Math Matters

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The math behind... Knots

Technical terms used:

Topology, writhing number, ideal crossing number, unknotting number, topoisomerases

Uses and applications:

Tangled earphone wires are perhaps the most common example in which we experience how complicated knots can be. But knots are not always a problem. Various types of knots have been used since ancient times for a variety of applications, from tying shoelaces, knitting and sewing for making garments, hairstyles, and sailors' rope knots to the sutures used by surgeons. More recently, knot theory has found applications in understanding folding and unfolding of DNA and RNA molecules.

How it works:

Most everyday knots deal with open-ended threads. A mathematical knot, however, is formed by a single closed loop of string. Simply speaking, a loop has a knot if it cannot be made into a single circular loop without cutting and rejoining the string. Mathematicians have designed fairly simple ways to represent and quantify properties of knots using diagrams on paper. The crossing points of the strings are assigned a + symbol when upper string seems to be rotated counterclockwise with respect to the string below it, and a – symbol otherwise. Adding these signs for all crossovers in a knot gives the writhing number. The writhing number can be misleading because it is not unique for a given knot topology. To solve this problem, mathematicians defined two other numbers, called the ideal crossing number (C) and the unknotting number (U). C is the number of crossovers for a knot in its simplest form, and U is the number of times one has to cut and rejoin the strings at crossovers in order to remove the knot. C and U together can unambiguously identify any type of knot.

Interesting fact:

Surprisingly, this pure mathematical concept is reflected in many natural phenomena. A most rigorously studied example of naturally occurring knots would be the DNA topology. More amazing are the molecules called topoisomerases, which can create and remove the knots in a couple-of-meters-long DNA folded inside our tiny cells. Apart from the knots at molecular level, people have observed a few animals that can form knots. For example, a marine fish called hagfish folds its body into a sliding knot which can move from head to tail. The hagfish manipulates the knot to catch its prey or to clean its own body. Sometimes, snakes have been observed to tie themselves into a knot. Studies later proved that an Ebola-like virus is the culprit of this mysterious disease.

References:

[1] http://www.tiem.utk.edu/~gross/bioed/webmodules/DNAknot.html

[2] V. Zintzen, C. D. Roberts, M. J. Anderson, A. L. Stewart, C. D. Struthers, E. S. Harvey, Hagfish predatory behaviour and slime defence mechanism. *Scientific Reports*. **1,131** (2011).

[3] http://news.nationalgeographic.com/news/2012/08/120822-snakes-virus-ibd-ebola-animals-science/

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