

Apply It.



The math behind... Pattern Formation

Some Technical terms used:

Reaction-diffusion, Turing instability, diffusion, stability analysis, periodicity, morphogens

Uses and applications:

Many experiments involving interaction between chemicals lead to the observation of patterns with an almost periodic nature. These patterns can be used to explain how animals such as zebra, tigers, fish and giraffe among others obtain their distinctive patterns. The same theory may also be used to explain stripes in sand dunes, patterns in tree bark and human fingerprints.

How it works:

Alan Turing hypothesized that the markings in the coats of different animals are caused by variation in the levels of chemicals called morphogens. These chemicals are found in cells deep within the skin of animals. Colored pigments are produced in these skin cells if the morphogen is present above some critical level. If the level is too low, then no pigment will be produced. This can be used to explain the patterns seen in various different creatures.

Mathematically we explain this by considering a situation in which two chemicals interact to change their concentrations and diffuse within some limited space. This is known as a system of reaction-diffusion equations. Suppose that one of these chemicals were a catalyst and the other an inhibitor. If these two chemicals have different diffusion coefficients, i.e. they move around at different speeds then the system may undergo a bifurcation. Put simply, the system then exhibits instability and this causes a pattern to form in the concentration of the morphogen. This then causes certain areas to produce pigment, thus creating the familiar patterns seen in many creatures.

The size of the limited space (domain) in which these chemicals diffuse is important. If the domain is very small then the inhibitor will diffuse across it fairly quickly. As a result of this, we tend to see very little pattern formation. Alternatively, if the domain in which the chemicals diffuse is very large, then there can be a pattern formed. However, it will most likely be too small to see. It has been argued that this explains why very small and very large animals are not usually patterned.

Interesting facts:

Mathematical biology was an interest of Alan Turing's in the latter years of his life. He only formally published one paper on the subject [1], but his contribution is considered a seminal piece of work.

Turing's main interest in pattern formation was in explaining the occurrence of Fibonacci numbers and the so called golden ratio within certain plant features. In order to study this phenomenon, he used a system of reaction-diffusion equations. The study of this system leads to his theory of diffusion-driven instability.

References:

[1] Turing, A. M. (1952), The chemical basis of morphogenesis, *Phil. Trans. R. Soc. B*, Vol. 237, No. 641, pp. 37-72.

[2] Murray, J. D. (2003), *Mathematical Biology II: Spatial Models and Biomedical Applications* (3rd edition), Springer, New York.

Submitted by Matthew Edgington, University of Reading, United Kingdom.
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