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IP1

Hypergraph Decompositions and their Applications

Many combinatorial objects can be thought of as a hypergraph decomposition, i.e. a partition of (the edge set of) one hypergraph into (the edge sets of) copies of some other hypergraphs. For example, a Steiner Triple System is equivalent to a decomposition of a complete graph into triangles. In general, Steiner Systems are equivalent to decompositions of complete uniform hypergraphs into other complete uniform hypergraphs (of some specified sizes). The Existence Conjecture for Combinatorial Designs, which I proved in 2014, states that, bar finitely many exceptions, such decompositions exist whenever the necessary 'divisibility conditions' hold. I also obtained a generalisation to the quasirandom setting, which implies an approximate formula for the number of designs; in particular, this resolved Wilson's Conjecture on the number of Steiner Triple Systems. A more general result that I proved in 2018 on decomposing lattice-valued vectors indexed by labelled complexes provides many further existence and counting results for a wide range of combinatorial objects, such as resolvable designs (the generalised form of Kirkman's Schoolgirl Problem), whist tournaments or generalised Sudoku squares. In this talk, I plan to review this background and then describe some more recent and ongoing applications of these results and developments of the ideas behind them.

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IP2

Combinatorial Theories in Machine Learning

Recent years have witnessed tremendous progress in the field of Machine Learning (ML). However, many of the recent breakthroughs demonstrate phenomena that lack explanations, and sometimes even contradict conventional wisdom. One main reason for this is because classical ML theory adopts a worst-case perspective which seems too pessimistic to explain practical ML: in reality data is rarely worst-case, and experiments indicate that often much less data is needed than predicted by traditional theory. In this talk we will discuss two variations of classical learning theory. These models are based on a distribution- and data-dependent perspective which complements the distribution-free worst-case perspective of classical theory, and is suitable for exploiting specific properties of a given learning task. A common theme of these models is their combinatorial nature. This can be seen as a continuation of the fruitful link between machine learning and combinatorics, which goes back to the discovery of the VC dimension more than 50 years ago.

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IP3

Irregular Triads in 3-uniform Hypergraphs

Szemerdi's celebrated regularity lemma states, roughly speaking, that the vertex set of any large graph can be partitioned into a bounded number of sets in such a way that all but a small proportion of pairs of sets from this partition induce a 'regular' graph. The example of the half-graph shows that the existence of irregular pairs cannot be ruled out in general. Recognising the half-graph as an instance of the so-called 'order property' from model theory, Malliaris and Shelah proved in 2014 that if one assumes that the large graph contains no half-graphs of a fixed size, then it is possible to obtain a regularity partition with no irregular pairs. In addition, the number of parts of the partition is polynomial in the regularity parameter, and the density of each regular pair is either close to zero or close to 1. This beautiful result exemplifies a long-standing theme in model theory, namely that so-called stable structures (which are characterised by an absence of large instances of the order property), are extremely well-behaved. In this talk I will present recent joint work with Caroline Terry (OSU), in which we define a higher-arity generalisation of the order property and prove that its absence characterises those large 3-uniform hypergraphs whose regularity decompositions allow for particularly good control of the irregular triads.

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IP4

Nibble and Absorption Methods for Colourings and Decompositions

Since its inception in the 1980's (by Ajtai-Komlos-Szemerdi as well as Rdl), the 'nibble' or 'semirandom method' has had a transformative impact e.g. on Ramsey theory, design theory as well as graph and hypergraph colouring. More recently, its potential has been further enhanced by combining the nibble with absorption approaches. The latter aim to 'improve' a near-optimal structure (obtained e.g. by the nibble) into an optimal one, usually via local transformations. I will illustrate this with the proof of the Erds-Faber-Lovsz conjecture on colouring linear hypergraphs (joint work with Dong-yeap Kang, Tom Kelly, Daniela Khn, Abhishek Methuku). Our proof draws on edge-colouring and vertex-colouring results obtained via the 'nibble as well as absorption arguments which turn almost perfect matchings into 'optimal' ones. Our argument also applies an earlier result on Hamilton decompositions (joint with Daniela Khn), whose proof in turn first introduced absorption methods to decomposition problems. I will also discuss some related results and open questions e.g. on large matchings in hypergraphs as well as colourings of locally sparse graphs.

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IP5

The Erds-Ginzburg-Ziv Problem

The Erdös-Ginzburg-Ziv Problem is a classical extremal problem in discrete geometry. For given numbers m and n, it asks the following question: What is the minimum number s such that among any s points in the integer lattice Z^n there are m points whose centroid is also a lattice point? It turns out that it essentially suffices to consider the case where m=p is a prime number, and that the problem then naturally translates into a problem over the finite field F_p. Surprisingly, a wide range of different algebraic techniques can be used to approach this problem in different ranges for p and n. This talk will give an overview of the known results and bounds for the Erdös-Ginzburg-Ziv

Problem, and of the different techniques that were used to obtain them. A particular focus of the talk will be the case where m=p is a fixed prime and n is large with respect to p. In this case, the relevant techniques are related to the slice rank polynomial method, which appeared first in the context of the famous cap-set problem about subsets of F_3^n without arithmetic progressions.

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IP6

On Learning in the Presence of Biased Data and Strategic Behavior

In this talk I will discuss two lines of work involving learning in the presence of biased data and strategic behavior. In the first, we ask whether fairness constraints on learning algorithms can actually improve the accuracy of the classifier produced, when training data is unrepresentative or corrupted due to bias. Typically, fairness constraints are analyzed as a tradeoff with classical objectives such as accuracy. Our results here show there are natural scenarios where they can be a win-win, helping to improve overall accuracy. In the second line of work we consider strategic classification: settings where the entities being measured and classified wish to be classified as positive (e.g., college admissions) and will try to modify their observable features if possible to make that happen. We consider this in the online setting where a particular challenge is that updates made by the learning algorithm will change how the inputs behave as well.

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IP7

Localization Schemes: A Framework for the Analysis of Sampling Algorithms

Two recent and seemingly-unrelated techniques for proving mixing bounds for Markov chains are: (i) the framework of "Spectral Independence", introduced by Anari, Liu and Oveis Gharan, and its numerous extensions, which have given rise to several breakthroughs in the analysis of mixing times of discrete Markov chains and (ii) the Stochastic Localization technique which has proven useful in establishing mixing and expansion bounds for both log-concave measures and for measures on the discrete hypercube. In this talk, I'll present a framework which aims to both unify and extend those techniques, thus providing an approach that gives bounds for sampling algorithms in both discrete and continuous settings. In its center is the concept of a "localization scheme" which, to every probability measure on some space Ω (which will usually be either the discrete hypercube or R^n), assigns a martingale of probability measures which "localize" in space as time evolves. As it turns out, every such scheme can be associated with a Markov chain, and many chains of interest (such as Glauber dynamics) appear naturally in this framework. This viewpoint provides tools for deriving mixing bounds for the dynamics through the analysis of the corresponding localization process. Generalizations of the concept of Spectral Independence naturally arise from our definitions, and in particular we will show how to recover the main theorems in the spectral independence framework via simple martingale arguments (completely bypassing the need to use the theory of high-dimensional expanders). We demonst how to apply our machinery towards simple proofs to m mixing bounds in the recent literature. We will br discuss some applications, among which are obtaining first $O(n \log n)$ bound for mixing time of the hardc model (of arbitrary degree) in the tree-uniqueness reg under Glauber dynamics and to proving a KL-diverg decay bound for log-concave sampling via the Restri Gaussian Oracle, which achieves optimal mixing under $\exp(n)$-warm start. Based on a joint work with Yu Chen.

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IP8

The Promise and Perils of Parameterized Al rithms

Parameterized graph algorithms offer a tantalizing t box for designing efficient solutions in the face of comp tionally intractable problems. Despite this, they are in quently employed in real-world network analysis, for a r iad of not-entirely-invalid reasons. In this talk, we su the key challenges in bridging this theory-practice gap describe the landscape of recent work, highlighting sev successful collaborations in computational genomics. prior knowledge of parameterized algorithms/comple is assumed.

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SP1

2022 Dnes Knig Prize Lecture: Quantita Bounds in the Polynomial Szemerdi Theorem Related Results

Let $P_1, \ldots, P_m$ be polynomials with integer coefficients zero constant term. Bergelson and Leibmans polyno generalization of Szemerdis theorem states that any set A of $\{1, \ldots, N\}$ that contains no nontrivial prog sions $x, x + P_1(y), \ldots, x + P_m(y)$ must satisfy $|A| = o($ In contrast to Szemerdis theorem, quantitative bound Bergelson and Leibmans theorem (i.e., explicit bound this $o(N)$ term) are not known except in very few spe cases. In this talk, I will discuss recent progress on pro a quantitative version of the polynomial Szemerdi theo and related problems in additive combinatorics, harm analysis, and ergodic theory.

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CP1

Cliques in Realization Graphs of Degree Seque

The realization graph $\mathcal{G}(d)$ of a degree sequence d is graph whose vertices are the labeled realizations of d which edges join realizations that differ by switching a gle pair of edges. We describe a structure in realizat of d that determines precisely whether $\mathcal{G}(d)$ has a cliq size n, for any n. This allows us also to characterize degree sequences d having complete graphs as their ization graphs.

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CP1

Odd Covers of Graphs

The "odd cover problem" of finding the minimum number of complete bipartite graphs, or bicliques, which cover every edge of the complete graph an odd number of times was proposed by Babai and Frankl in 1992. A more general question asks, given a simple graph G, for the minimum number of bicliques such that each edge of G is in an odd number of bicliques and each non-edge in an even number, denoted $b_2(G)$. This talk will examine each of these problems. We show that $b_2(G)$ is at least half of the rank over $\mathbb{F}_2$ of the adjacency matrix of G, and that this bound is tight for bipartite graphs. We also provide new bounds for the odd cover problem, and solve the problem for a density 3/8 portion of complete graphs. Joint work with Alexander Clifton, Eric Culver, Jiaxi Nie, Jason O'Neill, Puck Rombach, and Mei Yin, began at GRWC 2021.

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CP2

Reinforcement Learning Model for a Grid-Based Cops Vs Robbers

Decision-making models require the ability to determine efficient optimized actions across multiple entities in complex environments. In recent years, researchers have applied various approaches to multi-agent environments, including game theory, optimization, and more recently, deep reinforcement learning. Most deep reinforcement learning algorithms treat agents as decentralized with rewards that do not require cooperation across agents. In this talk, we present our development of a grid-based board game of cops vs. robbers to benchmark various existing and novel multi-agent learning techniques. We model this multi-agent environment as a stochastic game with side payments to help incentivize team members to work together. Our work is based on cooperative-competitive (COCO) values (Kalai & Kalai, 2010) as a solution for a two-player normal form game that leverages side payments. We aim to integrate the Q-learning algorithm extension of COCO values (Sodomka et al., 2010) with multi-agent neural networks to handle the difficult problem of state-space estimation as the number of features increase.

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CP2

A Parallel Variant of the Tower of Hanoi Puzzle

A parallel variant of the Tower of Hanoi Puzzle is summarized. Within this parallel context, two theorems on minimal walks in the state space of configurations, along with their constructive proofs, are presented. These proofs are used to describe a method for identifying and eliminating sub-optimal transfers within an arbitrary, valid sequence of disk configurations (as per the rules of the Puzzle). Potential applications of this method to the area of reinforcement learning will be discussed.

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CP3

Fibonacci Cordial Labelling of Some Special Families of Planar Graphs

An injective function f from vertex set $V(G)$ of a graph G to the set $\{F_0, F_1, F_2, \cdots, F_n\}$, where F_i is the i^{th} Fibonacci number $(i = 0, 1, \cdots, n)$, is said to be Fibonacci cordial labeling if the induced function f^* from the edge set $E(G)$ the set $\{0, 1\}$ defined by $f^*(uv) = (f(u) + f(v))\ (mod 2)$ satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1. A graph that admits Fibonacci cordial labeling is called a Fibonacci cordial graph. We will discuss Fibonacci cordial labeling of triangular families of graphs, such as friendship graphs and triangular snake graphs, and of related families of planar graphs, such as double triangular snake and quadrilateral snake graphs. In triangular graphs, we found that whether the graph is Fibonacci cordial is dependent on the modulus of the number of the number of triangles in the graph.

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CP3

Hidden Ancestor Graphs with Assortative Vertex Attributes

Synthetic vertex-labelled graphs play a valuable role in development and testing of graph machine learning algorithms. The hidden ancestor graph is a new stochastic model for a vertex-labelled multigraph G in which the observable vertices are the leaves L of a random rooted tree T, whose edges and non-leaf nodes are hidden. The likelihood of an edge in G between two vertices in L depends on the height of their lowest common ancestor in T. The label of a vertex $v \in L$ depends on a randomized label inheritance mechanism within T such that vertices with the same parent often have the same label. High label assortativity, high average local clustering, heavy tailed vertex degree distribution, and sparsity, can all coexist in this model. Subgraphs consisting of the agreement edges (end point labels agree), and the conflict edges (end point labels differ), respectively, play an important role in testing anomaly correction algorithms. Instances with a hundred million edges can be built in minutes on an average workstation with sufficient memory.

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CP3

Degree Sequence Theorems for Measures of Graph Connectivity

Given a finite simple graph G, we can define many measures of connectivity. Classic examples include vertex connectivity and edge connectivity. If we consider the minimum number of vertices (resp. edges) that must be removed from G so that the remaining components all have order less than a fixed $k \geq 1$, we get *k-component order connectivity* (resp. *k-component order edge connectivity*). We will begin with a brief survey of existing results that use the degree sequence of a graph to determine a lower bound on its connectivity, edge connectivity, or k-component order connectivity. From there, we present recent results for k-component order edge connectivity. Throughout, we will discuss relevant corollaries, interesting features, and computational complexity for these theorems.

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CP4

Cheeger Constant of Cartesian Products of Cayley Graphs Arising from Generalized Dihedral Groups

Let n, x be positive integers satisfying $1 < x < n$. Let $H_{n,x}$ be a group of the form $\langle a, b | a^n = b^2 = (ba)^x = 1 \rangle$. $H_{n,x}$ will be referred to as the generalized dihedral groups. It is possible to associate a cubic Cayley graph to each such group. Bounds on the Cheeger constant, $h(G)$, of these graph is known. We consider the problem of finding the Cheeger constant, $h(G \square G)$, of the Cartesian product of these graphs and comparing it to the known bounds on $h(G)$.

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CP4

The Unlabeled Barcode Poset

A barcode is a finite multiset of intervals on the real line, $B = \{(b_i, d_i)\}_{i=1}^n$. Barcodes are important objects in topological data analysis, where they serve as summaries of the persistent homology groups of a filtration. Recently, researchers have begun studying barcodes as combinatorial objects. Most notably, in [Kanari et al., 2020] the authors present a natural mapping between the space of barcodes with $n+1$ bars and the symmetric group $\mathfrak{S}_n$. In this paper, we first define our own map, between the space of barcodes with n bars and the set of permutations of the multiset $\mathcal{M}_{n,2}$, composed of the integers $[n] = \{1, 2, \ldots n\}$ such that every element has multiplicity 2. From this mapping, we define an equivalence class on the space of barcodes and define a partial order on these classes. This order is the weak Bruhat order applied to our multiset permutations, but we show that it has an interesting interpretation in terms of the barcodes themselves. We call this poset the unlabeled barcode poset, and show that it possesses several interesting properties.

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CP4

On New Results on Algebraic Constructions of Extremal Graph Theory and Their Applications.

New explicit constructions of infinite families of finite small world graphs of large girth with well defined projective limits which is an infinite tree are described. Members of this family are not edge transitive graphs.The applications of these objects to constructions of LDPC codes and cryptographic algorithms are shortly observed. We define families of homogeneous algebraic graphs of large girth over commutative ring K. For each commutative integrity ring K with $|K| > 2$ we introduce new family of bipartite homogeneous algebraic graphs of large girth over K formed by graphs with sets of points and lines isomorphic K^n, $n > 1$ such that their projective limit is well defined and isomorphic to an infinite forest.

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CP5

Developing Compressed Sensing Techniques for Discrete Signal

Compressed sensing is a relatively new mathematical paradigm that shows a large dimensional signal can be efficiently reconstructed from a small number of linear measurements under the assumption that the signal is sparse. To put it another way, given a noisy signal $x \in R^n$, the goal is to precisely reconstruct x from its measurements $b = Ax + e$. In this case, $A \in R^{m \times n}$ is an underdetermined matrix, where m is much smaller than n, and $e \in R^m$ represents the vector modelling the noise in the system. Since the system is underdetermined, there are infinitely many solutions to this system. However, if additional constraints were added, such as the signal x obeying a sparsity constraint, it would be possible to uniquely recover it. We say x is s-sparse when it has at most s nonzero entries. The poster will present a condition for the unique recovery using the idea of restricting the original signal space. Additionally, some limitations on the sensing matrix A will be discussed.

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CP5

Resonance Structures of Capped Carbon Nanotubes

A *fullerene* is a 3-regular plane graph with only hexagonal and pentagonal faces, and models a pure carbon molecule. Nanotubes are a class of fullerenes that are cylindrical in shape and extremely useful in applications. The *Clar number* of a fullerene is a parameter related to its aromaticity and stability. In this talk, we partition nanotubes into two classes, those with relatively small and with relatively large Clar numbers. We describe the double bond structures, or perfect matchings, capable of forming in these two classes. This is joint work with Jack Graver (Syracuse University) and Aaron Williams (Williams College).

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CP5

Low-Order Divergence-Free Approximations for the Stokes Problem on Worsey-Farin and Powell-Sabin Splits

We derive low-order, inf-sup stable and divergence-free finite element approximations for the Stokes problem using Worsey-Farin splits in three dimensions and Powell-Sabin splits in two dimensions. The velocity space simply consists of continuous, piecewise linear polynomials, whereas the pressure space is a subspace of piecewise constants with weak continuity properties at singular edges (3D) and singular vertices (2D). We discuss implementation aspects that arise when coding the pressure space, and in particular, show that the pressure constraints can be enforced at an algebraic level.

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CP6

Partial Rainbows

The forbidden subgraph problem is among the classics in its area – how many edges can an F-free graph have? The maximum number of edges in such an order n graph yields the extremal number; the minimum gives the saturation number. Recent work by numerous authors has explored a coloring variation on this problem – how many edges can there be in a properly edge-colored graph avoiding a rainbow copy of F (where rainbow means each of its edges receives its own color)? Again, one can ask for a maximum or minimum here, and one discovers the rainbow extremal and rainbow saturation numbers (introduced by Keevash, Mubayi, Sudakov, and Versträete, and B., Johnston, Rombach, respectively). In this talk, we cross the space between monochrome and rainbow, creating a spectrum of extremal functions (and saturation functions) with varying amounts of distinct colors. This is joint work with Vic Bednar and Moheng Zhang.

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CP6

New Bounds on Diffsequences

For an infinite set of positive integers D, a k-term D-diffsequence is a sequence of positive integers $a_1 < a_2 < \cdots < a_k$ such that $a_i - a_{i-1}$ is in D for $i = 2, 3, \cdots, k$. We say that D is r-accessible if every r-coloring of the positive integers contains arbitrarily long monochromatic D-diffsequences. In this talk, we consider which classes of sets are 2-accessible. In particular, we determine which sets of the form $D := \{d_1, d_2, \cdots\}$ with $d_i \mid d_{i+1}$ for all i are 2-accessible.

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CP6

Eigenvalues, Forbidden Subgraphs, and Ramsey

Given a real number λ, what can we say about the family $G(\lambda)$ of graphs with eigenvalues bounded from below by $-\lambda$. The Cauchy interlacing theorem implies that the family $G(\lambda)$ is closed under taking (induced) subgraphs. Similar to Wagners theorem, which describes the family of planar graphs by finite forbidden minors, it is natural to ask for which λ the family $G(\lambda)$ has a finite forbidden subgraph characterization. In this talk, I will illustrate the key Ramsey theoretic ideas in answering this question.

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CP6

Rainbow Saturation

A graph G is rainbow H-saturated if there is some proper edge coloring of G which is rainbow H-free (that is, it has no copy of H whose edges are all colored distinctly), but where the addition of any edge makes such a rainbow H-free coloring impossible. Taking the maximum number of edges in a rainbow H-saturated graph recovers the rainbow Turán numbers whose systematic study was begun by Keevash, Mubayi, Sudakov, and Verstraëte. In this talk, we introduce and examine the corresponding *rainbow saturation number* – the minimum number of edges among all rainbow H-free graphs.

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CP7

Approval Ballot Triangles and Tsscpps

We discuss a family of binary triangular arrays, called approval ballot triangles (ABTs), that are in bijection with totally symmetric self-complementary plane partitions (TSSCPPs). These triangles correspond to a ballot process in which voters select their collection of approved candidates rather than voting for a single person. We situate ABTs within the ballot problem literature and explain how they generalize some known combinatorial families. We deepen the connection between TSSCPPs and ballot problems by giving a decomposition of a strict-sense ballot into a list of sequentially compatible ABTs.

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CP7

On the Stability of Graph Independence Number

Let G be a graph on n vertices of independence number $\alpha(G)$ such that every induced subgraph of G on $n-k$ vertices has an independent set of size at least $\alpha(G)-\ell$. What is the largest possible $\alpha(G)$ in terms of n for fixed k and ℓ? We show that $\alpha(G) \leq n/2 + C_{k,\ell}$, which is sharp for $k-\ell \leq 2$. We also use this result to determine new values of the Erdős–Rogers function.

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CP7

Oriented Discrepancy of Hamilton Cycles

One of the sufficient conditions for Hamiltonicity in graphs, obtained by Dirac in 1952, asserts that every n-vertex graph of minimum degree at least $n/2$ is Hamiltonian. We conjecture the following generalization: in any orientation of the edges of an n-vertex graph with minimum degree $\delta \geq n/2$ there exists a Hamilton cycle in which at least δ edges are pointing forward. In this talk, I will present an approximate version of this conjecture, according to which a minimum degree of $(n+O(k))/2$ suffices to guarantee a Hamilton cycle with at least $(n+k)/2$ edges pointing forward. I will also discuss an analogous problem for random graphs, showing that above the Hamiltonicity threshold, any orientation contains, with high probability, an "almost-directed" Hamilton cycle, namely, one in which almost all the edges point in the same direction. The talk is based on joint work with Lior Gishboliner and Michael Krivelevich.

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CP7

Homotopy and the Homomorphism Threshold of Odd Cycles

Consider a family $\mathcal{F}$ of C_{2r+1}-free graphs, where $r \geq 2$. Suppose that each graph in $\mathcal{F}$ has minimum degree linear in its number of vertices. Thomassen showed that such a family has bounded chromatic number, or, equivalently, that all graphs in $\mathcal{F}$ are homomorphic to a complete graph of bounded size. Considering instead homomorphic images which are themselves C_{2r+1}-free, we construct a family of dense C_{2r+1}-free graphs with no C_{2r+1}-free homomorphic image of bounded size. This provides the first nontrivial lower bound on the homomorphism threshold of longer odd cycles and answers a question of Ebsen and Schacht. Our proof introduces a new technique to describe the topological structure of a graph. We establish a graph-theoretic analogue of homotopy equivalence, which allows us to analyze the relative placement of odd closed walks in a graph. This notion has surprising connections to the neighborhood complex, and opens many further interesting questions.

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CP8

A Modification of the Random Cutting Model

We propose a modification to the random destruction of graphs: Given a finite network with a distinguished set of sources and targets, remove (cut) vertices at random, discarding components that do not contain a source node. We investigate the number of cuts required until all targets are removed, and the size of the remaining graph. This model interpolates between the random cutting model going back to Meir and Moon and site percolation. We prove several general results, including that the size of the remaining graph is a tight family of random variables for compatible sequences of expander-type graphs, and determine limiting distributions for specific classes of trees.

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CP8

Applications of Berry-Esseen Theorems in Extremal Combinatorics

A set $S \subset [n]$ is a $B_k[g]$ set if $|S_n| \leq g$ for every n where $S_n := \{(s_1, ..., s_k) \mid s_1 + \cdots + s_k = n,\ s_i \in S, s_1 \leq \cdots \leq s_k\}$. This is a natural generalization of Sidon sets, which are exactly $B_2[1]$ sets. Dubroff, Fox and Xu recently used the Berry-Esseen theorem to prove the best known lower bound on the largest element in a subset of positive integers with distinct subset sums. We show in [J.-Tait-Timmons, Upper and lower bounds on the maximum size of $B_k[g]$ sets] that the Berry-Esseen Theorem can also be used to improve the best known upper bound on the maximum size of a $B_k[g]$ set in $[n]$. As the Berry-Esseen Theorem quantifies how "close" a probability distribution is to the normal distribution, it is natural to suppose that it may be a successful tool in other combinatorial settings where the distributions that show up in probabilistic arguments behave like a normal distribution.

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CP8

Subtrees of Graphs

A subtree of a graph is any (not necessarily induced) subgraph that is also a tree. In this talk, different questions concerning the distribution of subtrees in deterministic and random graphs will be discussed. For instance: what can we say about the distribution of the subtree sizes, or the average size? What is the probability that a random subtree is spanning (covers all vertices)?

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CP9

Nonorientable Embeddings of Graphs and Digraphs with Large Faces

In 1965 Edmonds gave a characterization of when two connected graphs can be embedded in some surface as duals of each other. We show that Edmonds' characterization can be strengthened slightly. As a corollary, Edmonds observed that every connected graph has a one-face embedding in some surface. Ringel (and independently others) proved that the surface can be guaranteed to be nonorientable unless the graph is a tree. Fijavž, Pisanski and Rus showed that an especially nice nonorientable one-face embedding can be constructed when the graph is eulerian. We extend these results to directed embeddings of digraphs, where the boundary of every face must be a directed walk. Edmonds also observed that every eulerian graph has a two-face embedding in some surface, where both faces are bounded by euler circuits. We call this a *bi-eulerian* embedding. We show that the surface can be guaranteed to be nonorientable unless the graph is a cycle. In fact, we prove a more general result on extending a circuit decomposition of a graph to an embedding using an euler circuit.

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CP9

Extremal Properties of the Sphere for Weighted Cone-Volume Functionals

Weighted cone-volume functionals are introduced for the convex polytopes whose vertices lie in the unit sphere $\mathbb{S}^{n-1}$ in $\mathbb{R}^n$. For these functionals, geometric inequalities are proved and the equality conditions are characterized. Several interesting corollaries are derived and an old problem on surface area maximization is settled. Connections are made to the Orlicz theory of convex geometry, and applications to crystallography and quantum theory are presented.

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CP9

Take-Away Impartial Combinatorial Game on Different Geometric and Discrete Structures

Following from the winning strategy for a Take-Away Impartial Combinatorial Game on only Oddly Uniform or only Evenly Uniform Hypergraphs in the Ph.D. Dissertation of Dr. Kristen Barnard (an Assistant Professor of Mathematics at Berea College), T. H. Molena found the new winning strategy for a Take-Away Game on neither Oddly nor Evenly Uniform Hypergraphs during her Undergraduate Independent Research opportunity. However, those neither Oddly nor Evenly Uniform Hypergraphs must meet the given special requirements. In a Take-Away Game on hypergraphs, two players take turns to remove the vertices and the hyperedges of a hypergraph. In each turn, a player must remove either only one vertex or only one hyperedge. When a player chooses to remove one vertex, all of the hyperedges that contain the chosen vertex are also removed. When a player chooses to remove one hyperedge, only that one chosen hyperedge is removed. Whoever removes the last vertex wins the game. All of the new theorems in this research paper are in agreement with the previous theorems in Dr. Kristen Barnards Ph.D. Dissertation. KEYWORDS: Take-Away, combinatorial games, hypergraphs, geometric structures, discrete structures

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CP10

Extending McKay's Canonical Isomorph Algorithm to C-Sets

Many scientific computing packages externally call the program Nauty, which implements McKay's celebrated algorithm for computing the canonical isomorph of a graph. However, scientists are not often concerned with graphs but rather much richer data structures (amino acids, chemical reaction networks, social networks) which they wish to compute with up to isomorphism. Traditionally one must encode one's high level model into a graph in order to leverage Nauty; however, we develop an alternative which is to implement McKay's algorithm for attributed C-sets, which generalize a broad class of data structures, including many generalizations of graphs (e.g. directed, symmetric, reflexive), tabular data (e.g. data frames), and combinations of the two (e.g. weighted graphs, relational databases). We discuss the extent to which ideas of McKay's algorithm translate naturally to the more general setting and show how working at this higher level offers potential for more transparent code and computational advantages.

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CP10

Extending Deming's KE Graph Recognition Algorithm

A Knig-Egervry (KE) graph is a generalization of a bipartite graph having the property that the independence number plus the matching number equals its order. Deming (1979) gave an efficient algorithm that produces either a certificate that a graph is KE (together with a maximum independent set) or a certificate that it is not. Here we extend this algorithm to decompose a graph into one (possibly empty) KE subgraph together with some number (possibly zero) subgraphs whose independence number is one less than its matching number. For some graphs the independence number of the parent graph equals the sum of the independence number of these subgraphs. This is joint work with Mark Kayll.

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CP10

Characterizing and Benchmarking Qubo Reformulations of the Knapsack Problem

It has been shown that quantum computers can outperform classical computers in solving some instances of NP-hard problems, for instance, the Graph partitioning problem. To do this, a Quadratic Unconstrained Binary Optimization (QUBO) formulation is needed, but most of the literature regarding QUBO reformulations for constrained optimization problems is centered around equality constrained problems. Here, we focus on the "simplest' inequality constrained problem: the knapsack problem (KP). Specifically, we derive different QUBO formulations for the KP, characterize the range of their associated penalty constants, as well as computationally benchmark them through experiments using the quantum approximate optimization algorithm (QAOA) on a gate-based quantum computer. As a byproduct, we correct some erroneous results regarding QUBO reformulations for the KP reported in the literature.

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MS1

Recent Results on Size Ramsey Numbers

The size Ramsey number of a graph H is the smallest number of edges in a graph G with the property that every two-colouring of the edges of G contains a monochromatic copy of H. In this talk, we will discuss recent progress on several classical questions about size Ramsey numbers. For instance, what is the size Ramsey number of the complete bipartite graph $K_{s,t}$? And what is the size Ramsey number of a graph with maximum degree three? Parts of this talk represent joint work with Rajko Nenadov and Milos Trujic and also with Jacob Fox and Yuval Wigderson.

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MS1

Monochromatic Components with Many Edges

Given an r-edge-coloring of the complete graph K_n, what is the largest number of edges in a monochromatic connected component? In this talk we introduce a general framework for studying this natural question and apply it to fully resolve the $r = 3$ and $r = 4$ cases, showing an asymptotically tight lower bound of $\frac{1}{r(r-1)}\binom{n}{2}$ on the number of edges in both cases.

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MS1

Friendly Bisections of Random Graphs

Resolving a conjecture of Fredi from 1988, we prove that with high probability, the random graph $\mathbb{G}(n, 1/2)$ admits a friendly bisection of its vertex set, i.e., a partition of its vertex set into two parts whose sizes differ by at most one in which $n - o(n)$ vertices have at least as many neighbours in their own part as across. Our proof is constructive, and in the process, we develop a new method to study stochastic processes driven by degree information in random graphs; this involves combining enumeration techniques with an abstract second moment argument.

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MS1
On Some Ramsey Multiplicity Question

Ramsey's Theorem guarantees for every graph H that any 2-edge-coloring of a sufficiently large complete graph contains a monochromatic copy of H. In 1962, Erdős conjectured that the random 2-edge-coloring minimizes the number of monochromatic copies of K_k, and the conjecture was extended by Burr and Rosta to all graphs. In the late 1980s, the conjectures were disproved by Thomason and Sidorenko, respectively. A classification of graphs whose number of monochromatic copies is minimized by the random 2-edge-coloring, which are referred to as common graphs, remains a challenging open problem. If Sidorenko's Conjecture, one of the most significant open problems in extremal graph theory, is true, then every 2-chromatic graph is common, and in fact, no 2-chromatic common graph unsettled for Sidorenko's Conjecture is known. While examples of 3-chromatic common graphs were known for a long time, the existence of a 4-chromatic common graph was open until 2012, and no common graph with a larger chromatic number is known. We construct connected k-chromatic common graphs for every k. This answers a question posed by Hatami, Hladky, Kral', Norine and Razborov [Combin. Probab. Comput. 21 (2012)], Conlon, Fox and Sudakov [London Math. Soc. Lecture Note Ser. 424 (2015), Problem 2.28], and Jagger, Stovicek and Thomason [Combinatorica 16, (1996)] whether there exists a common graph with chromatic number at least four.

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MS2
Sfcdecomp: Multicriteria Optimized Tool Path Planning in 3d Printing Using Space-Filling Curve Based Domain Decomposition

We explore efficient optimization of toolpaths based on multiple criteria for large instances of 3D printing problems. We first show that the minimum turn cost 3D printing problem is NP-hard, even when the region is a simple polygon. We develop SFCDecomp, a space filling curve based decomposition framework to solve large instances of 3D printing problems efficiently by solving these optimization subproblems independently. For the Buddha model, our framework builds toolpaths over a total of 799,716 nodes across 169 layers, and for the Bunny model it builds toolpaths over 812,733 nodes across 360 layers. Building on SFCDecomp, we develop a multicriteria optimization approach for toolpath planning. We demonstrate the utility of our framework by maximizing or minimizing tool path edge overlap between adjacent layers, while jointly minimizing turn costs. Strength testing of a tensile test specimen printed with tool paths that maximize or minimize adjacent layer edge overlaps reveal significant differences in tensile strength between the two classes of prints. A preprint is available at Preprint: https://www.arxiv.org/abs/2109.01769.

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MS2
Parametric Growth Processes for Metamaterial Design

A substantial amount of research in additive manufacturing is devoted to designing mechanical metamaterials that derive their physical properties from the particular arrangement of their small-scale geometry. In this talk, I will present recent works that use parametric growth processes to generate mechanical metamaterials. One of the main advantages of using parametric growth processes is that they allow the implicit grading of mechanical properties. Seamlessly grading different target material properties is essential for various domains such as Functionally Graded Materials (FGM).

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MS3
Pivot Gray Codes for the Spanning Trees of a Graph

There are many algorithms for exhaustively listing the spanning trees of an undirected graph. Some of the more interesting algorithms list the trees in a "revolving-door" order: successive trees differ by the deletion of an edge and the addition of a new one. However, none of them have the stronger property that the two edges share a common vertex. A listing with this "pivot" property is called a "pivot Gray code". Pivot Gray codes via fast algorithms are known to exist for the Fan and Wheel graphs; however, little is known about other graph classes. We will discuss pivot Gray codes more generally and demonstrate their relationship to spanning arborescences of a directed graph and a corresponding open problem by Knuth.

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MS3
Cut-Down De Bruijn Sequences

We introduce combinatorial generation and Gray codes through the lens of cut-down de Bruijn sequences. A cut-down de Bruijn sequence is a cyclic string of length $1 \leq L \leq k^n$ such that every substring of length n appears at most once. We recall a construction by Etzion for $k = 2$

(1986) and outline how the approach can be simplified and extended for $k > 2$. We also review some recent results that put further restrictions on the number of substrings of lengths ¡ n. We conclude with an open question regarding the efficient construction of such restricted cut-down de Bruijn sequences by Nellore and Ward (2021).

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MS3

Cool-Lex Order for Ordered Trees and Lukasiewicz Paths

Cool-lex order differs from most Gray codes in that it does not reflect sublists. Instead, sublists are rotated upward (i.e., the first string is listed last). Variations of this idea have been used to create shift Gray codes and efficient algorithms for a variety of combinatorial objects, including what is arguably the most practical approach for generating multiset permutations. In this talk, we show that cool-lex order creates simple Gray codes for ordered trees and Lukasiewicz paths. In the first case, we also provide a loopless generation algorithm. In the second case, our algorithm can be viewed as a new generalization of the cool-lex algorithm for generating multiset permutations.

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MS3

An Easy Way to Generate Matroid Basis and Independent Set Gray Codes

The bases of a matroid generalize the spanning trees of a graph, and the bases of the column space of a matrix. In this talk, we provide a simple method for constructing a basis exchange Gray code for the bases of a matroid: Start from any basis, and then greedily apply any exchange in which the larger element involved in the exchange is as small as possible. In the special case of graphic matroids, this means that you can generate the spanning trees of an edge labeled graph via edge exchanges very easily: Start from any spanning tree, and then greedily exchange any pair of edges (i.e., one edge enters and one edge exists) in which the larger label of an edge involved in the exchange is minimized. Similar results are provided for the independent sets of a matroid (e.g., the forests of a graph). Joint work with Arturo Merino (TU Berlin) and Torsten Mutze (University of Warwick).

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MS4

A Fourth Moment Phenomenon for Normal Approximation of Monochromatic Subgraphs

Given a graph sequence $\{G_n\}_{n\geq 1}$ and a simple connected subgraph H, we denote by $T(H, G_n)$ the number of monochromatic copies of H in a uniformly random vertex coloring of G_n with $c \geq 2$ colors. In this article, we prove a central limit theorem for $T(H, G_n)$ with explicit error rates. The error rates arise from graph counts of collections formed by joining copies of H that we call *good joins*. Counts of good joins are closely related to the fourth moment of a normalized version of $T(H, G_n)$, and that connection allows us to show a *fourth moment phenomenon* for the central limit theorem. For $c \geq 30$, we show that $T(H, G_n)$ (appropriately centered and rescaled) converges in distribution to $\mathcal{N}(0, 1)$ whenever its fourth moment converges to 3 (the fourth moment of the standard normal distribution). We show the convergence of the fourth moment is necessary to obtain a normal limit when $c \geq 2$. The combination of these results implies that the fourth moment condition characterizes the limiting normal distribution of $T(H, G_n)$ for all subgraphs H, whenever $c \geq 30$.

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MS4

Extremal Results on Feedback Arc Sets in Directed Graphs

For a directed graph G, let $\beta(G)$ denote the size of a minimum feedback arc set, a smallest subset of edges whose deletion leaves an acyclic subgraph. A result of Berger and Shor implies that any oriented graph G with m edges satisfies $\beta(G) = m/2 - \Omega(m^{3/4})$. Here, we improve the exponent $3/4$ for B-free oriented graphs when B is bipartite (when B is not bipartite, the exponent $3/4$ is best possible). We show that for every rational number r between $3/4$ and 1, there is a finite collection of digraphs $\mathcal{B}$ such that every $\mathcal{B}$-free digraph G with m edges satisfies $\beta(G) = m/2 - \Omega(m^r)$ (this bound is best possible up to the implied constant factor). Finally, we give a characterization of quasirandom directed graphs via minimum feedback arc sets.

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MS4

Large Deviations in Random Latin Squares

We resolve up to constants the question of large deviations of the number $\mathbf{N}$ of intercalates (2×2 combinatorial subsquares which are themselves Latin squares) in a random $n \times n$ Latin square. In particular, for constant $\delta > 0$ we prove that $\Pr(\mathbf{N} \leq (1-\delta)n^2/4) = \exp(-\Theta(n^2))$ and $\Pr(\mathbf{N} \geq (1+\delta)n^2/4) = \exp(-\Theta(n^{4/3}\log n))$. As a consequence, we deduce that a typical order-n Latin square has $(1+o(1))n^2/4$ intercalates, matching a lower bound due to Kwan and Sudakov and resolving an old conjecture of McKay and Wanless. We additionally show that in almost all order-n Latin squares, the number of cuboctahedra (i.e., the number of pairs of possibly degenerate 2×2 subsquares with the same arrangement of symbols) is of order n^4, which is the minimum possible. As observed by Gowers and Long, this number can be interpreted as measuring "how associative' the quasigroup associated with the Latin square is. This talk covers two works by the speaker.

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MS5

Coloring Graphs with Forbidden Subgraphs

A celebrated conjecture of Alon, Krivelevich, and Sudakov states that, for any graph F, there is a constant $c(F) > 0$ such that $\chi(G) \leq (c(F) + o(1))\Delta/\log\Delta$ for all F-free graphs G of maximum degree Δ. The only graphs F for which this conjecture has been verified so farby Alon, Krivelevich, and Sudakov themselvesare the so-called *almost bipartite* graphs, i.e., graphs that can be made bipartite by removing at most one vertex. Equivalently, a graph is almost bipartite if it is a subgraph of the complete tripartite graph $K_{1,t,t}$ for some $t \in \mathbb{N}$. The best heretofore known upper bound on $c(F)$ for almost bipartite F is due to Davies, Kang, Pirot, and Sereni, who showed that $c(K_{1,t,t}) \leq t$. The main result of this talk is a *uniform* upper bound on $c(F)$ for these graphs. Namely, we prove that $c(F) \leq 4$ for any almost bipartite graph F and, moreover, $c(F) \leq 1$ for bipartite F.

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MS5

3-Coloring Near-Quadrangulations of the Plane and the Torus

A graph drawn on a surface is a near-quadrangulation if the sum of the lengths of its faces of length other than four is bounded by a constant. We give efficient algorithms for the 3-precoloring extension problem in the planar near-quadrangulations and for the 3-coloring of near-quadrangulations of the torus, based on nowhere-zero flows. We also present some structural corollaries.

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MS5

Proper Orientations and Proper Chromatic Number

The proper chromatic number of a graph G is the minimum k such that there exists an orientation of the edges of G with all vertex-outdegrees at most k and such that for any adjacent vertices, the outdegrees are different. Two major conjectures about the proper chromatic number are resolved. First, it is shown, that the proper chromatic number of any planar graph is bounded (in fact, it is at most 14). Secondly, it is shown that for every graph, the proper chromatic number is at most $O(\frac{r\log r}{\log\log r}) + \frac{1}{2}MAD(G)$, where $r = \chi(G)$ is the usual chromatic number of the graph, and $MAD(G)$ is the maximum average degree taken over all subgraphs of G. Several other related results are derived. Our proofs are based on a novel notion of fractional orientations.

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MS5

Counting Cycle Double Covers

We count the number of ways to double-cover edges of a bridgeless graph by cycles (even subgraphs) and by circuits (2-regular connected subgraphs). We prove that every bridgeless cubic planar graph with n vertices has at least $(5/2)^{n/4-1/2}$ circuit double covers. For graphs on a fixed surface we have a weaker result, namely an exponential in $\sqrt{n}$, provided the embedding has representativity at least 4. We conjecture that every bridgeless cubic graph has at least $2^{n/2-1}$ circuit double covers.

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MS6

Powers of Hamiltonian Cycles in Randomly Augmented Graphs

It follows from the theorems of Dirac and of Komlós, Sarközy, and Szemerédi, who confirmed the Posá-Seymour conjecture, that for every $k \geq 1$ and sufficiently large n already the minimum degree $\delta(G) \geq \frac{k}{k+1}n$ for an n-vertex graph G alone suffices to ensure the existence of the k-th power of a Hamiltonian cycle. In this talk we will study the number of random edges one has to add to a graph G with minimum degree $\delta(G) \geq \left(\frac{k}{k+1} + \varepsilon\right) n$ (with $\varepsilon > 0$) in order to create an ℓ-th power of a Hamiltonian cycle, where $\ell \geq k + 1$. This talk is based on three projects obtained together with Sylwia Antoniuk, Christian Reiher, Andrzej Ruciński and Mathias Schacht.

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MS6

On Randomly Weighted, Randomly Perturbed Dense Graphs

We consider the following probabilistic model of some classical combinatorial optimization problems. We are given a n-vertex graph with minimum degree cn for some absolute constant $c > 0$. We then add $o(n^2)$ random edges and then give each edge a random weight. For minimum spanning tree, shortest path and assignment problems we give asymptotic estimates of the value of the optimum solution. For the asymmetric travelling salesperson problem, we show that Karp's patching heuristic is asymptotically optimal.

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MS7

Counting Colorings of Triangle-Free Graphs

By a theorem of Johansson, every triangle-free graph of maximum degree Δ has chromatic number at most $O(\Delta/\log\Delta)$. Using the entropy compression method, Molloy recently sharpened this bound to $(1 + o(1))\Delta/\log\Delta$. We use Rosenfeld's enumerative approach to entropy compression in order to establish an optimal lower bound on the number of proper q-colorings of a triangle-free graph of maximum degree Δ when $q \geq (1 + o(1))\Delta/\log\Delta$. This yields a common strengthening of Molloy's result and the lower bound on the number of independent sets in triangle-free graphs due to Davies, Jenssen, Perkins, and Roberts.

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MS7

Turan Numbers of Sunflowers

A collection of distinct sets is called a sunflower if the intersection of any pair of sets equals the common intersection of all the sets. Sunflowers are fundamental objects in extremal set theory with relations and applications to many other areas of mathematics as well as theoretical computer science. A central problem in the area due to Erdos and Rado from 1960 asks for the minimum number of sets of size r needed to guarantee the existence of a sunflower of a given size. Despite a lot of attention, including a polymath project, even the asymptotic answer remains unknown. We study a related problem first posed by Duke and Erdos in 1977 which requires that in addition the intersection size of the desired sunflower be fixed. This question is perhaps even more natural from a graph theoretic perspective since it asks for the Turan number of a hypergraph made by the sunflower consisting of k edges, each of size r and with common intersection of size t. For a fixed size of the sunflower k, the order of magnitude of the answer has been determined by Frankl and Furedi. In the 1980's Chung, Erdos and Graham and Chung and Erdos considered what happens if one allows k, the size of the desired sunflower to grow with the size of the ground set. We resolve this problem for any uniformity, by determining up to a constant factor the n-vertex Turan number of a sunflower of arbitrary uniformity r, common intersection size t and with the size of the sunflower k allowed to grow with n.

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MS7

Multicolor List Ramsey Numbers Grow Exponentially

The list Ramsey number $R_\ell(H, k)$, recently introduced by Alon, Bucic, Kalvari, Kuperwasser, and Szabo, is a list-coloring variant of the classical Ramsey number. They showed that if H is a fixed r-uniform hypergraph that is not r-partite and the number of colors k goes to infinity, $e^{\Omega(\sqrt{k})} \leq R_\ell(H, k) \leq e^{O(k)}$. We prove that $R_\ell(H, k) = e^{\Theta(k)}$ if and only if H is not r-partite.

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MS7

Lower Bounds for Multicolor Ramsey Numbers

A recent breakthrough of Conlon and Ferber yielded new lower bounds on multicolor Ramsey numbers, via an elegant construction combining algebra and randomness. In this talk, I will explain how their bounds can be further improved with additional probabilistic tools, focusing on the method of random homomorphisms, which can be viewed as a common generalization of well-known probabilistic techniques.

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MS9

New Results and Open Problems on the Cop Number When Forbidding An Induced Subgraph

The game of Cops and Robbers on graphs has been studied for over 40 years. The main focus has been on studying the cop number (usually finding upper and lower bounds), for instance by relating it to other graph properties and parameters. Joret, Kaminski and Theis (2010) were the first to study the cop number when forbidding an induced subgraph, inspired by Andreae (1986) who considered forbidding a minor. We will survey the various results on the topic that have been found since. The outlines of the speaker's proof that the cop number of $2K_2$-free graphs is at most 2 will be presented, as well as some tight examples for the known upper bounds for $2K_1 + K_2$-free graphs and $4K_1$-free graphs. Finally, we will discuss some open problems.

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MS10

Decidability and Periodicity of Translational Tilings

Translational tiling is a covering of a space using translated copies of some building blocks, called the tiles, without any positive measure overlaps. Which are the possible ways that a space can be tiled? In the talk, we will discuss the study of this question as well as its applications, and report on recent progress, joint with Terence Tao.

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MS10

Popular Differences for Matrix Patterns

The following combinatorial conjecture arises naturally from recent ergodic-theoretic work of Ackelsberg, Bergelson, and Best. Let M_1, M_2 be $k \times k$ integer matrices, G be a finite abelian group of order N, and $A \subseteq G^k$ with $|A| \geq \alpha N^k$. If M_1, M_2, $M_1 - M_2$, and $M_1 + M_2$ are automorphisms of G^k, is it true that there exists a popular difference $d \in G^k \setminus \{0\}$ such that

$$\#\{x \in G^k : x, x+M_1 d, x+M_2 d, x+(M_1+M_2)d \in A\} \geq (\alpha^4 - o(1))N^k.$$

We show that this conjecture is false in general, but holds for $G = \mathbb{F}_p^n$ with p an odd prime given the additional spectral condition that no pair of eigenvalues of $M_1 M_2^{-1}$ (over the algebraic closure $\overline{\mathbb{F}}_p$) are negatives of each other. In particular, the "rotated squares' pattern does not satisfy this eigenvalue condition, and we give a construction of a set of positive density in $(\mathbb{F}_5^n)^2$ for which that pattern has no nonzero popular difference. This is in surprising contrast to three-point patterns, which we handle over all compact abelian groups and which do not require additional spectral conditions.

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MS10

Gowers Uniformity of Thin Subsets of Primes

A celebrated theorem of Green-Tao asserts that the set of primes contains arbitrarily long arithmetic progressions. More specifically, they count asymptotically the number of k-term arithmetic progressions in primes up to a threshold. Their work involves discorrelation estimates between primes and nilsequences, which imply that the set of primes is Gowers uniform. In this talk I will discuss results of this type for primes restricted to short intervals and in arithmetic progressions. For example, we prove that the set of primes in $(X, X+H]$ with $H > X^{5/8+o(1)}$ is Gowers uniform; we also prove that, for almost all $q < X^{1/4-o(1)}$, the set of primes up to X in a coprime residue class $a \pmod q$ is Gowers uniform. This is based on joint works with K. Matomaki, J. Teravainen, T. Tao.

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MS10

On Product Sets of Arithmetic Progressions

We prove that the size of the product set of any finite arithmetic progression $\mathcal{A} \subset \mathbb{Z}$ satisfies

$$|\mathcal{A} \cdot \mathcal{A}| \geq \frac{|\mathcal{A}|^2}{(\log|\mathcal{A}|)^{2\theta+o(1)}},$$

where $2\theta = 1-(1+\log\log 2)/(\log 2)$ is the constant appearing in the celebrated Erdős multiplication table problem. This confirms a conjecture of Elekes and Ruzsa from about two decades ago. If instead $\mathcal{A}$ is relaxed to be a subset of a finite arithmetic progression in integers with positive constant density, we prove that

$$|\mathcal{A} \cdot \mathcal{A}| \geq \frac{|\mathcal{A}|^2}{(\log|\mathcal{A}|)^{2\log 2-1+o(1)}}.$$

This solves the typical case of another conjecture of Elekes and Ruzsa on the size of the product set of a set $\mathcal{A}$ whose sum set is of size $O(|\mathcal{A}|)$. Our bounds are sharp up to the $o(1)$ term in the exponents. We further prove asymmetric extensions of the above results.

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MS11

On the Average Genus of Graphs

It's a classical problem to study the minimum and maximum genus of a graph G; We will study the average genus of G across all of it's 2-cell embeddings. By Euler's formula this is equivalent to studying the average number of faces. We show that although the number of faces of an embedding may be often as large as $\Theta(n^2)$, for any simple graph G the average number of faces is linear. In the case where the vertex degrees are large we show this is sub-linear, and we also extend this result to all multigraphs, showing the average number of faces is at most logarithmic in edge multiplicity.

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MS11

Non-Hamiltonian Cubic Planar Graphs

In 1880, Peter Gutherie Tate conjectured that every planar cubic graph was hamiltonian. He knew the conjecture was false but hoped to modify it and created a true statement. We consider several modifications and present minimal counterexamples to each. A certain amount of historical data will be included.

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MS11

Approximating TSP Walks in Subcubic Graphs

There has been extensive research on approximating TSP walks in subcubic graphs. We show that if G is a 2-connected simple subcubic graph on n vertices with n_2 vertices of degree 2, then G has a TSP walk of length at most? $\frac{5n}{4} + \frac{n_2}{4} - 1$, establishing a conjecture of Dvořák, Král', and Mohar. This upper bound is best possible. Our proof implies a quadratic-time algorithm for finding such a TSP walk, thereby giving a $\frac{5}{4}$-approximation algorithm for the graphic TSP on simple cubic graphs and improving on the previously best-known approximation ratio of $\frac{9}{7}$. This is joint work with Michael Wigal and Youngho Yoo.

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MS13

Sums of Linear Transformations

We show that if L_1 and L_2 are linear transformations from $\mathbb{Z}^d$ to $\mathbb{Z}^d$ satisfying certain mild conditions, then, for any finite subset A of $\mathbb{Z}^d$,

$$|L_1A + L_2A| \geq (|\det(L_1)|^{1/d} + |\det(L_2)|^{1/d})^d|A| - o(|A|).$$

This result corrects and confirms the two-summand case of a conjecture of Bukh and is best possible up to the lower-order term for many choices of L_1 and L_2. As an application, we prove a lower bound for $|A + \lambda \cdot A|$ when A is a finite set of real numbers and λ is an algebraic number.

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MS13

Higher Order Generalizations of Stability and Arithmetic Regularity

We present recent work, joint with J. Wolf, in which we define a natural notion of higher-order stability and show that subsets of $\mathbb{F}_p^n$ which are tame in this sense can be approximately described by a union of low-complexity quadratic subvarieties up to linear error. This generalizes previous joint work with Wolf on arithmetic regularity lemmas for stable subsets of $\mathbb{F}_p^n$ to the realm of higher-order Fourier analysis.

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MS13

The Quantitative U^4 Inverse Theorem and Symmetrization of Tensors in Low Characteristic

We give the first quantitative bounds for the inverse theorem for the Gowers U^4-norm over $\mathbb{F}_p^n$ for $p = 2, 3$. This builds upon earlier work of Gowers and Milicevic that solved the problem for $p \geq 5$. One problem that arises as a key step is the following symmetrization problem for low-characteristic tensors. Given a tensor T that is "close to symmetric" in the sense that permuting the variables gives another tensor that is close in partition rank, is T itself close in partition rank to a genuinely symmetric tensor? This question is fairly easy to answer in the affirmative for 3-tensors, but recent work of Milicevic shows that this fails for higher tensors. This implies that a more intricate proof strategy is necessary to give quantitative bounds for the U^k-inverse theorem in low-characteristic when $k \geq 5$.

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MS13

Discrepancy in Modular Arithmetic Progressions

Celebrated theorems of Roth and of Matousek and Spencer together show that the discrepancy of arithmetic progressions in the first n positive integers is $\Theta(n^{1/4})$. We study the analogous problem in the $\mathbb{Z}_n$ setting. We asymptotically determine the logarithm of the discrepancy of arithmetic progressions in $\mathbb{Z}_n$ for all positive integer n. We further determine up to a constant factor the discrepancy of arithmetic progressions in $\mathbb{Z}_n$ for many n. For example, if $n = p^k$ is a prime power, then the discrepancy of arithmetic progressions in $\mathbb{Z}_n$ is $\Theta(n^{1/3+r_k/(6k)})$, where $r_k \in \{0, 1, 2\}$

is the remainder when k is divided by 3. This solves a problem of Hebbinghaus and Srivastav.

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MS14

Random Walks on Hypergraphs with Edge-Dependent Vertex Weights

Hypergraphs are used in machine learning to model higher-order relationships in data. While spectral methods for graphs are well-established, spectral theory for hypergraphs remains an active area of research. In this talk, I will discuss recent work on hypergraphs with edge-dependent vertex weights: hypergraphs where every vertex v has a weight $\gamma_e(v)$ for each incident hyperedge e, describing the contribution of v to the hyperedge e. In particular, I will discuss using random walks to develop a spectral theory for such hypergraphs by deriving a random walk-based hypergraph Laplacian and bounding the mixing time of random walks on such hypergraphs. Moreover, I will discuss conditions under which random walks on such hypergraphs are equivalent to random walks on graphs. As a corollary, I will show that current machine learning methods that rely on Laplacians derived from random walks on hypergraphs with edge-*independent* vertex weights do not utilize higher-order relationships in the data. If time permits, I will also describe recent work on learning higher-order relationships (hyperedges) from unstructured data.

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MS14

Generative Stochastic Blockmodels for Hypergraph Modularity Clustering

We present a new approach for hypergraph clustering based on a higher-order generalization of the stochastic block model, which allows us to model nodes with heterogeneous node degrees and hyperedge sizes. Applying approximate maximum likelihood inference in this model naturally leads to a higher-order generalization of the popular modularity graph clustering objective. We develop a Louvain-style algorithm for the general version of our objective, as well as a very fast algorithm for the "all-or-nothing" special case, in which hyperdges are expected to lie fully within clusters. Ours is not the first notion of a higher-order generalization of modularity; this talk will compare and contrast our approach with other methods that are not explicitly tied to a generative model for community structure in hypergraphs. In practice, our approach allows us to capture higher-order clustering structure in school contact networks, U.S. congressional bill cosponsorship, U.S. congressional committees, product categories in co-purchasing behavior, and hotel locations from web browsing sessions. Implementations for our hypergraph clustering methods are available to use as part of a registered Julia package HyperModularity.jl.

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MS15

Bayesian Phylogenetic Inference of Stochastic Block Models on Infinite Trees

This project involves a classification problem on a deep network, by considering a broadcasting process on an infinite communication tree, where information is transmitted from the root of the tree to all the vertices with certain probability of error. The information reconstruction problem on an infinite tree, is to collect and analyze massive data samples at the nth level of the tree to identify whether there is non-vanishing information of the root, as n goes to infinity. Its connection to the clustering problem in the setting of the stochastic block model, which has wide applications in machine learning and data mining, has been well established. For the stochastic block mode, an "information theoretically solvable but computationally hard" region, or say "hybrid-hard phase", appears whenever the reconstruction bound is not tight of the corresponding reconstruction on the tree problem. Inspired by the recently proposed $q_1 + q_2$ stochastic block model, we try to extend the classical works on the Ising model and the Potts model, by studying a general model which incorporates the characteristics of both Ising and Potts through different in-community and out-community transition probabilities, and rigorously establishing the exact conditions for reconstruction.

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MS15

Big Data Information Reconstruction on the Infinite Communication Networks

Finding information about the source of massive data as time evolves, is one of the big data challenges. We consider a broadcasting process in which information is transmitted from a given root node on a noisy d-ary tree network, and explore the reconstruction of some information on the root based on the configuration of the deep leaves. It is shown that the reconstruction bound determines the efficiency of the Glauber dynamics on trees and random graphs. The reconstruction threshold is also believed to play an important role in a variety of other contexts, such as the efficiency of reconstructing phylogenetic ancestors in evolutionary biology, communication theory in the study of noisy computation, network tomography, etc.

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MS15

Reconstruction of DNA Evolution Models

The purpose of this project is to establish the exact reconstruction threshold of the Tamura 1992 DNA evolution models on regular d-ary trees. This reconstruction problem has wide applications in various fields such as biology, information theory and statistical physics, and its close connections to cluster learning, data mining and deep learning have been well established in recent years. We study the T92 DNA model, taking into consideration of the Chargaff's second parity rule by allowing the existence of a guanine cytosine content bias. The corresponding information reconstruction problem in molecular phylogenetics will be explored, by means of refined analyses of moment recursion and an asymptotic 4-dimensional nonlinear second order dynamical system. We will establish the threshold of the reconstruction under the unequal base frequencies of adenine and thymine.

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MS16

Sharp Density Bounds on the Finite Field Kakeya Problem

A set is Kakeya if it contains a line in every direction. We prove that every Kakeya set in the n-space over $\mathbb{F}_q$ has at least $2^{-n+1}q^n$ elements. This is sharp up to the lower-order terms. Joint work with Ting-Wei Chao.

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MS16

Expanders and Related Geometric Results

Using new techniques in higher convexity, we establish superquadratic expansion results leading to improved lower bounds on the number of dot products determined by a finite set in the plane.

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MS16

Homogeneous Structures in Subset Sums and Non-Averaging Sets

Recently, we introduced new techniques to show the existence of long homogeneous arithmetic progressions in the set of subset sums of sequences of positive integers, leading to the solutions of several longstanding open problems. Extending this result to the high-dimensional setting, we prove optimal conditions that guarantee the existence of large homogeneous generalized arithmetic progressions in subset sums, strengthening previous results of Szemerdi and Vu. As an application, we make progress on the ErdősStraus non-averaging sets problem, showing that every subset A of [n] of size at least $n^{\sqrt{2}-1+o(1)}$ contains an element which is the average of two or more other distinct elements of A. This gives the first polynomial improvement on a result of Erdős and Srkzy from 1990.

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MS16

Non-Classical Polynomials and the Inverse Theorem

In this note we characterize when non-classical polynomials are necessary in the inverse theorem for the Gowers U^k-norm. We give a brief deduction of the fact that a bounded function on $\mathbb{F}_p^n$ with large U^k-norm must correlate with a classical polynomial when $k \leq p+1$. To the best of our knowledge, this result is new for $k = p+1$ (when $p > 2$). We then prove that non-classical polynomials are necessary in the inverse theorem for the Gowers U^k-norm over $\mathbb{F}_p^n$ for all $k \geq p+2$, completely characterizing when classical polynomials suffice.

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MS17

Hypergraph Random Walks: Applications

In the last twenty years network science has proven its strength in modelling many real-world systems, involving generic agents connected by edges to represent their pairwise interactions. Yet, in many relevant cases, this is only a first order step, because interactions involve larger sets of nodes, at a time. The framework of hypergraphs, whose hyperedges effectively account for many-body interactions, is thus better suited to deal with such systems. We hereby study a class of random walks defined on such hypergraphs, grounded on a microscopic physical model where many-body proximity is associated to highly probable exchanges among agents belonging to the same hyperedge. We provide an analytical characterisation of the process by introducing a generalised random walk Laplace operator that reduces to the standard random walk Laplacian once the interactions are pariwise. We illustrate our results on synthetic models for which we have a full control of the high-order structures, and real-world networks where higher-order interactions are at play. As a first application we compare the random walk on hypergraphs with respect to the one on networks, drawing interesting conclusions on node rankings. Secondly, we show how random walk on hypergraphs can be successfully used for classification tasks involving objects with several features. We then conclude by showing the impact of the high-order interactions on the detection of the community structures existing among the nodes.

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MS17

Hypergraph Eigenvarieties and Geometric vs Algebraic Multiplicity

In spectral graph theory, eigenspaces have been paid little attention because they do not add much to the story painted by eigenvalues. However, for hypergraph (homogeneous adjacency) spectra, the set of vectors corresponding to an eigenvalue are more complicated: they are projective varieties instead of linear spaces. For example, almost nothing is known about the relationship between the algebraic multiplicities of eigenvalues and, generalizing geometric multiplicity, the dimensions of the irreducible components constituting their 'eigenvarieties'. We investigate 'nullvarieties' – eigenvarieties corresponding to the zero eigenvalue – of paths, linear trees, and other simple classes of hypergraphs, enumerating irreducible components, connecting their supports with edge transversals, and verifying special cases of a conjecture of Hu-Ye.

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MS17

Community Detection for Hypergraph Networks via Regularized Tensor Power Iteration

We propose a new method for community detection that operates directly on the hypergraph. At the heart of our method is a regularized higher-order orthogonal iteration (reg-HOOI) algorithm that computes an approximate low-rank decomposition of the network adjacency tensor. Compared with existing tensor decomposition methods such as HOSVD and vanilla HOOI, reg-HOOI yields better performance, especially when the hypergraph is sparse. Given the output of tensor decomposition, we then generalize the community detection method SCORE (Jin, 2015) from graph networks to hypergraph networks. We call our new method Tensor-SCORE. In theory, we introduce a degree-corrected block model for hypergraphs (hDCBM), and show that Tensor-SCORE yields consistent community detection for a wide range of network sparsity and degree heterogeneity. As a byproduct, we derive the rates of convergence on estimating the principal subspace by reg-HOOI, with different initializations, including the two new initialization methods we propose, a diagonal-removed HOSVD and a randomized graph projection. We apply our method to several real hypergraph networks which yields encouraging results. It suggests that exploring higher-order interactions provides additional information not seen in graph representations.

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MS17

Submodular Random Walks

Recently Li Milenkovic and Yoshida independently developed the idea of a Laplacian for a collection submodular functions over the subset lattice which generalizes existing Laplacians over graphs, directed graphs, and hypergraphs. Along with the Laplacian they both define similar versions of the Cheeger ratio and inequality and provide means of efficiently approximating spectra of the Laplacians. In this presentation we explore extending the work of Li Milenkovic and Yoshida to define a random walk on the base elements of the subset lattice whose evolution is governed by a collection of submodular functions. Joint work with Sinan Aksoy and Bill Kay.

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MT1

Inspiring and Training Young Mathematicians

Input your abstract, including TeX commands, here. The abstract should be no longer than 1500 characters, including spaces. Only input the abstract text. Don't include title or author information here.

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MT2

Stochastic Localization and Concentration Inequalities

One of the most successful techniques in obtaining concentration bounds is called localization, a technique developed by Gromov and Milman (87), Lov asz and Simonovits (93). This technique allows to reduce inequalities in high dimensional space to corresponding low-dimensional bounds via a geometric bisection procedure which "localizes" a high-dimensional object. It was used to derive the first nontrivial bounds on the Kannan-Lovasz-Simonovits (KLS) conjecture which asserts that convex sets exhibit dimension-free expansion. The Stochastic Localization technique is an extension which attempts to localize a measure via an analytic (rather than a geometric) procedure, which iterates by applying tilting the density in continuous time, via in a stochastic process driven by a Brownian motion. This extension has been useful in producing concentration bounds in both continuous and discrete settings. In partic-

ular, this technique was used in a recent breakthrough by Chen on the aforementioned KLS conjecture. In this minitutorial, we will go over the basics of this technique, see how it works and how it can be used to produce concentration bounds. The technique is based on stochastic calculus, but the minitutorial will only assume basic knowledge in probability theory.

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MT2

Stochastic Localization and Concentration Inequalities

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MT2

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