\}, 0\}.$$

Let ϑ_{B_1} be the production generated from the block 1, then, the production generated from the block 2 is given by

$$\begin{aligned} W_2 = \min\{\vartheta_{B_1}, \vartheta_3\} &= \min\{\min\{\vartheta_1, \vartheta_2\}, \vartheta_3\} \\ &= \min\{\vartheta_1, \vartheta_2, \vartheta_3\}, \end{aligned}$$

and its mathematical expectation is now

$$\begin{aligned} \mathbf{E}[W_2] &= \min\{\min\{\vartheta_1, \vartheta_2, \}, \vartheta_3\} \left(\mathbf{P}[\pi = 1]\right)^3 \\ &= \min\{\vartheta_1, \vartheta_2, \vartheta_3\} \left(\tfrac{\mu}{\mu+\lambda}\right)^3. \end{aligned}$$

The average number of pieces inside the buffer of machine 3 is

$$\mathbf{E}(\mathcal{Q}) = \frac{\mu}{\mu+\lambda} \left(\min\{\vartheta_1, \vartheta_2, \} - \vartheta_3\right).$$

From the previous results it is evident that, if $\gamma(n-1) = \min\{\vartheta_1, \ldots, \vartheta_{n-1}\}$, then

$$\Omega_n = \{\gamma(n-1) - \vartheta_n, -\vartheta_n, \gamma(n-1), 0\}, \tag{3.16}$$

$$W_{n-1} = \min\{\vartheta_1, \ldots, \vartheta_n\}, \tag{3.17}$$

$$\mathbf{E}[W_{n-1}] = \min\{\vartheta_1, \ldots, \vartheta_n\} \left(\frac{\mu}{\mu+\lambda}\right)^n, \tag{3.18}$$

and

$$\mathbf{E}(\mathcal{Q}) = \frac{\mu}{\mu+\lambda} \left(\gamma(n-1) - \vartheta_n\right). \tag{3.19}$$

Note that, in any case, the efficiency η_{n-1} of the production of block $(n-1)$, can be measured as a function of $\mathbf{E}(\pi)$ as follows

$$\frac{\text{Real production}}{\text{Nominal production}} = \frac{\mathbf{E}[W_{n-1}]}{W_{n-1}}, \tag{3.20}$$

where we have approximated the real production by their mathematical expectation. Obviously, the greatest efficiency (and the best precision of the estimators) is reached when $\lambda << \mu$, and n is small (not greater than 5, for example).

Analogously, the rate of time in which the system formed by n machines remains in the state 1 does not depend on ϑ_j, $j = 1, \ldots, n$. This is easily seen because (2.11) is the relevant equation and it has no explicit

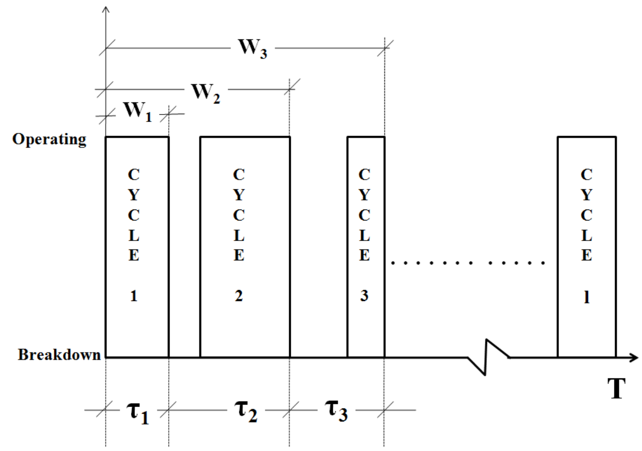

Figure 3: Cycles of waiting time for the lth failure

dependence upon ϑ_j, since, μ and λ can be defined in terms of the sum of μ_j and λ_j as follows:

$$\mu = \sum_{j=1}^{n} \mu_j = n\mu, \quad \text{and} \quad \lambda = \sum_{j=1}^{n} \lambda_j = n\lambda.$$

When the μ's and λ's are not equal, the quotient $\mu/(\mu+\lambda)$ must be replaced by

$$\frac{\sum_{j=1}^{n} \mu_j}{\sum_{j=1}^{n} \mu_j + \sum_{j=1}^{n} \lambda_j}.$$

Thus, the same result is obtained replacing μ and λ with their corresponding sums in (2.11). Therefore

$$\mathbf{E}[\kappa] \approx \frac{\mu}{\mu+\lambda}.$$

4 The expected time to l failures

It is actually important to evaluate the elapsed time from starting the system until the first failure appears, or to enter the number of failures that appear in a certain interval of time. In this section we are interested in determining the waiting time for the occurrence of l failures of the system, where l is integer-valued.

Consider again a system formed by n machines connected in series with identical rates λ and μ. We assume that these failures are independent and they occur in each machine composing the system. The reliability of each machine is defined as the probability that it operates without failure for a length of time t, and this may be expressed in terms of the random variable $\mathcal{T}$, the time to machine failure as

$$R(t) = \mathbf{P}[\mathcal{T} > t] = \int_t^{\infty} f(t^{'})dt^{'} = e^{-\lambda t},$$

Table 1: Results of the numerical example

Stage	i		Parameters measured $\mathbf{E}[\pi]$	W_i	$\mathbf{E}[W_i]$	$\mathbf{E}[\mathcal{Q}]$	η	$\mathbf{E}(A_r)$	$\mathbf{Var}(A_r)$
Block	1	Analytical	0.9900	14	13.7241	0	0.9802	500	250,000
		Real	0.9861	13.7065	-	-0.0499	–	–	-
Block	2	Analytical	0.9900	14	13.5882	-1.9801	0.9705	333.33	111,111.1111
		Real	0.9915	13.6128	-	-2.1150	–	–	-
Block	3	Analytical	0.9900	14	13.5882	-0.9900	0.9705	250	62,500
		Real	0.9910	13.4317	-	-1.2983	–	–	-

where $f(t^{'}) = \lambda e^{-\lambda t^{'}}$, for $t^{'} \geq 0$. Thus, for a system with n machines connected in series we have

$$R_n(t) = \Pi_{i=1}^{n} e^{-\lambda_i t} = e^{-n\,\lambda\,t} \; = \; e^{-\hat{\lambda} t}, \tag{4.21}$$

hence, $\hat{\lambda} = n\lambda$ represents the total rate of failure (hazard function) of the system, i.e., $\hat{\lambda}^{-1}$ is the expected value of the system failure time.

Since failures of the total system occur in the interval 0 to ∞, we define the successive inter-arrival times $\tau_1, \tau_2, \ldots,$ as follows [20]: τ_1 is the time from 0 to the first failure, and for $j > 1$, τ_j is the time from the $(j-1)$st to the jth failure (Figure 3). Let A_l be the random variable called the waiting time to the l-th failure, which represents the time it takes to register l cycles of failures. Then

$$\tau_1 = A_1, \tau_2 = A_2 - A_1, \ldots, \tau_l = A_l - A_{l-1}, \tag{4.22}$$

from (4.22) its clear that, for $l \geq 1$ the following equation holds

$$A_l = \tau_1 + \tau_2 + \ldots + \tau_l.$$

In order to simplify our proposal we assume that the random variable A_l is defined for $t \geq 0$ in accordance with a Poisson process at mean rate $\hat{\lambda}$. Then, A_l is given by the density gamma probability with parameters l and $\hat{\lambda}$, and therefore

$$\begin{aligned} f_{A_l}(t) &= \hat{\lambda} e^{-\hat{\lambda} t} \tfrac{(\hat{\lambda} t)^{l-1}}{(l-1)!}, \quad && t > 0 \\ &= 0, && t < 0. \end{aligned} \tag{4.23}$$

For $t > 0$, the cumulative density function (cdf) of A_l, is

$$F_{A_l}(t) = 1 - e^{-\hat{\lambda} t} \left(\sum_{i=0}^{(l-1)} \frac{\left(\hat{\lambda} t\right)^i}{(i)!} \right). \tag{4.24}$$

From (4.24) we consider the reliability function, which represents the probability that l failures occur after t; i.e., the possible amount of allowed cycles of failure in the system before pausing itself.

$$R_{A_l}(t) = e^{-\hat{\lambda} \tau} \left(\sum_{i=0}^{(l-1)} \frac{\left(\hat{\lambda} t\right)^i}{(i)!} \right), \tag{4.25}$$

and

$$\mathbf{E}\left[A_l\right] = \frac{l}{\hat{\lambda}}, \tag{4.26}$$

$$\mathbf{Var}\left[A_l\right] = \frac{l}{\hat{\lambda}^2}, \tag{4.27}$$

where $\hat{\lambda}$ is defined as before, if all the machines have identical rates λ and μ. In another case, $\hat{\lambda} = \sum_{i=1}^{n} \lambda_i$.

5 Numerical example

To illustrate our results we developed a simulation model using a spreadsheet. We considered a time horizon of 2242 hours, the corresponding values of the production rates were: $\vartheta_1 = 14, \vartheta_2 = 14$, $\vartheta_3 = 16$ and $\vartheta_4 = 15$. The the intensities evaluated were $\lambda = 0.001$ and $\mu = 0.1$; with $l = 1$ cycle. The simulation results, which are shown in Table 1, suggested that our mathematical model gives a good approximation of the performance estimators of the activity of a FMS under the assumptions made here.

6 Conclusions

The study of the buffered flows is an important topic not only in production lines, but also in data transmission, telephone traffic, dam problems, etc. The dynamics of the population level of a buffer can be described by stochastic differential equations in which the noise is continuous Markov chains.

In this paper we assume that the system is nearly balanced. This concept means that every isolated machine works with an identical mean throughput [19]. We restricted our analysis to the Markov dynamics in which the time between failure and the time to repair are exponentially distributed, and it is only required to evaluate the λ and μ intensities. Our models demonstrate that the throughput of a FMS is not easy to predict without using models that recognize the stochastic effects owing to the mix of jobs and their random processing times.

References

[1] G. J. Miltenburg, and I. Krinsky. Evaluating flexible manufacturing systems, *IIE Transactions*, Vol. 19, No. 2, pp. 222–232, 1987.

[2] N. Kulatilaka. Valuing the flexibility of flexible manufacturing systems. *IEEE Transactions on Engineering Management*, Vol. 35, No. 4, pp. 250–257, 1988.

[3] G. E. Monahan, and T. L. Smunt. Optimal acquisition of flexible manufacturing processes, *Operations Research*, Vol. 37, No. 2, pp. 288–300, 1989.

[4] M. C. Burstein, and M. Talbi, M. Economic justification of the introduction of flexible manufacturing technology: Traditional procedures versus a dynamics based approach, *Proceedings of the First ORSA/TIMS Conference on Flexible Manufacturing Systems*, edited by K.E. Stecke and R. Suri, pp. 100–106, 1984.

[5] C. Gaimon. Dynamic game results of the acquisition of new technology, *Operations Research*, Vol. 37, No. 3, pp. 410–425, 1989.

[6] L. C. Leung, and J. M. A. Tanchoco. Multiple machine replacement within an integrated system framework, *The Engineering Economist*, Vol. 32, No. 2, pp. 89–114, 1987.

[7] V. M. Ozernoy. Choosing the "best" multiple criteria decision-making method, *Infor*, Vol, 30, No. 2, pp. 159–171, 1992.

[8] N. C. Suresh. An extended multi-objective replacement model for flexible automation investments, *International Journal of Production Research*, Vol. 29, No. 9, pp. 1823–1844, 1991.

[9] J. R. Canada, and W. G. Sullivan. *Economic and Multiattribute Evaluation of Advanced Manufacturing Systems,* Prentice-Hall, Englewood Cliffs, N.J., 1989.

[10] H. R. Parsaei, and M. R. Wilhelm. A justification methodology for automated manufacturing technologies, *Computers and Industrial Engineering*, Vol. 16, No. 3, pp. 363–373, 1989.

[11] S. U. Randhawa, and T. M. West. A multi-attribute methodology for the evaluation of automated assembly systems, *Proceedings of the Industrial Engineering Conference*, May 11-15, Dallas, TX, 1986, pp. 526–533, 1986.

[12] H. R. Parsaei, M. R. Wilhelm, and S. S. Kolli. Application of outranking methods to economic and financial justification of CIM systems, *Computers and Industrial Engineering*, Vol. 25, Nos.-4, pp. 357–360, 1993.

[13] J. W. Troxler, L. Blank. A comprehensive methodology for manufacturing system evaluation and comparison, *Journal of Manufacturing Systems*, Vol. 8, No. 3, pp. 175–183, 1989.

[14] D. Sipper. and R. L Bulfin Jr. *Production: Planning, Control, and Integration*. New York: The McGraw-Hill Companies, Inc., 1997.

[15] J. D. Thompson. *Organizations in Action*, The McGraw Hill Companies, Inc., New York, 1967.

[16] P. Naor. On machine interference, *Journal of the Royal Statistical Society*. Series B (Methodological), Vol. 18, No. 2, pp. 280–287, 1956.

[17] A. T. Bharucha-Reid. *Elements of the Theory of Markov Processes and Their Applications*, Dover Publications, New York, 1988.

[18] J. O. McClain and D. R. Moodie. A comment on "buffer space allocation in automated assembly lines", *Operations Research*, Vol. 39, No. 5, pp. 857–860, 1991.

[19] M. O. Hongler. *Chaotic and Stochastic Behaviour in Automatic Production Lines,* New Series m: Monographs, Lecture Notes in Physics, Springer-Verlag, 1994.

[20] E. Parzen. *Stochastic Process*, Classics in Applied Mathematics, SIAM, Philadelphia, 1999.

[21] G. P. Lechuga, A. A. Karelyn, M. G. M. Orán, and L. M. PLá. Stochastic analysis of operations decoupling in flexible manufacturing systems, *WSEAS Transactions on Systems*, Issue 11, Vol. 4, pp. 2138–2143, 2005.