The Effect of Vibration on Flow Rate of Non-Newtonian Fluid

Mohsen Asle Zaeem* Sergey Lapin† Konstantin Matveev‡

Abstract

Acoustic stimulation is a promising method for increasing drainage of non-Newtonian fluids through porous structures in various applications. In this study, a mathematical model is developed for unsteady flow of a bi-viscous incompressible fluid in a circular straight channel. Longitudinal vibrations are superimposed on the flow driven by changing pressure gradients along the channel. Simulations are carried out for a range of relevant dimensionless parameters. Effects of vibration amplitude, frequency, and fluid viscosity ratio on the enhancement of mean flow rate are discussed.

1 Introduction

Flows of non-Newtonian fluids in narrow channels and porous structures are encountered in the vast majority of chemical and process industries. The areas of applications are very broad and diverse. They include various applications in petroleum industries [3], biomedical and processing industries [8, 9] as well as important applications in food engineering and processing [10].

One of the practical problems encountered with high-viscosity non-Newtonian fluids is augmenting the mean flow rate in a channel at a given mean pressure gradient.

It is known that by using sound or vibrations it is possible to increase the time-averaged flow of shear-thinning and viscoplastic fluids. Barnes et al. [1] observed an increase of flow rates of a polymer solution in a pipe under pulsating pressure gradients. Kazakia and Rivlin [7] calculated the mean flow enhancement for a slightly non-Newtonian fluid.

Deshpande and Barigou [5] carried out simulations of oscillatory flows of non-Newtonian fluids using a CFD code. While they observed changes of flow rates of shear-thinning and shear-thickening fluids, they found no effect from oscillations on the mean flow rate of a Bingham fluid. This observation contradicts a quasi-steady analytical solution of Iassonov and Beresnev [6] that shows the mean flow rate amplifications for a Bingham fluid in the low-frequency limit.

In this study, an unsteady flow of a non-Newtonian incompressible fluid is analyzed in a circular cross section of a channel under the presence of an oscillating pressure gradient. Results of numerical simulations carried out for a range of relevant dimensionless parameters are presented.

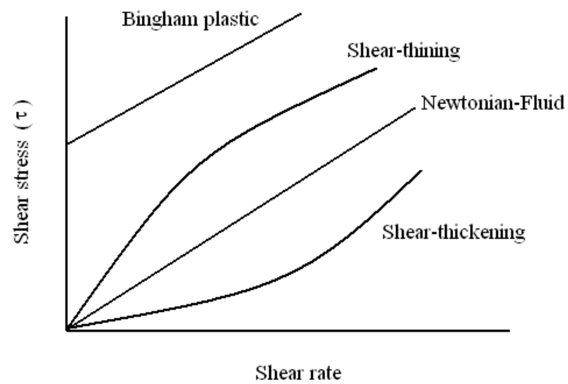

Figure 1: Dependence of shear stress on shear rate for different type of fluids.

2 Methodology

In this work we propose a model which effectively approximates shear-thinning and shear-thickening fluids and a Bingham fluid by using a linear functions approximations.

The governing equation for the fluid flow is the momentum equation [4]:

$$\rho\frac{\partial\vec{v}}{\partial t}+\rho(\vec{v}\cdot\nabla)\vec{v}=-\nabla p+\nabla\cdot\tau, \quad (2.1)$$

where $\vec{v}$ is the fluid velocity, p is the fluid pressure, ρ is the fluid density, and τ is the stress tensor.

We consider an axisymmetric flow in a tube with a constant radius. If we assume parallel flow, we can neglect the pressure gradient and velocity components normal to the wall of the tube, so the effect of the nonlinear term in Equation 2.1 becomes negligible. By considering these assumptions, the momentum equation in the cylindrical coordinate system has the following form:

*School of Mechanical and Materials Engineering, Washington State University, Pullman, WA
†Department of Mathematics, Washington State University, Pullman, WA
‡School of Mechanical and Materials Engineering, Washington State University, Pullman, WA

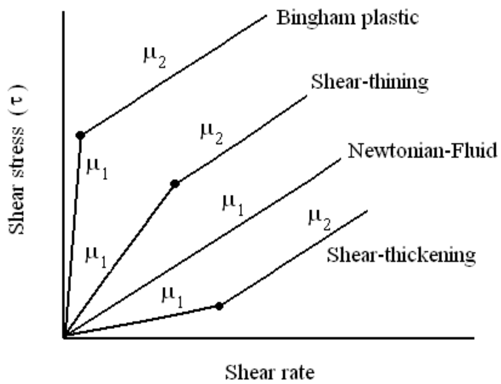

Figure 2: Dependence of shear stress on shear rate for non-Newtonian fluids by linear functions.

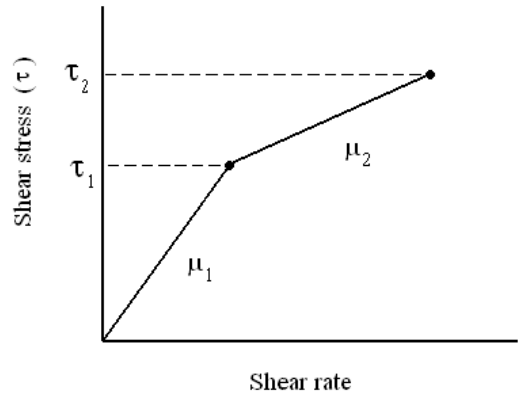

Figure 3: The parameters for Equation 2.3.

$$\rho \frac{\partial v}{\partial t} = \left(-\frac{\partial p}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}(r\tau),\ 0 \le r < R, \tag{2.2}$$

where R is the radius of the tube and v is the axial velocity.

The constitutive law relating the stress and strain rate of the fluid is given by

$$\tau = \begin{cases} \mu_1 \frac{\partial v}{\partial r} & \text{if} \quad \tau < \tau_1, \\ \mu_2 \frac{\partial v}{\partial r} + \tau_1 & \text{if} \quad \tau_1 \le \tau < \tau_2, \end{cases} \tag{2.3}$$

where μ_1 and μ_2 are dynamic viscosities and τ_1 and τ_2 are yield stresses (see Fig. 3).

By assuming constant pressure gradient $\frac{\partial p}{\partial z} = G$ and employing the Equation 2.3 we obtain

$$\rho \frac{\partial v}{\partial t} = -G + \frac{\mu_n}{r}\frac{\partial v}{\partial r} + \mu_n \frac{\partial^2 v}{\partial r^2} + \frac{\tau_n}{r},\ 0 \le r < R. \tag{2.4}$$

where $n = 1, 2$ based on Equation 2.3.

The described linear functions approximation can be easily extended to the multiviscous fluids case.

To model the effects of unsteady pressure gradient on the fluid flow inside the tube, longitudinal vibrations are applied to the channel wall parallel to the direction of the fluid flow. The displacement of the wall is given by the following relation:

$$w = be^{i\omega t}, \tag{2.5}$$

where b is the displacement amplitude and ω is the angular frequency of the vibration.

Now we can define the relative velocity of the fluid inside the tube with respect to the wall movement:

$$U = (v - \dot{w}). \tag{2.6}$$

Substituting Equation 2.6 into Equation 2.4, the governing equation for the flow takes the following form:

$$\rho \frac{\partial U}{\partial t} = \rho b \omega^2 e^{i\omega t} - G + \frac{\mu_n}{r}\left(\frac{\partial U}{\partial r}\right) + \mu_n \left(\frac{\partial^2 U}{\partial r^2}\right) + \frac{\tau_n}{r}, \tag{2.7}$$

where n depends on Equation 2.3. The no-slip boundary condition for a tube with a constant radius R becomes:

$$U(R, t) = 0. \tag{2.8}$$

Instantaneous volume flow rate through the tube is defined as

$$Q(t) = 2\pi \int_0^R U(r, t) r dr. \tag{2.9}$$

To find the mean flow rate, the average of the instantaneous flow rate over a period of oscillation can be calculated by:

$$Q_m = \frac{1}{T}\int_0^T Q(t) dt. \tag{2.10}$$

In the present paper the value of μ_2 is considered to be a constant and it is the viscosity of crude oil.

For numerical simulations it is convenient to use non-dimensional variables. Let us first denote by $Q_p(G)$ the flow rate of the Poiseuille flow [2], which is given by

$$Q_p(G) = -\frac{\pi R^4}{8\mu} G. \tag{2.11}$$

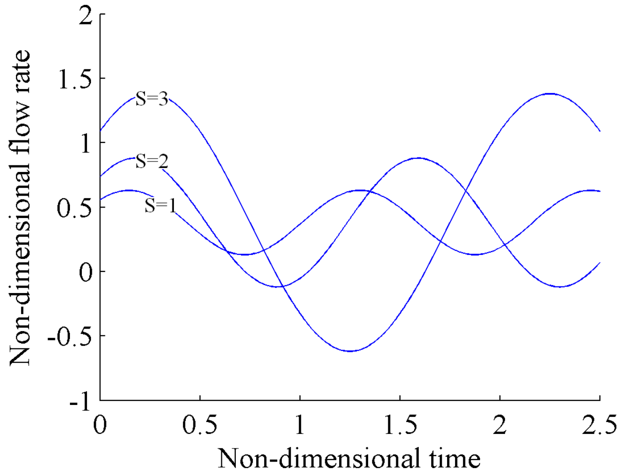

Figure 4: Flow rates for different values of effective amplitude of oscillation; Newtonian fluid, $\Theta = 1, g = 1$.

Now we can introduce a non-dimensional flow rate

$$q = \frac{Q}{Q_p(G_c)}, \tag{2.12}$$

where

$$G_c = \frac{2\tau_1}{R}$$

is the minimal external pressure gradient required to mobilize the yield stress fluid in channel of given radius R.

We also define non-dimensional pressure gradient

$$g = \frac{G}{G_c}. \tag{2.13}$$

In order to see dependence of flow rate on vibrations of the tube we introduce the dimensionless "effective" amplitude of oscillations as:

$$s = \frac{\rho b \omega^2}{G_c}. \tag{2.14}$$

3 Results

A bi-viscous fluid is defined by the specific dependence of its shear stress on the shear rate. At low absolute values of shear rates, the fluid viscosity is greater than at higher shear rates; and their viscosity ratio is defined by the parameter $\Theta = \mu_1/\mu_2$ (Fig. 3).

In our numerical simulations we consider the following fluids:

- Newtonian fluid .

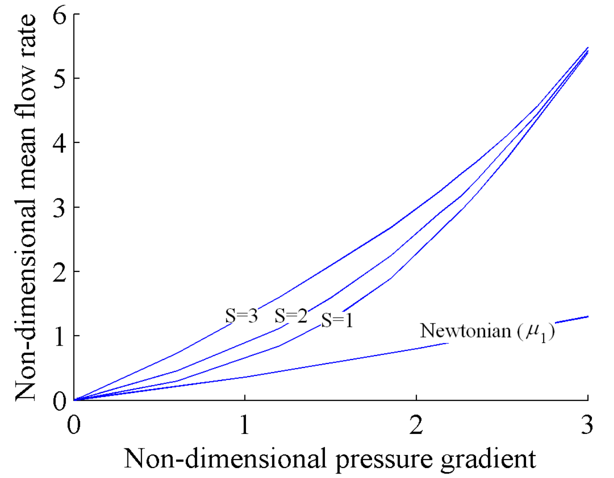

Figure 5: Flow rate versus pressure gradient for different values of effective amplitude of oscillation; Shear-thinning fluid, $\Theta = 4$.

	g =1	g =2	g =3
$s = 1$	1.86	2.86	4.15
$s = 2$	2.50	3.25	4.18
$s = 3$	3.67	3.75	4.21

Table 1: Ratios of shear-thinning fluid flow rate to Newtonian flow rate for different values of gradient g and amplitude of oscillations s.

- Shear-thickening fluid.
- Shear-thinning fluid.
- Approximation to Bingham fluid.

In Figure 4, flow rate of a Newtonian fluid is plotted for different values of the effective amplitude of oscillation, s. It can be seen in Figure 4 that for a constant pressure gradient, the same mean flow rate, equal to 0.4, is obtained for different values of s. It means that the oscillatory displacement of the wall does not affect the mean flow rate of the Newtonian fluid in the pores.

Figure 5 and Figure 6 show flow rates of shear-thinning and shear-thinking fluids, respectively. In Table 1 and Table 2 we present ratios of computed vibrational flow rate to Newtonian flow rate for different values of gradient and amplitude of oscillations. As it can be seen in the Figure 5 and Table 1, for a constant pressure gradient, the mean flow rate of a shear-thinning fluid will increase by increasing the effective amplitude of oscillations. We see completely opposite behavior for

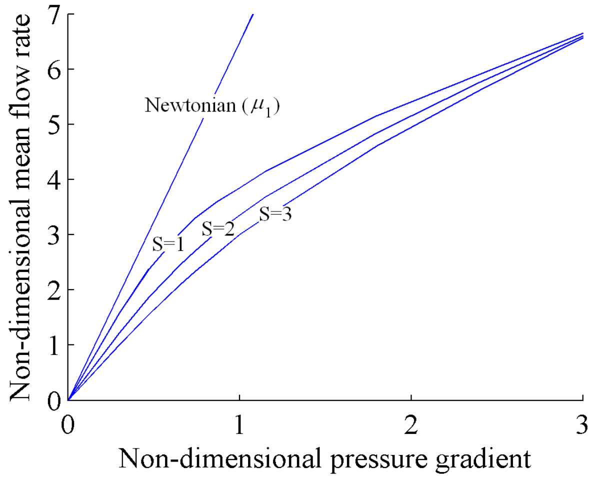

Figure 6: Flow rate versus pressure gradient for different values of effective amplitude of oscillation; Shear-thickening fluid, $\Theta = 0.254$.

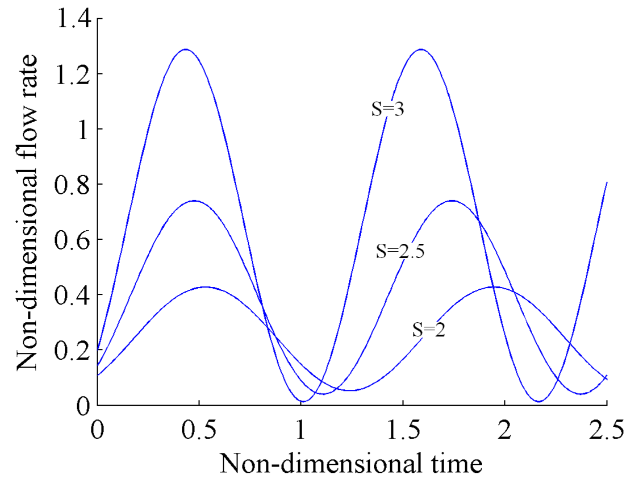

Figure 7: Flow rates for different values of effective amplitude of oscillations; Bingham fluid, $\Theta = 100, g = 1$.

	g =1	g =2	g =3
$s = 1$	0.592	0.404	0.332
$s = 2$	0.515	0.380	0.330
$s = 3$	0.416	0.361	0.318

Table 2: Ratios of shear-thickening fluid flow rate to Newtonian flow rate for different values of gradient g and amplitude of oscillations s.

a shear-thickening fluid (Fig.6, Table 2). For high values of effective amplitude of oscillation, applying different frequencies will not affect the mean flow rate of neither shear-thinning nor shear-thickening fluids.

Figure 7 shows flow rate of a Bingham fluid [1] for three different values of effective amplitude s. It can be seen in Figure 7 that for a constant pressure gradient, the mean flow rate will be different for different values of s. For higher values of s, the oscillatory displacement will boost the mean flow rate of the Bingham fluid in the pores.

Figure 8 shows flow rate of Bingham fluid versus pressure gradient for different values of effective amplitude of oscillations. In Table 3 we also summarized values for ratio of Bingham flow to Newtonian flow for various representative values of gradient and amplitude of oscillations. One can clearly observe that by increasing the frequency of oscillations s we can improve the flow performance of Bingham fluids. Especially notice the improvement we obtain for lower values of pressure gradient g, for instance $g = 1$. However, for higher values of g the oscillations have almost no effect on the flow rate of Bingham fluid.

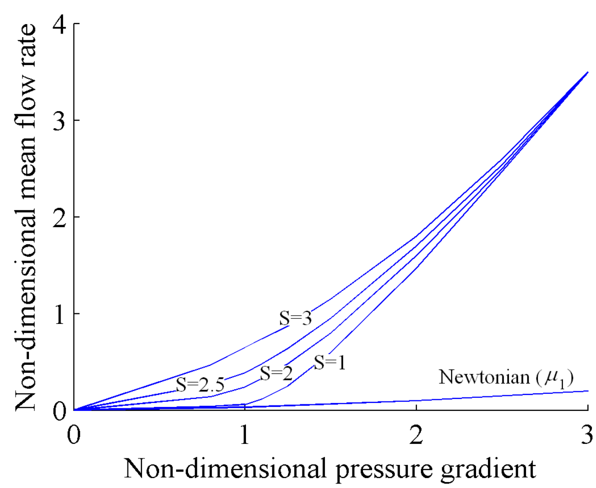

Figure 8: Flow rate versus pressure gradient for different values of effective amplitude of oscillations; Bingham fluid, $\Theta = 100$.

4 Conclusion

Calculations have been carried out to observe the effect of oscillations of longitudinal pressure gradient on the mean flow rate of non-Newtonian fluids. It is found that oscillations enhance flow rate of a bi-viscous shear-

[1] To approximate Bingham fluid we essentially use biviscous model with very high viscosity ratio Θ.

	g =1	g =2	g =3
$s = 1$	2	14.7	17.45
$s = 2$	8	16	17.50
$s = 3$	18	19	17.50

Table 3: Ratios of Bingham flow rate to Newtonian flow rate for different values of gradient g and amplitude of oscillations s.

thinning fluid while the effect of oscillations on the shear-thickening fluid is the opposite.

The Bingham fluid exhibits even more significant augmentation of the mean flow rate when subjected to vibrations. These effects are observed in a certain range of the mean pressure gradient.

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