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Special Issue on Dynamical Systems

*This **special issue** highlights research from the 2021 SIAM Conference on Applications of Dynamical Systems, as well as other timely developments and trends in the field.*

Figure 1. *Sea ice off the coast of Greenland, which has been diminished by warming Arctic temperatures. The infusion of cold water into the ocean risks disrupting the Atlantic meridional overturning circulation. Public domain image courtesy of NASA.*

In an article on page 6, Matthew Francis overviews the research of Christopher Jones, who works with mathematical models that explore rate-induced tipping in climate systems, the weakening Atlantic meridional overturning circulation, and the effects of human-driven climate change.

Maintaining Biodiversity in an Increasingly Variable World

By Sebastian J. Schreiber

All living organisms experience fluctuations in environmental conditions like temperature, precipitation, nutrient availability, and predation risk. Because these conditions impact survival, growth, and reproduction, the environmental fluctuations cause additional fluctuations in population densities. As a result, environmental fluctuations can affect the viability of populations and the dynamics of interacting species. The most recent report from the Intergovernmental Panel on Climate Change[1] found that the frequency and intensity of heavy precipitation events—as well as concurrent heatwaves and droughts—have increased since the 1950s. Understanding the ecological impacts of these growing environmental variabilities is critical for managing populations and conserving biodiversity. Recent mathematical developments in the analysis of stochastic models [2, 4, 5] allow researchers to study these impacts.

Environmental fluctuations' effect on populations is known as *environmental stochasticity*. "[It is] obviously true that the numbers in most natural populations are sometimes increasing and sometimes decreasing, and that these fluctuations may be enormous," Herbert Andrewartha wrote in the 1950s. "One is thus led to expect that the so-called stochastic models of populations might be more realistic and therefore more successful than the deterministic models" [1]. Richard Lewontin and Dan Cohen answered Andrewartha's call [7] and studied the simplest possible model that accounts for environmental stochasticity:

$$x(t+1)=x(t)f(t). \qquad (1)$$

Here, $x(t)$ is the population density at time t and $f(t)$ corresponds to the fitness of individuals at time t (the average contribution of an individual to the population size over a single time step). If $f(t)$ is a sequence of independent and identically distributed (i.i.d.) random variables, then $\lim_{t\to\infty}\frac{1}{t}\ln x(t)=\mathbb{E}[\ln f(1)]$ with probability 1 by the law of large numbers. The population thus grows exponentially quickly if the per capita growth rate $r=\mathbb{E}[\ln r]$ is positive; if r is negative, the population decays

[1] https://www.ipcc.ch/report/sixth-assessment-report-working-group-i

See **Biodiversity** *on page 3*

Phase Transitions in the Heart and the Genesis of Cardiac Arrhythmia

By D'Artagnan Greene and Yohannes Shiferaw

Approximately one in five human deaths occur when the periodic beating of the heart suddenly transitions into a chaotic rhythm [7]. This transition—called a cardiac arrhythmia—prevents the heart from effectively pumping blood and can have deadly consequences. To identify the cause of arrhythmias, one must understand the basic physiology of heart cells and tissue [4]. Heart cells regulate voltage across their membranes via the complex interplay of millions of nanometer-scale proteins called ion channels. Ion channels are sensitive to the voltage gradient; their nonlinear response to this gradient endows the cells with the properties of an excitable system. Heart cells are excitable in the sense that injection of a small initial current can induce a much larger secondary current flow that generates a rapid increase in the voltage across the cell membrane.

Cells are connected to each other via ion channels, which means that an excitation of one cell can rapidly excite its nearest neighbors. As with falling dominoes, this process can rapidly spread the signal outward across many cells to eventually excite all cardiac tissue. This rapid propagation of electrical excitation orchestrates the heart's rhythmic beating. In an unhealthy heart, however, the typically organized electrical excitation can break down into a swirl of turbulent electrical activity that overruns the heart's intrinsic rhythm and causes an arrhythmia.

In recent decades, biologists have worked closely with applied mathematicians and physicists to decipher the mechanisms that drive cardiac arrhythmias. The mathematical perspective has yielded important insights about the structure of coherent excitations in tissue—such as spiral and scroll waves—that practitioners have experimentally observed in the heart [2]. Additionally, molecular biologists have discovered that arrhythmias can arise due to defects at the level of individual amino acids in proteins that regulate the voltage across cell membranes. However, the mechanism through which nanometer-scale molecular defects influence the spatiotemporal activity of an entire heart remains unknown. This is because sub-microsecond time scale fluctuations between stable conformational states—such as an ion channel's open and closed state—dictate the function of proteins at the nanometer scale. Furthermore, an electrical activation in tissue involves the excitation of millions of electrically coupled cells. Understanding the cause-and-effect relationships across these vast length and time scales is a daunting yet necessary challenge that will allow researchers to fully comprehend the mechanisms of cardiac arrhythmias.

In order for a cardiac cell to function properly, a vast array of proteins and ions must work together like a finely tuned instrument. Cells contribute to this partnership by utilizing signal transduction — a process that transports and delivers information between different parts of the cell with precise timing. The most ubiquitous signaling messenger is the calcium (Ca) ion, which regulates a broad range of cellular processes. Cardiac cells have subcellular compartments that store Ca ions at concentration levels that are roughly four orders of magnitude higher than those in the cell. To induce a cellular response, a small initial increase in Ca ions triggers the release of a large amount of Ca from these compartments into the cell interior, which in turn diffuses and activates an array of Ca-sensitive proteins. This architecture also endows the cell interior with the properties of an excitable system, since a signal can spread in the interior much like electrical activity spreads between cells in tissue.

See **Cardiac Arrhythmia** *on page 4*

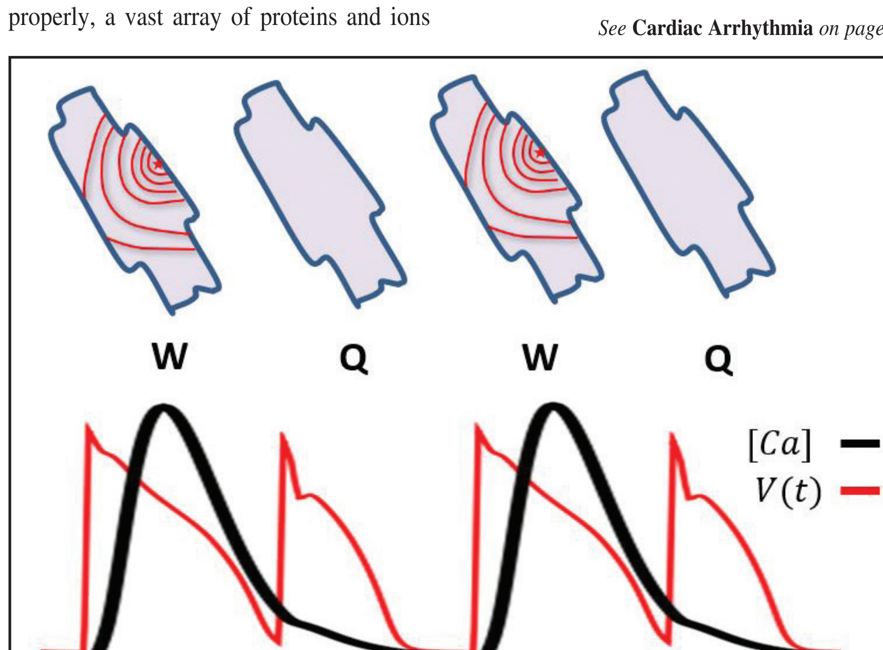

Figure 1. *A paced cardiac cell can exhibit a sustained alternating pattern in which a quiescent beat (Q) follows a calcium (Ca) wave (W). A large amount of Ca is released into the cell during the wave, which generates an alternating Ca and voltage signal. Figure courtesy of the authors.*

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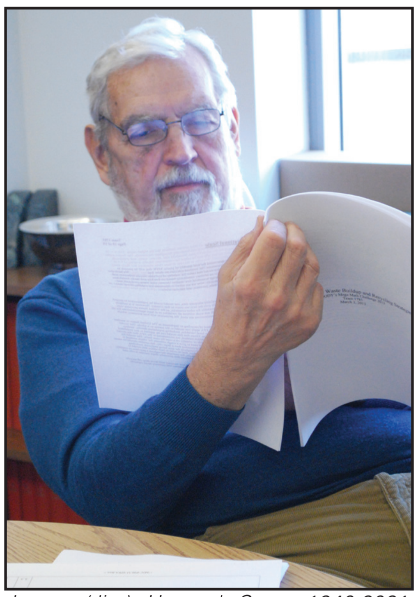
Obituary: James Howard Case

James (Jim) Howard Case passed away unexpectedly on September 1, 2021, at the age of 80. A longtime member of SIAM, Jim was perhaps best known within the SIAM community for his thoughtful, eloquent book reviews in *SIAM News*. He was also an early and ardent supporter of the MathWorks Math Modeling Challenge (M3 Challenge),[1] a program of SIAM.

A man of diverse interests who was admired for his sense of humor, Jim was born in Rochester, NY, in 1940 to Charles Zopher Case and Mary Proctor Case. He grew up on a dairy farm in nearby Avon, NY, and graduated from Groton School in Massachusetts in 1958. Jim went on to earn a bachelor's degree in mathematics at the University of Rochester. As an undergraduate, he was a three-sport varsity athlete in football, swimming, and baseball; in 2013, he was inducted into Rochester's Athletic Hall of Fame.

Judges of the 2011 MathWorks Math Modeling Challenge (M3 Challenge), a program of SIAM, gather at SIAM headquarters in Philadelphia, Pa., to evaluate submitted papers from competing student teams. James Case (seventh from right, in the back) served as both a triage and contention judge for more than 10 years. Photo courtesy of SIAM.

Jim's overriding passion was to become a professional baseball pitcher, and he enthusiastically signed with the Los Angeles Dodgers organization. He was assigned to the Panama City Fliers (an affiliate of the Dodgers) and attended spring training in Vero Beach, Fla., but suffered a shoulder injury and was released in the spring of 1962. After the sudden end to his baseball career, Jim took a tramp steamer to Cannes, France; bought a bicycle; and headed north to Paris. He stopped along the way to enjoy Michelin-starred restaurants and spoke enough French to successfully order a meal. Following this experience, his favorite vacation was always a trip through the French countryside in search of good food and wine, and he remained a devoted patron of La Pyramide in Vienne.



James (Jim) Howard Case, 1940-2021. Photo courtesy of SIAM.

Jim completed his master's and doctoral degrees at the University of Michigan in 1967 and wrote a dissertation on the equilibrium points of *n*-person differential games under the supervision of Robert McDowell Thrall. He was elected to Sigma Xi, The Scientific Research Honor Society, while at Michigan. After graduation, Jim conducted postdoctoral research at Princeton University and the University of Wisconsin's Mathematics Research Center. In 1970, he moved to Baltimore, Md., and joined the Operations Research and Industrial Engineering Department (now the Department of Applied Mathematics and Statistics) at Johns Hopkins University as an assistant professor. He left the school in 1976, later becoming a lecturer at Towson University and working for the Federal Trade Commission and American Petroleum Institute.

Jim authored three books throughout his professional career, including *Competition: The Birth of a New Science*,[2] as well as a number of book chapters and peer-reviewed journal papers on topics such as game theory and finance. In fact, his first published peer-reviewed paper appeared in the *SIAM Journal on Control* in 1969 [1]. In addition, Jim wrote for *Mathematical Reviews* and served as an associate editor of *Operations Research* from 1974-1979 and the *American Mathematical Monthly* from 1995-2001. He was also a member of the Mathematical Association of America and the International Society of BioPhysical Economics.

Jim was a frequent contributor to *SIAM News* and published both book reviews and freelance pieces based on SIAM conference lectures or other noteworthy topics. In the last 10 years alone, he wrote nearly 50 articles for *SIAM News* that covered a wide variety of subjects, including baseball statistics, communication networks, probability, big data algorithms, artificial intelligence, and a number of mathematical biographies; a complete archive of his work from June 2012 onward is available online.[3] Jim's most recent submission—a book review of *Bounded Gaps Between Primes* by Kevin Broughan—appeared in the September 2021 issue.[4]

Jim was also actively involved with the M3 Challenge, an annual mathematical modeling competition for high school students, and served as both a triage and contention judge for over 10 years. In 2016, a participating M3 team that had received an honorable mention prize reached out to SIAM's M3 Challenge staff and asked if anyone in the SIAM or M3 community would be willing to visit the school and present the team certificates. Jim eagerly volunteered and drove 75 miles from Baltimore to Ashburn, Va., to attend the award ceremony at Stone Bridge High School — a reflection of his unwavering desire to inspire young people with a burgeoning interest in math.

Upon moving to Baltimore, Jim became an ardent fan of the Orioles, Colts, and eventually the Ravens. He was an avid downhill skier and instilled a passion for the sport in his children, their spouses, and his grandchildren, all of whom treasure memories of annual family ski vacations.

While teaching in British Columbia for a semester, Jim took a liking to hard cider. When he returned to Baltimore, he founded the Chesapeake Hard Cider Company in 1983. Jim owned and operated the company until 1991. In his spare time, he enjoyed membership with the L'Hirondelle Club, the 14 West Hamilton Street Club, and the Wednesday Club.

During his undergraduate years, Jim's Rochester swim coach, Roman "Speed" Speegle, introduced him to Patricia (Pat) deYoung, a member of the women's swim team; they married in 1962. Jim is survived by his wife Pat; children Martha, Caroline, and Charles, as well as their spouses; five grandchildren; and his sister Elizabeth. He will be greatly missed by his family, friends, and colleagues; the *SIAM News* staff; and the entire SIAM community.

1 https://m3challenge.siam.org

2 https://www.amazon.com/Competition-Birth-Science-James-Case/dp/0809035782

3 https://sinews.siam.org/About-the-Author/james-case

4 https://sinews.siam.org/Details-Page/prime-gap-breakthrough

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SIAM News would like to thank the family of Jim Case for their contributions to this article.

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Biodiversity

Continued from page 1

exponentially quickly toward extinction. In contrast, the expected population size $\mathbb{E}[n(t)]$ grows exponentially like $\mathbb{E}[f(1)]^t$. Lewontin and Cohen highlighted a surprising prediction from this simple model, based on the fact that the geometric mean $\exp(\mathbb{E}[\ln f(1)])$ is less than the arithmetic mean $\mathbb{E}[f(1)]$: They noted that "even though the expectation of population size may grow infinitely large with time, the probability of extinction may approach unity" [7].

Mathematical models of population growth often account for density dependence in the fitness f of the population:

$$x(t+1) = x(t)f(x(t),\xi(t+1)), \qquad (2)$$

where $\xi(t)$ is a sequence of i.i.d. random variables representing demographic impacts of environmental fluctuations. Under appropriate assumptions about the behavior near infinity, these models exhibit statistically bounded fluctuations. Furthermore, if the per capita growth rate when rare (GRWR) $r=\mathbb{E}[\ln f(0,\xi(1))]$ is positive, then the population is stochastically persistent — it tends to spend little time near 0. Alternatively, the population tends toward extinction if $r<0$. This simple characterization of extinction versus persistence provides a mathematic proof that increased variability in precipitation may have led to the local extinction of two populations of Bay checkerspot butterflies [8] (see Figure 1).

Species are not isolated from each other; they interact through a complex web of direct and indirect pathways. Moreover, individuals within a species may differ from one another in demographically important ways due to variations in behavior, morphology, physiology, or spatial location. To account for diversity of species interaction and individual differences, consider Markovian models in which $x=(x_1,\ldots,x_n)$ is the vector of species' densities and $y\in\mathbb{R}^k$ are auxiliary variables [4]:

$$\begin{aligned} x_i(t+1) &= x_i(t)f_i(x(t),y(t),\xi(t+1)) \\ & \qquad i=1,\ldots,n \qquad (3) \\ y(t+1) &= G(x(t),y(t),\xi(t+1)). \end{aligned}$$

The auxiliary variables describe population structure (i.e., keep track of each species' frequency in a patch, age class, or stage), capture feedback variables (e.g., trait evolution and plant-soil feedbacks), or allow for structure in environmental fluctuations (e.g., autocorrelation).

Characterizing coexistence and extinction in these types of models is a much more delicate process than in the single-species model. However, studies have extended an approach that is inspired by Josef Hofbauer's work with deterministic models [6] to these stochastic models, as well as to stochastic differential equations and piecewise deterministic Markov processes (PDMPs) [2, 4, 5]. Like the single-species models, this approach relies on GRWRs. But unlike the single-species models, there are multiple contexts in which a species may become rare. These contexts are given by ergodic measures $\mu(dxdy)$ of (3) that support a subset of species, i.e., $\mu(\{(x,y):\min_i x_i=0\})=1$. For such an ergodic measure, the GRWR of a missing species i when introduced at infinitesimally small densities is

$$r_i(\mu)=\int \mathbb{E}[\ln f_i(x,y,\xi(1))]\mu(dxdy).$$

This is the per capita growth rate $\ln f_i$ averaged over the fluctuations in x, y, and ξ. The Hofbauer condition ensures coexistence (i.e., all species' densities tend to stay away from low values) if fixed weights $w_i>0$ exist, such that

$$\begin{aligned} &\sum_i w_i r_i(\mu)>0 \text{ for all ergodic } \mu \\ &\text{with } \mu((x,y):\min_i x_i=0)=1. \end{aligned} \qquad (4)$$

The function $V(x,y)=-\Sigma_i w_i \ln x_i$ then acts like a type of average Lyapunov function near the extinction set — i.e., V tends to increase along trajectories when $\min_i x_i$ is sufficiently small. Related conditions help researchers identify when one or more species goes towards extinction exponentially quickly. These results collectively allow one to determine whether all modeled species coexist, or whether one or multiple stochastic attractors are present wherein one or more species are excluded.

Application of these results to multispecies models has yielded new, mathematically rigorous insights into environmental stochasticity's impact on the dynamics of communities of interacting species. For example, one study examines the way in which autocorrelated fluctuations in fecundity or survival determine the fate of competing species [9]. For these models, $f_i(x,y)=\lambda_i(y_i)/(1+x_i+x_2)+s_i(y_i)$, where λ_i and s_i are the maximal fecundity and survivorship of species i and $y=(y_1,y_2)$ is a multivariate autoregressive process. In the absence of environmental fluctuations (i.e., when y remains constant), the species with the higher reproductive number $\lambda_i/(1-s_i)$ excludes the other (see Figure 2a). Accounting for environmental fluctuations can shift the exclusionary dynamics to stochastic coexistence (see Figure 2b) or a stochastic priority effect (see Figure 2c and 2d), whereby there is a positive probability that one species drives the other to extinction. These outcomes have a delicate dependence on whether survival or fecundity fluctuates, and the sign of its temporal autocorrelation. For example, positively autocorrelated fluctuations in fecundity promote coexistence and positively autocorrelated fluctuations in survival promote stochastic bistabilities. Equally surprising is Michel Benaïm and Claude Lobry's use of PDMP Lotka-Volterra models to show that random switching between two environments—both of which favor the species in question—can lead to that species' extinction [3].

Indirect species interactions are more challenging to study but can yield additional unexpected results. For example, analyses of the dynamics of two prey species that share a common predator revealed that an increase in the density of one prey species leads to an increase in the predator's density and a

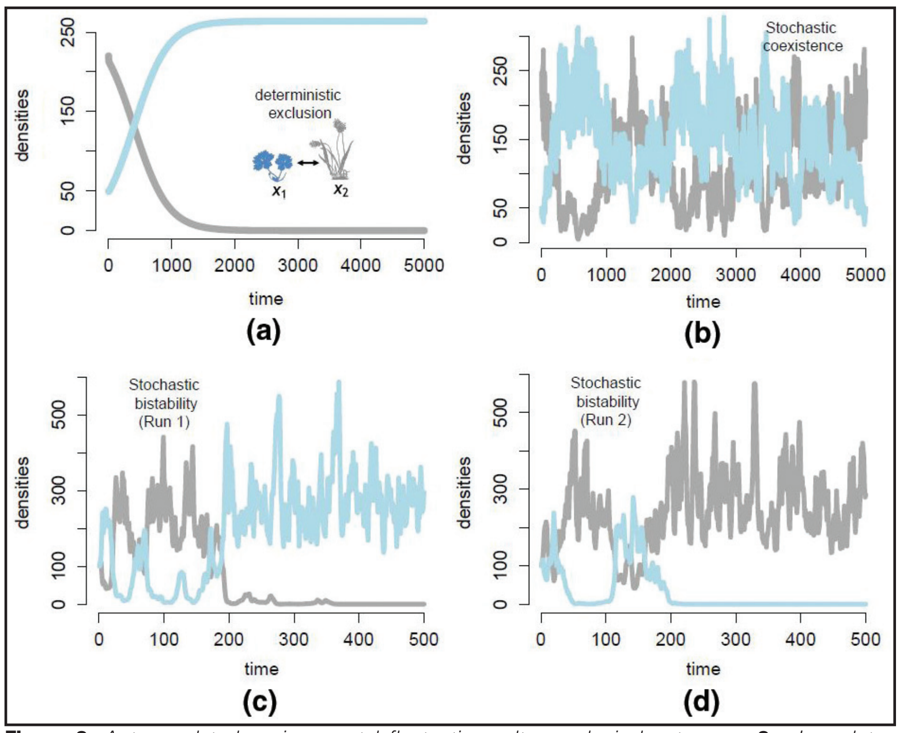

Figure 2. *Autocorrelated environmental fluctuations alter ecological outcomes.* **2a.** *In a deterministic model of two competing species, one species always excludes the other.* **2b-2d.** *Autocorrelated fluctuations in demographic parameters can alter this outcome, allowing for stochastic coexistence (2b) or a stochastic bistability (2c and 2d) for which there is a positive probability of losing either species for the same initial conditions [9]. Figure courtesy of Sebastian Schreiber.*

corresponding decrease in the other prey species [10, 11]. Although the prey are not actually competing, they *appear* to be due to the indirect effect of the predator. A simple rule of dominance emerges in the absence of autocorrelated environmental fluctuations: the prey species that supports the higher mean predator density excludes the other prey [11]. Autocorrelated fluctuations in predator attack rates can shift the exclusionary dynamics to stochastic coexistence or a stochastic priority effect [10]. Yet unlike the competition models, the intermediary species (the predator) may exhibit varied responses to environmental perturbations on shorter and longer time scales. Highly autocorrelated environmental fluctuations can therefore generate different ecological outcomes (stochastic bistability) than those that result from weakly autocorrelated environmental fluctuations (coexistence).

Despite this progress, many challenges remain. Significant gaps endure between the necessary and sufficient conditions for stochastic coexistence, and a deeper biological understanding of environmental stochasticity's impact on the ecological dynamics of more diverse communities remains elusive. However, a growing number of talented mathematicians are currently tackling these problems; further insight is sure to follow.

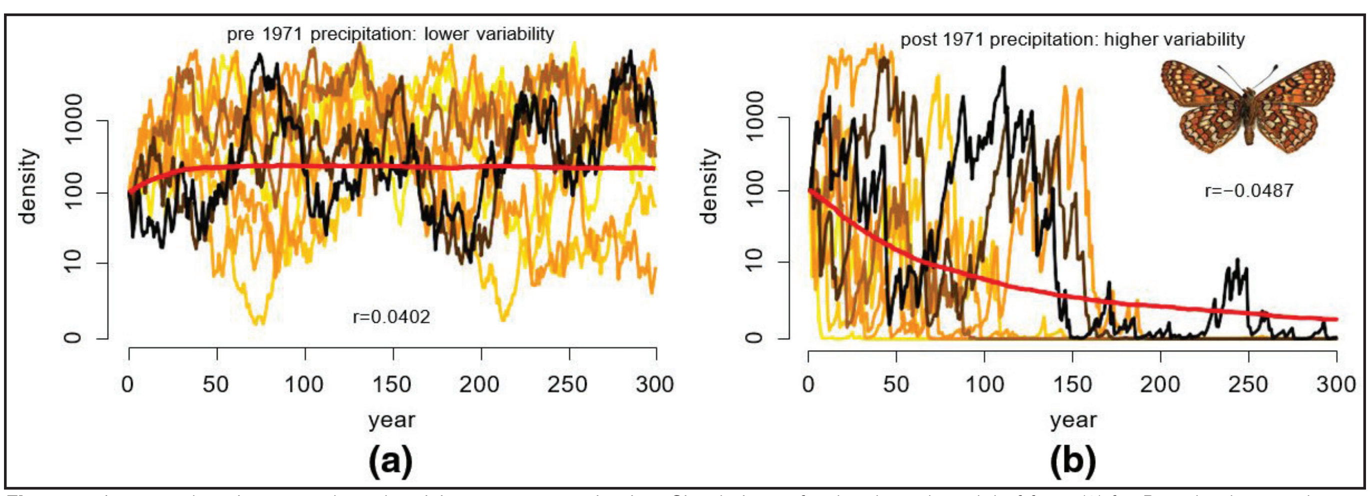

Figure 1. *Increased environmental stochasticity promotes extinction. Simulations of a data-based model of form (2) for Bay checkerspot butterflies that are experiencing variability in precipitation.* **1a.** *For precipitation variability before 1971, $r>0$ and stochastic persistence occurs.* **1b.** *For precipitation variability after 1971, $r<0$ and asymptotic extinction occurs. These predictions are consistent with the hypothesis that increased precipitation variability caused the local extinction of two butterfly populations [8]. Figure courtesy of Sebastian Schreiber.*

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Sebastian J. Schreiber is a professor of evolution and ecology at the University of California, Davis. He uses the theories of stochastic processes and dynamical systems to tackle questions in ecology, evolution, and epidemiology.

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Cardiac Arrhythmia

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As a result, waves of released Ca—which occur randomly in cardiac cells—can propagate within a cell under appropriate conditions. These Ca waves are dangerous because Ca wave propagation is a highly nonlinear function of the Ca concentration in intracellular stores, and the waves disrupt the rhythmic responses of cardiac cells. Nucleation of these waves is also highly sensitive to local fluctuations, meaning that their timing and extent is stochastic. Consequently, there is now a consensus in the cardiac community that Ca waves can cause a wide range of cardiac arrhythmias [8].

Although biologists have clearly demonstrated that a disruption in Ca signaling can trigger cardiac arrhythmia, researchers still do not understand how a molecular-scale defect can prompt a breakdown of electrical activity for the whole heart. Our group has recently found that Ca waves can synchronize across millions of cells in cardiac tissue. Ca waves fire in unison when this synchronization occurs, which dramatically amplifies their effect on the tissue scale. To discern the impetus behind such synchronization, we must apply concepts from the theory of phase transitions — an elaborate mathematical framework that explains sudden changes in material properties with temperature [1].

The concept of symmetry plays a fundamental role in phase transitions. This symmetry arises in the heart because periodically driven cells can acquire temporal patterns in which Ca waves occur in the cells on alternate beats (see Figure 1, on page 1). A sequence of Ca waves (W) followed by quiescence (Q) produces this alternating pattern, so that a given cell can alternate with a sequence

$$...Q\ W\ Q\ W...$$

However, simply shifting the sequence by one beat yields

$$...W\ Q\ W\ Q...,$$

which is dynamically equivalent. Paced cardiac cells thus possess a subtle symmetry—first described in a different context [6]—that has important consequences at the tissue scale. To explore these consequences, we introduce an order parameter that measures the phase of the alternating sequence. On a two-dimensional (2D) lattice, we can therefore describe the cell at site ij with an order parameter

$$s_{ij} = \begin{Bmatrix} +1 & \rightarrow & \ldots QWQW\ldots \\ -1 & \rightarrow & \ldots WQWQ\ldots \end{Bmatrix}.$$

On a lattice of coupled cells, the probability of observing sequence $\{s_{ij}\}$ is the same as observing $-\{s_{ij}\}$. This phenomenon—known as Ising symmetry—is shared by the interacting spin-$1/2$ systems that characterize ferromagnetic materials. The "spins" s_{ij} interact in a manner that is governed by the interplay between the voltage and Ca signals in cardiac tissue. By accounting for these interactions, we realize that we can map the organization of Ca waves in tissue to an equivalent statistical mechanics problem with a Hamiltonian, given by

$$H = \frac{\epsilon}{N}\left(\sum_{ij} s_{ij}\right)^2.$$

N refers to the total number of cells and ϵ is a parameter whose sign is determined by the way in which voltage couples to Ca according to the multitude of ion channels that regulate the cell membrane. This model is analogous to the classic Curie-Weiss model for ferromagnetism, which exhibits an order-disorder phase transition at a critical temperature T_C [5]. In the context of the heart, the transition maps directly to a synchronization transition wherein Ca waves self-organize across millions of cells in cardiac tissue. Here, the role of thermal fluctuation corresponds to the stochasticity of Ca wave formation and a critical pacing rate replaces the critical temperature. Below the critical pacing rate, the voltage nudges the stochastic subcellular Ca signal to synchronize millions of cells so that they fire Ca waves in unison. Numerical simulations of simplified 2D systems reveal that when this transition commences, a large piece of cardiac tissue coarsens into regions of synchronized waves (see Figure 2a). These regions of synchronized waves can induce large voltage perturbations in that tissue, potentially leading to wave break and reentry (see Figure 2b). In systems with disrupted Ca signaling, we find that arrhythmias only occur at parameter regimes where the synchronization transition has transpired.

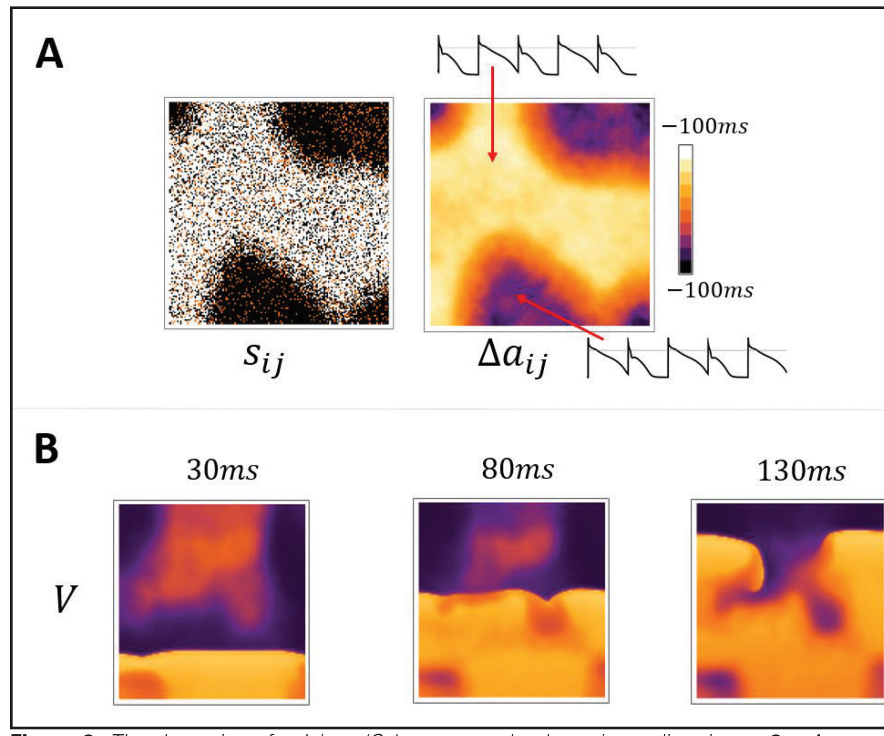

Figure 2. *The dynamics of calcium (Ca) waves and voltage in cardiac tissue.* **2a.** *At steady state, the order parameter s_{ij} that measures the phase of the alternating waves can synchronize across large patches of cardiac tissue. Here, black regions denote cells with $s_{ij}=1$ and white regions denote cells with $s_{ij}=-1$. Orange points denote cells wherein no Ca wave occurs. This pattern then drives spatial patterns of voltage that are quantified by the action potential amplitude Δa_{ij}, which measures beat-to-beat differences in the voltage time course.* **2b.** *When the same tissue is paced from the bottom edge, planar waves undergo wave break that is caused by the spatial patterns that form due to the synchronization transition. Here we simulate a piece of cardiac tissue that consists of a 150×150 cell grid. The wave break occurs after 30 beats when the spatial patterns have developed. Figure courtesy of [3].*

Our results highlight new relationships between biological tissue and material science systems. Under certain conditions, biological tissue can share symmetries that also exist in unrelated systems (like ferromagnetic materials). Furthermore, the phase transition that we identify here has direct physiological relevance — it provides a precise mechanism that relates subcellular defects in Ca cycling to a tissue-scale phenomenon that involves millions of cells. In the future, researchers might be able to utilize this novel perspective to develop new therapeutics that prevent the synchronization transition from occurring.

This article is based on Yohannes Shiferaw's invited talk at the 2021 SIAM Conference on Applications of Dynamical Systems,[1] which took place virtually this May.

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Machine Learning and Dynamical Systems

By Qianxiao Li and Weinan E

Mathematical modeling of dynamical systems (DS) is a central goal of the quantitative sciences. Although machine learning (ML) technologies are modern inventions, the interaction of data and dynamics has a long history. One of data science's first forays into modeling dynamics perhaps began with astronomy — particularly with Ptolemy's archaic but instructive geocentric model of the cosmos, which culminated in Kepler's laws of planetary motion. These laws ultimately laid the empirical basis for Newton's landmark contributions. Since then, the interactions of DS and data science have matured in both breadth and depth. Recent innovations in ML—particularly deep learning (DL)—have yielded further insights into the connection between these fascinating fields. Here we introduce some recent lines of work at the intersection of ML and DS.

Three Types of Connections

To organize ideas, one can classify the types of interactions between ML and DS into three complementary directions: machine learning *of* dynamical systems, *by* (or *via*) dynamical systems, and *for* dynamical systems (see Figure 1 for some examples). The first direction is perhaps most familiar, as it concerns the question of how one can obtain mathematical models from observations of dynamical processes (much like the works of Ptolemy and Kepler). A key issue involves learning such dynamics from data while still retaining physical insights. The second direction pertains to the study of the theory and algorithms for modern ML methods that one applies to data from dynamical problems. Examples include the well-known recurrent neural networks (RNNs) and their extensions (such as long short-term memory networks (LSTMs) and gated recurrent units) as well as complex mechanisms (such as attention) whose theoretical understanding remains quite limited. Finally, recent work in the last direction shows that deep neural networks (DNNs) are themselves akin to DS; viewing them in this way allows one to employ dynamics-based mathematical ideas and tools to advance theory and algorithms for DL [3]. We now discuss selected works in each of these directions.

ML of DS	ML by DS	ML for DS
System identification	Control formulation of deep learning	Recurrent neural networks
Learning and control	Architectures via discretization	LSTM, GRU
Model reduction	Approximation theory of composition/dynamics	Attention, transformers

Figure 1. *Examples of interactions between machine learning (ML) and dynamical systems (DS).*

Machine Learning by Dynamical Systems

Despite widespread practical success, DL still requires further theoretical progress. For instance, researchers seek to develop a succinct mathematical setting with which to view DL that focuses on newly arising phenomena. As an example, one significant novel aspect of DL is the presence of compositional structures in the model; stacked layers achieve complexity through repeated composition. An important unresolved question explores the way in which this new compositional structure changes the model's behavior in terms of approximation, optimization, and generalization.

DS theory offers a promising framework for carrying out such an analysis. The connection between DS and these compositional structures was first publicized in 2017 [3]; one can regard deep (residual) neural networks (NNs) as finite-difference discretizations of a continuous-time DS (see Figure 2, on page 7). This outlook—which is popularized in ML literature as neural ordinary differential equations [2]—provides a convenient language with which to capture the key features of DL. Studies have explored the immediate consequences of this viewpoint in terms of network stability [4] and training methods [5, 6]. On the optimization side, these factors set forth the connection between DL and optimal control theory. In particular, researchers can regard the latter as a form of *mean field* optimal control; one can derive similar optimality conditions—such as Pontryagin's maximum principle and the Hamilton-Jacobi-Bellman equations—in the mean field setting.

Another interesting angle is approximation theory, which focuses on how one can build a complex hypothesis space through composition or dynamics. Complexity in shallow NNs arises from the linear combination of a large number of adaptive basis functions (or neurons/nodes). DNNs seem to require another angle, and several relevant results appear in the continuous setting [7]. However, a comprehensive understanding of the approximation properties of compositional or dynamical hypothesis spaces is currently limited. In particular, a general characterization of the aspects that make a "nice" function for approximation through composition/dynamics remains an interesting open question. The resolution of this question might explain why the performance of deep models is very problem-dependent.

Machine Learning for Dynamical Systems

A reverse scenario encourages researchers to comprehend the application of modern ML techniques to problems that entail dynamics, such as time series forecasting and sequence-to-sequence models for language or engineering applications. In the specific context of DL, a key mathematical problem involves elucidating the interaction of compositional or dynamical structures in DNNs as well as the dynamical structures that are present in the data or data generation process. This method offers researchers the unique opportunity to couple data and model structure in the analysis.

Consider RNNs, which are among the simplest ways to model sequential relationships. Yet many issues plague RNN applications in practice, especially their inability to handle long sequences. Researchers have made various improvements from a practical angle, including the now-popular LSTM. However, a complete understanding of the theory and limitations of recurrent architectures for time series modeling remains fragmented. This restraint is an obstacle for practitioners, who often must rely on trial and error to select the correct model architecture for the task at hand.

To address this issue, a functional approximation framework can serve as a general formulation on which one analyzes time series modeling via NNs [8]. Use of this framework proves that a "curse of memory" is associated with RNNs, even in linear settings. This finding parallels the investigation of the "curse of dimensionality" and demonstrates that both approximation and optimization become exceedingly difficult when memory increases in the

See **Machine Learning** *on page 7*

It's Not the Heat, It's the Rate

Rate-Inducted Tipping's Relation to Climate Change

By Matthew R. Francis

For many years, scientists have warned that the Atlantic meridional overturning circulation (AMOC)—the thermal cycle that drives currents in the Atlantic Ocean—is getting weaker [1]. Among other effects, the AMOC carries warm water to Ireland and the U.K. and returns cooler water from the north to southern regions. Instability in this circulation cycle could result in its complete collapse and cause widespread disruptions in temperature, changes in rain and snowfall patterns, and other natural disasters.

The potential loss of the AMOC represents a possible tipping point due to human-driven climate change. Global increases in temperature lead to warmer ocean water and melting polar ice, both of which decrease water density (see Figure 1, on page 1). The subsequent lower-density water does not sink as much as it cools, thus disrupting the thermal cycle. When the AMOC collapsed in the prehistoric past, it jolted Earth's climate and affected every ecosystem.

Many researchers, including Christopher K. Jones of the University of North Carolina at Chapel Hill, have searched for mathematical models to describe processes that resemble the AMOC collapse. Jones delivered an invited talk about this subject at the 2021 SIAM Conference on Applications of Dynamical Systems (DS21),[1] which took place virtually in May. "Climate change is all about the *rate* at which something is changing," Jones said. "The impacts of climate change may be triggered not just because the state of the system you're looking at is reaching a certain threshold, but because the *rate* at which the state of the system is changing is triggering the event."

In other words, the rapidity of climate change might be influencing the likelihood of AMOC collapse, severe droughts, extreme hurricane seasons, and other phenomena more than a static temperature or greenhouse-gas concentration threshold. For instance, some studies suggest that a fairly rapid infusion of cold freshwater into the ocean could have triggered a 1,000-year cold snap known as the Younger Dryas [2]. According to this theory, a slower introduction of the same amount of water would not have created such a large disruption; it was the *rapidity* of the cold water infusion that made the difference.

Jones and his graduate student Katherine Slyman are therefore examining a particular class of dynamical system models that involve rate-induced tipping (R-tipping), which differs from the better-known bifurcation-based tipping (B-tipping). In B-tipping, the dynamical system has different steady-state configurations; the one in which the system resides depends upon the value of state parameters, not the rate at which those parameters change. "There are identifiable different stable states that are quite distinct from each other," Jones said. "The transition from one to the other can be abrupt." It need not be a small perturbation either; the trigger for the transition could be large in some cases.

In her own minisymposium presentation at DS21, Slyman used the analogy of a well-known magic trick wherein a magician sets dishes on a tablecloth and quickly pulls the cloth away. If the magician is too slow in yanking the cloth, the dishes fall on the floor and break; with sufficient speed they remain on the table. The rate of change makes all the difference.

Time Is (Not) On Our Side

Sebastian Wieczorek of the University of Cork and his colleagues were the first researchers to work out much of the R-tipping formalism. They were examining the "compost bomb instability," a phenomenon that causes peatlands to spontaneously catch fire. "They're still below their ignition temperature," Jones said. "But the *rate* at which they've been warmed causes them to catch fire. So, the rate has reached a certain threshold and not the state itself."

One challenge of R-tipping compared to B-tipping is that the systems are non-autonomous, which means that the equations include explicit time dependence. A generic way to write a system of n equations with a single time-dependent parameter Λ is $\dot{x} = f(x, \Lambda(t))\, x \in \mathbb{R}^n$, $\Lambda \in \mathbb{R}$. "When you introduce rate-induced tipping, you have this parameter that now changes in time," Slyman said. "That makes this problem hairy to solve and kind of unpleasant."

The particular behavior of some non-autonomous R-tipping systems allows for simplifications that exploit the essential two-state nature of tipping points. As is common with other non-autonomous systems, one must first define a new variable $s = rt$ with a constant rate parameter $r > 0$:

$$\dot{x} = f(x, \Lambda(s))$$

$$\dot{s} = r.$$

This formulation makes the system autonomous, albeit by increasing the model's dimension from n to $n+1$. However, the choice of the function Λ also plays an important role in the model's manageability. "It's important to understand that this is not an instantaneous thing and not a very long-term thing," Jones said. "It's kind of an intermediate time thing. The rate is changing rapidly but it's not changing instantaneously. And it's not *not* changing at all; it's somewhere in the middle."

Hyperbolic tangent is a simple function that behaves in this way. It has a finite range $[-1, 1]$ for its domain and is invertible, differentiable, and ramp-like in shape:

$$\Lambda(t) = 1 + \tan h(rt).$$

The rate r controls the ramp function's steepness and the limits as t approaches $\pm\infty$ define the system's behavior before and after the transition. Researchers use a topological trick known as compactification—which attaches points at infinity onto the phase space $\{x, \Lambda\}$—to extract this asymptotic behavior [3]. Since the derivative of the ramp function is zero at $\Lambda(\pm\infty) = \lambda_{\pm}$ in the compactified space, the new task involves identifying places where $\dot{x} = 0$ as well; these spots are asymptotic fixed points (see Figure 2, on page 8).

See **Rate-Induced Tipping** *on page 8*

[1] https://www.siam.org/conferences/cm/conference/ds21

Student-Centric Graduate Training: Challenges and Opportunities

By Yara Skaf and Reinhard Laubenbacher

In part 1 of this article,[1] which appeared in the October 2021 issue of SIAM News, the authors discussed prospective changes that would make mathematics Ph.D. programs more student-centric. Here they address some of the challenges and opportunities that coincide with customized course plans.

Individuals with Ph.D.s in the mathematical sciences have a variety of career opportunities available to them that range from the traditional tenure-track path to the private sector and academic research organizations. To best prepare students for this rapidly evolving job market, we advocate for a student-centric graduate training approach that tailors program content and structure to each student's unique needs. As an intermediate step, we proposed the introduction of four different tracks with distinct roles for coursework, teaching, and research that depend on student interest [4]. In this follow-up article, we discuss several challenges and opportunities that accompany such a structure.

1 https://sinews.siam.org/Details-Page/toward-student-centric-graduate-training

How might a student-centric program look? When new students arrive, they will receive a tool kit of valuable resources and learn about different faculty research areas and possible career trajectories. An initial mentoring team will help each student develop a career and training plan during the first year of the program. The plan can introduce potential study topics through a mixture of formal coursework, directed reading, and online resources, as well as a collection of activities to guide students as they narrow down future career objectives. Students will engage in research as early as possible—much like the laboratory rotation practice in the life sciences—then transition to a thesis project and select a Ph.D. advisor during their second year.

The adoption of this structure will have a profound impact on participating departments. It will more effectively assess specific student needs and plans and allow students to truly take a proactive role in their education. A richer palette of training options will likely attract a broader and more diverse group of participants to ultimately grow and enrich the departments. Because students and faculty will presumably engage with greater intensity than in a formal classroom setting, students become readily integrated into the intellectual and social life of the department. As a result, departmental faculty will obtain a more comprehensive view of the mathematical sciences and their applications and strengthen their connections to other departments and programs within the university.

Funding

The vast majority of mathematics Ph.D. students currently receive financial support through teaching assistantships. Under our proposed structure, fewer students might choose this teaching component. Those who are interested in careers in the private sector will instead seek support from relevant internships as well as research and other grants. How will we fund all of these needs? The new program structure will likely increase the graduate student population, particularly among students who are looking for "nontraditional" careers. As a result, the portion of the student population that pursues funding from teaching assistantships may only decrease marginally, thus allowing departments to still fulfill their teaching missions. Permitting select undergraduates to teach certain courses could also help mitigate this issue.

Nevertheless, finding financial support for students is still difficult. Resources include the National Science Foundation's Graduate Research Fellowship Program[2] as well as internships in both the non-academic sector and the academic research enterprise outside of mathematics departments. Such opportunities are not uncommon, particularly in the biomedical sector.

Equity, Diversity, and Inclusion

Despite widespread public awareness of and support for issues that pertain to equity, diversity, and inclusion (EDI), this area remains a major challenge for most U.S. graduate programs in science, technology, engineering, and mathematics (STEM) fields in general — and mathematics programs in particular. An extensive body of literature aims to document the state of diversity in STEM Ph.D. tracks, explore historical events that contribute to this state, and offer suggestions on how to foster a more supportive and inclusive climate in academia [1-3]. Application rates for graduate mathematics programs comparatively tend to be much lower for historically underrepresented groups like women, people of color, and sexual orientation or gender minorities. The reasons behind this trend are multifactorial but likely related

See **Graduate Training** *on page 9*

2 https://www.nsfgrfp.org

Machine Learning

Continued from page 5

target relationship. The concept of memory can be made mathematically precise in this context; Figure 3 provides a heuristic illustration. This framework's ability goes well beyond recurrent NNs and can compare and contrast different architectures for time series modeling, such as the WaveNet.[1]

Machine Learning of Dynamical Systems

Arguably the most studied interaction between ML and DS is the notion of learning dynamics from data. With the rapid adoption of data-driven methods in both computational and experimental sciences, this subject is becoming an increasingly important area of ML application. We subsequently focus on two dominant approaches that allow one to build dynamical models from data. The first is a *statistical* approach that uses generic model hypothesis spaces—e.g., sparse regression with polynomial functions—to regard recovery of a dynamical model as a regression problem [1]. One can further impart physical structure like symmetry and invariance to these models. The second technique is a *modeling* approach, which is more familiar in the fields of science and engineering. Here we derive a model space based on physical understanding of the dynamical process; data and learning are meant to fix unknown parameters and functions in the model parameterization. The key contribution of NNs is to approximate the unknown functions — e.g., free energy, Hamiltonian, or response coefficients. Examples of this approach include inverse problems with physics-informed NNs [9] and dissipative systems via the Onsager principle [10]. Although these methods differ in principle, they all lead to interesting interactions between learning and physics. The theoretical and algorithmic intricacies of these approaches thus constitute an active area of research.

1 https://deepmind.com/research/case-studies/wavenet

Outlook

Much is still unknown about the interface of dynamics and learning in each of the three aforementioned directions. A worthwhile overall question is as follows: Why might connecting dynamics and learning be fruitful to the research in each domain? Indeed, while the dynamical viewpoint of learning provides a familiar mathematical setting, it also captures certain key novelties that pertain to modern ML. In fact, it may provide an avenue wherein researchers can concretely explore why and when deep is better than shallow when it comes to NNs, or why certain recurrent architectures are better than others. Such progress will naturally inspire the design of better models that learn dynamics from data and balance approximation flexibility with the retention of physical insights. The system will in turn benefit one's overall understanding of dynamical processes through data. Ultimately, this line of work can help facilitate the principled adoption of ML in science and engineering workflows.

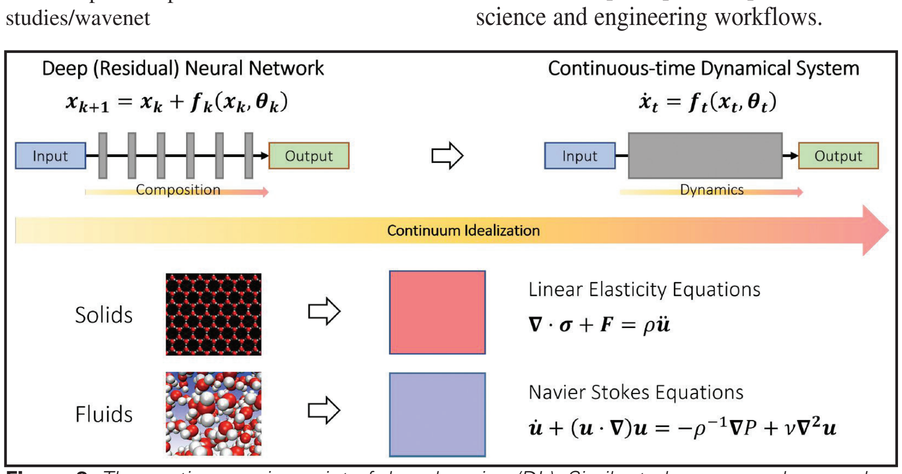

Figure 2. *The continuous viewpoint of deep learning (DL). Similar to how researchers analyze solid and fluid mechanics in the continuum limit, one can take a continuum idealization of DL that regards the layer structure as a discretization of a continuous dynamical system (DS). Here, the fictitious "time" parameter represents a continuous analogue of layers and the dynamics model layer composition. Conversely, one can regard deep residual neural networks as discretizations of a continuous-time DS. Figure courtesy of Qianxiao Li.*

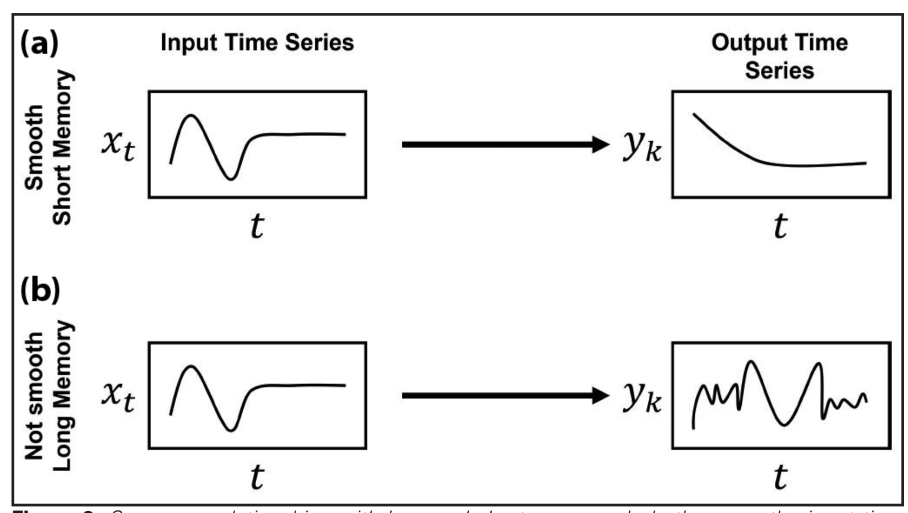

Figure 3. *Sequence relationships with long and short memory. In both cases, the input time series are identical: a smooth function of time that eventually stops varying.* **3a.** *In the smooth and short memory case, the output time series is also smooth in time and stops varying shortly after the input does. This observance means that the output does not depend on the input's values from the distant past.* **3b.** *In the not smooth and long memory case, long memory occurs as the output time series continues to vary irregularly. One can concretely define the concepts of smoothness and memory according to this heuristic illustration. Figure courtesy of Qianxiao Li.*

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Ulam's Ping-pong, Adiabatic Invariants, and Entropy

MATHEMATICAL CURIOSITIES
By Mark Levi

In the early days of quantum mechanics, physicists wondered why Planck's constant is constant — why is the energy/frequency ratio of photons fixed? After all, the atom that emits photons is buffeted by a surrounding electromagnetic field; it is surprising that this buffeting does not change the ratio.

At a Solvay Conference in 1911, Einstein showed that a slightly analogous phenomenon occurs for the mathematical pendulum. If the string's length is changed appreciably but slowly enough (taking a long time), then the ratio changes arbitrarily little. Quantities that behave in such a way are called *adiabatic invariants*.

Einstein derived the adiabatic invariance of the pendulum from the conservation of energy; *the work of pulling the string is spent on lifting the bob and on changing the bob's energy of oscillations*. With some massaging, this statement yields the conclusion that E/ω is an adiabatic invariant.

In addition to Einstein's physical explanation (which I believe can be made rigorous), another one exists that is based on the action-angle variables [1]; we can actually turn it into a geometrical proof that is almost free of formulas.

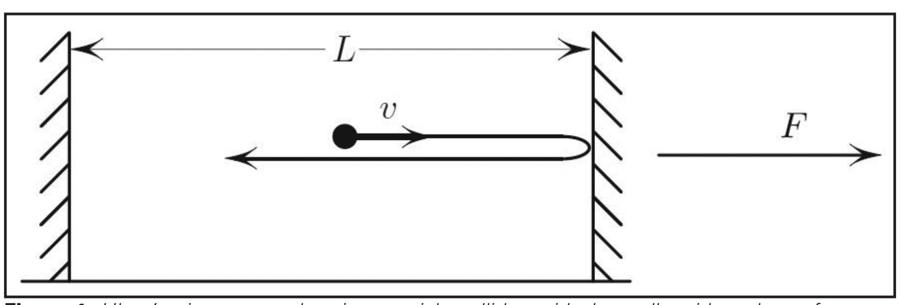

Figure 1. *Ulam's ping-pong, wherein a particle collides with the walls without loss of energy. There is no gravity. The particle exerts average force $F=mv^2/L$ on each wall: the analog of pressure in this single-atom gas.*

The coexistence of two seemingly unrelated explanations of the same effect suggests the need to rise above the maze to see the whole picture at a glance; as far as I know, this remains to be done. Instead, I would like to illustrate Einstein's idea on a simple toy model of Ulam's ping-pong: a particle bouncing between two walls — a baby model of ideal gas (see Figure 1).

Unlike in Figure 1, the wall in Figure 2 moves in slowly. The work spent on pushing the wall adds to the kinetic energy of the "molecule":

$$FdL \approx d\left(\frac{mv^2}{2}\right). \qquad (1)$$

Here, F is the averaged force of impacts on the wall in Figure 1 for the fixed wall; the "$\approx$" sign is due to the fact that I replaced the averaged force that the mover applied in Figure 2 with F — associated with the fixed wall in Figure 1. I claim that

$$F=\frac{mv^2}{L}; \qquad (2)$$

this is proven at the end.

Substitution of (2) into (1) gives

$$\left(-\frac{mv^2}{L}\right)dL \approx d(mv^2/2). \qquad (3)$$

The minus sign is due to the fact that the mover in Figure 2 pushes left, i.e., in the negative direction. Rearranging (3) yields

$$\frac{dL}{L}+\frac{dv}{v}\approx 0 \quad \text{or} \quad d\big(\underbrace{\ln vL}_{\text{entropy}}\big)\approx 0$$

so that $vL \approx$ const.

Here are some remarks:

1. vL is the area of the path in the phase plane when the walls are fixed (see Figure 3).
2. $\ln vL$, the log of the phase area, is the *entropy* of our "gas." It is additive because phase volumes multiply when we add more dimensions. Just like in this simple case, *the entropy of ideal gas is the log of the volume that is enclosed by the energy hypersurface in the phase space*. It is also an adiabatic invariant of the gas.
3. As with the pendulum, the energy/frequency interpretation for Ulam's ping-pong holds: $vL=\frac{E}{\omega}$, where $E=v^2/2$ is the energy of unit mass and $\omega=1/T=1/(2L/v)$.

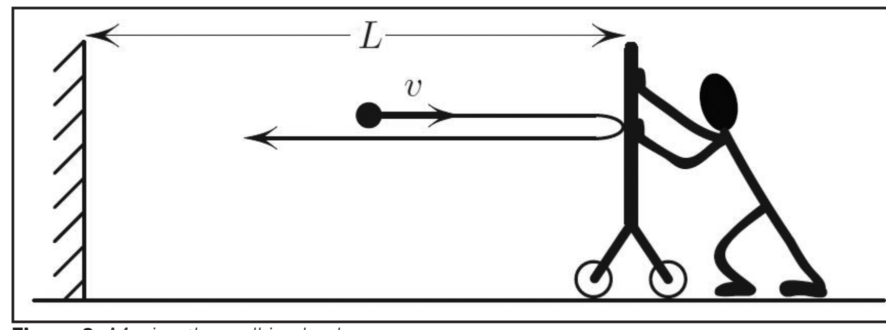

Figure 2. *Moving the wall in slowly.*

4. The adiabatic invariance of vL is the one-dimensional version of the adiabatic equation for ideal gas:

$$pV^{\gamma}=\text{const.}, \qquad (4)$$
$$\text{where} \quad \gamma=\frac{f+2}{f},$$

Here, f is the number of degrees of freedom; $f=1$ and $\gamma=3$ for our "gas." To see this connection, we note that pressure, volume, and temperature (p, V, T) in ideal gas correspond to our F, L, and $mv^2/2$ respectively. Since (4) involves pressure and volume, we also wish to express our adiabatic invariant vL in terms of "pressure" F and "volume" L. Substituting the v from (2) into vL yields

$$vL=\sqrt{FL^3/m},$$

so that $FL^3 \approx$ const. This is the exact counterpart of (4) for $f=1$.

5. pV^{γ}, or rather its power, has a geometrical interpretation as the volume enclosed by the energy surface in the phase space.

Proof of (2)

The momentum in each collision with the right wall in Figure 1 changes from mv to $-mv$, i.e., by $2mv$ at each impact. These impacts are spaced at times $T=2L/v$ apart. The change of momentum per unit time (the force) is hence

$$\frac{2mv}{T}=\frac{mv^2}{L},$$

as claimed.

Interestingly, this is exactly the same force as the centripetal force upon a mass m that moves in a circle of radius L with speed v; in fact, we can derive (2) by considering motion with speed v on a circle, but one of radius $L/2$; I omit this alternative derivation.

The figures in this article were provided by the author.

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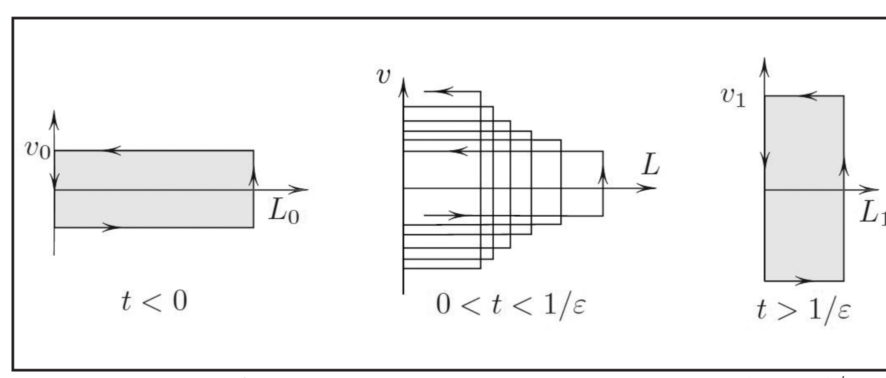

Figure 3. *Phase plane of Ulam's ping-pong. The wall moves only during time $0<t<1/\varepsilon$ and is otherwise fixed. The area changes minimally: $v_0L_0=v_1L_1+O(\varepsilon)$.*

Rate-Induced Tipping

Continued from page 6

Even with a ramp function, the system is not guaranteed to tip. In order for R-tipping to occur, the system must have qualitatively different outcomes based on r's value. The critical rate is the value of r at which there are heteroclinic orbits — trajectories that connect two saddle points in the phase space of the dynamical variable and the rate-dependent parameter. Above and below this critical value, the system variables x follow different trajectories that are unreachable from each other if r does not vary. R-tipping does transpire for many model problems, including some that correspond to real-world applications.

Bring in the Noise, Bring in the Tipping

In addition to B- and R-tipping, researchers have also investigated noise-induced tipping (N-tipping). As the name suggests, random fluctuations cause the dynamical system in these models to change state. Consider the standard Wiener process in the general case

$$dx=f(x,\Lambda)dt+\sigma dW.$$

Here, σ and W quantify the noise. Λ may be fixed for N-tipping alone or time dependent for a system with both noise and R-tipping; the latter case is especially interesting.

"What we see in the model problem is that you don't need to reach the r critical value if you put this addition of noise in the system," Slyman said. "The system can tip, and tip quite often—not even rarely—when you lower that ramp parameter and add noise to it." But if the ramp function is absent, noise alone cannot tip the model system in a reasonable amount of time. Noise and changing rates collectively produce qualitatively different behavior than either R- or N-tipping on their own.

The study of R-tipping is still a relatively new field, with most results in low-dimension systems. However, realistic climate change models—including those for AMOC—are very complex. Therefore, not all major climate-related changes are describable via R-tipping (with or without noise). For instance, Jones spoke at DS21 about a model for which R-tipping *can* describe hurricane dissipation but not hurricane genesis. Because climate change involves so many factors, it seems likely that the paradigm will prove successful for certain phenomena — particularly those driven by changes that commence more quickly than anything Earth has ever experienced.

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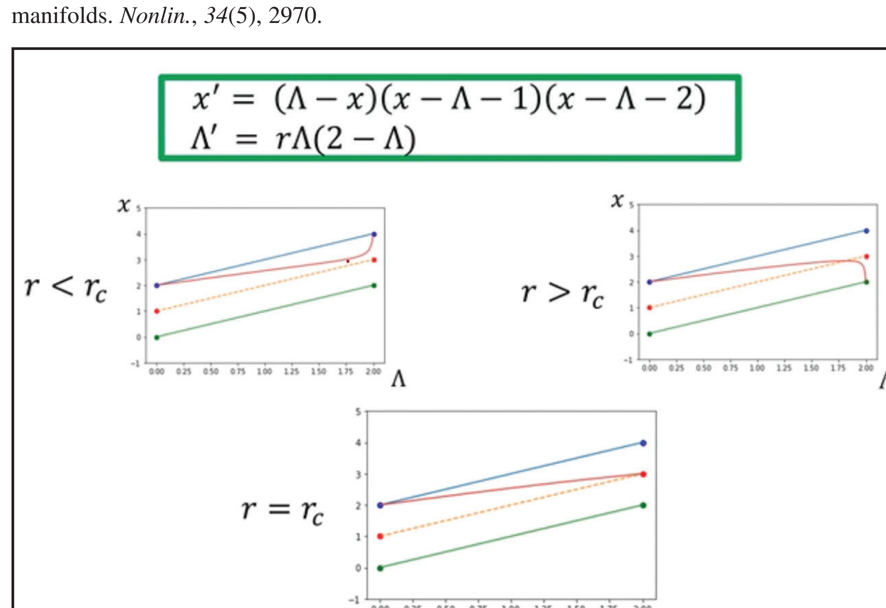

Figure 2. *A toy mathematical model that demonstrates compactification for rate-based tipping. Adjusting the rate at which the system changes leads to qualitatively different asymptotic behaviors with a critical value in between; compactification attaches those asymptotic fixed points to the phase space. Figure courtesy of [3].*

Graduate Training

Continued from page 7

to elements such as negative classroom environments, a lack of exposure to positive STEM experiences in early education, and blatant forms of discrimination. These scant application numbers give rise to low rates of matriculation and degree completion for members of underrepresented groups. For instance, women comprised only 24.4 percent of all mathematics Ph.D.s for U.S. citizens in the 2016-2017 academic year. Even more strikingly, African American students received just 2.1 percent of these Ph.D.s [1].

Our more customizable program structure could allow departments to facilitate the success of a much more heterogeneous cohort of graduate students by tailoring curricula to account for the unique needs and barriers that each individual faces. Furthermore, the process of restructuring longstanding elements of existing arrangements provides an ideal opportunity for departments to analyze other aspects of graduate training that may intentionally or unintentionally impact the educational environment for historically underrepresented or marginalized groups. Such efforts can help eventually produce a more inclusive educational environment that fosters a diverse mathematics workforce in the future.

Faculty and Mentoring

Faculty and peer mentoring contribute to a student-centric program's success. Upon entering the program, every student will ideally be assigned a mentoring committee with at least three members, including an advanced graduate student. This committee's composition may change over time as students progress. The committee members collectively assure that students' best interests remain at the center of all aspects of their personalized agendas. At least some committee members will interact frequently with the students, preferably on a weekly basis.

Peer mentoring is also crucial and will require the establishment of several new structures, which present a significant burden on the faculty. Faculty members who currently mentor Ph.D. students typically receive little to no credit for this activity, even though it is quite time consuming. Institutions should adopt different reward structures for faculty, perhaps by providing them with the equivalent of course credits or additional research funds from departmental grant overhead accounts or other sources. Some faculty may of course refrain from participating in this new structure; they can still serve as Ph.D. advisors to students who opt for the current training model.

Not all faculty are sufficiently familiar with the entire spectrum of career opportunities that are available to students. In-depth faculty training programs that address mentoring and career options beyond academia can mitigate this problem. Though such training would require a great deal of time, effort, and culture change within mathematics departments, active steps toward better student mentoring experiences will produce more qualified graduates whose future successes feed back to the program.

Concluding Thoughts

The mathematical sciences are crucial to human endeavors in all realms of society. They hold a unique place as both a universal language of science and technology and a research field in their own right; these two roles fundamentally depend on each other and have profound implications for the mathematical sciences workforce. We must implement training programs that account for mathematics' dual roles and do so in a unified and integrated environment. Our student-centric proposals are meant to address this need.

Ultimately, we have discussed several possible implementations for a student-centric graduate program. A continuum of possible program structures clearly exists between the current prevalent model and a fully customized program. Each department can develop its own resources, constraints, and strategic plans that dictate whether and what changes might be made. None of the aforementioned challenges have quick and easy solutions; the facilitation of more adaptable Ph.D. training requires substantial changes to many logistical structures, cultural attitudes, and long-standing departmental traditions. However, graduate programs must account for and adapt to evolving student interests and needs in order to remain competitive in today's fast-moving academic and professional environment. Re-imagining the traditional architecture also presents an exciting opportunity for departments to remove some of the deep-rooted impediments to EDI that Ph.D. programs in STEM fields—particularly mathematics—should address. Doing so will help create an educational environment that allows a greater number and variety of students to thrive within the mathematical community.

If readers have thoughts, questions, or suggestions about the aforementioned proposal, we encourage them to comment on the online version of this article or contact the authors directly at yara.skaf@ufl.edu and reinhard.laubenbacher@medicine.ufl.edu.

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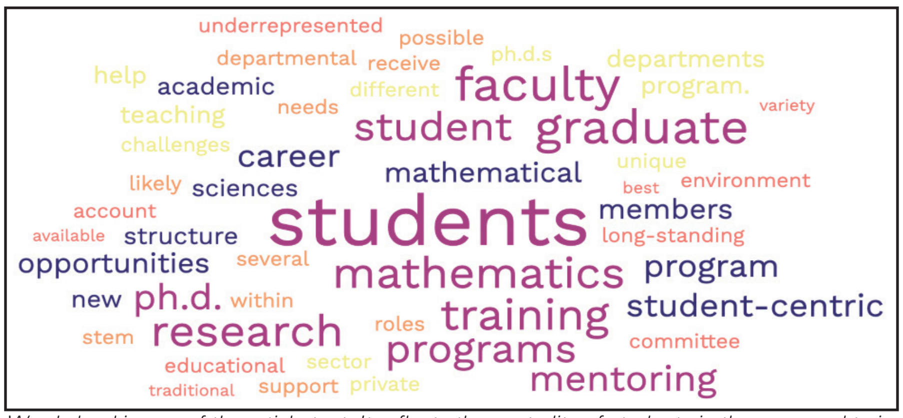

Word cloud image of the article text. It reflects the centrality of students in the proposed training paradigm as well as the paradigm's many connections within and outside of academics. Image courtesy of Free Word Cloud Generator.

Leveraging Diversity and Building Capacity for Sustained Collaboration

Mutual Learning and Immersion Activities Between the U.S., Latin America, and South Asia

By Anuj Mubayi, Aditi Ghosh, and Madhav Marathe

In 2015, the United Nations established a set of 17 global *Sustainable Development Goals* (SDGs)[1]—to be met by the year 2030—that will ultimately achieve a better and more sustainable future for everyone. However, the ongoing COVID-19 pandemic has complicated many of the goals and highlighted increasing concerns that are related to supply chain disruption, product development, environmental degradation, and inequality, among other topics. It has also amplified the importance of international research and training collaborations. Pandemics and other large systemic events—like financial contagions and the unfolding climate crisis—are universal problems that do not obey national boundaries. Solutions to such problems must therefore be global in nature and require cooperation from the global community.

Earlier this year, the U.S. Centers for Disease Control and Prevention (CDC) launched a national *Center for Forecasting and Outbreak Analytics*.[2] This center will tackle future health and social problems and include a group of multidisciplinary scientists from academia and the private sector. It will necessitate an influx of strong, well-rounded professionals in science, technology, engineering, and mathematics (STEM) who are trained in mathematics, computational science, and data science; this training will play an important role in the timely achievement of the global SDGs and inspire a workforce that addresses real situations in real time. To help future generations of STEM researchers recognize the usefulness of practical applied mathematics, we must instill cultural values, positive attitudes, and diversity acceptance among new trainees.

As the international community slowly adapts to a "new normal," the pace of global problem discussions and collaborations should accelerate to deliver much-needed change. Successful networking opportunities and partnerships allow researchers to access additional expertise, gain new perspectives, and build relationships with other individuals in their fields; the latter is crucial for early-stage career development. In short, collaborations across organizational, disciplinary, and cultural boundaries extend the possibilities of discovery [1]. Here we provide several examples of our international research collaborations in the form of capacity building, STEM education, and epidemiological modeling. These endeavors provide different types of global experiences for participants and expose STEM scholars to a more comprehensive worldview.

Collaborative Initiatives in Latin America

We have engaged in multiple research and training efforts with our counterparts in Latin American countries like Colombia, Peru, Ecuador, and Chile. Such collaborations involve students who are studying computational and mathematical methods and working on epidemic modeling. We also partnered with researchers at clinical institutions like the Instituto Nacional de Investigación en Salud Pública-INSPI in Ecuador and the Centro Nacional de Epidemiología, Prevención y Control de Enfermedades del Ministerio de Salud in Peru, where practical experiments for our research projects took place. Our group has even partook in multiple training series as speakers for tutorials on mathematical epidemiology and at the 2021 Mathematical Congress of the Americas.[3]

Moreover, we recently initiated a global consortium for the mathematical modeling community—the *Consortium of Modeling and Computations in Biosocial, Healthcare, and Sustainability Systems* (CMCBHS)—along with the Universidad de Medellín, Universidad del Valle, Universidad Nacional de Colombia (all of which are in Colombia), and Illinois State University. CMCBHS activities include multidisciplinary student conferences, research projects of global importance, and mathematical modeling training modules.

Several years ago, we participated in a unique international collaboration and research training workshop through a Partnerships for Enhanced Engagement in Research (PEER) grant[4] from the U.S. Agency for International Development to address crime and insecurity in El Salvador, which has one of the worst crime rates in the world. This fact, combined with the country's under-resourced educational foundation, inspired us to establish a joint workshop to build science and technology innovation capacity in El Salvador while exposing U.S. students to cultural values and resource limitations in a developing country. The workshop introduced participants—including university students, judicial institutions, and STEM teachers from secondary schools in El Salvador and the U.S.—to data science, statistical computing, and the STEM research process (see photo). It also helped create a crime data sciences laboratory to collect, model, and analyze real-time crime and violence data.

An international research and educational training workshop—funded by a Partnerships for Enhanced Engagement in Research (PEER) grant from the U.S. Agency for International Development—took place in El Salvador in January 2017. Anuj Mubayi (fourth from right), Oscar Picardo, and Victor Cuchillac organized the workshop activities, which addressed crime and insecurity in El Salvador. The workshop provided professionals and student mentors with a variety of high-impact teaching strategies and activities that prepared them to meet the deliverables of the PEER Project. Photo courtesy of Oscar Picardo.

Throughout the course of the aforementioned activities, we published a number of peer-reviewed articles with students and researchers from our partner institutes in Latin America. One study—which involved collaborators from the National Institute for Public Health Research of Ecuador, Universidad del Valle in Colombia, and Yachay Tech University in Ecuador—focused on transmission dynamics of leishmaniasis, a neglected tropical disease that poses a daily threat to millions of people around the world [2]. It utilized data from remote areas of Ecuador and ultimately estimated case underreporting in the country to be at least 38 percent. Unlike many mathematical studies, all coauthors were interdisciplinary and contributed to every aspect of the project: study design, data collection, lab experiments, mathematical modeling, and statistical inferences. This unique comprehensive experience improved participants' cross-cultural awareness.

Collaborative Initiatives in South Asia

We are presently working with the Royal University of Bhutan and Bhutan's Mongar Regional Referral Hospital on a PEER program[5] for research partnerships in the context of COVID-19 challenges. This project aims to positively impact Bhutan's education and research efforts while also addressing and learning from problems of global importance to train the new generation of STEM scientists. The collaboration is building a first-of-its-kind partnership between Bhutan and U.S. institutions, enhancing the investigative capacity of Bhutanese scientists, and familiarizing partners with global databases. Modeling exercises often require data-driven research from across the globe, which is crucial in today's world.

Some of our activities also involve institutions in India, such as the Indian Council of Medical Research's Rajendra Memorial Research Institute of Medical Sciences (RMRI) and the Sri Satya Satya Institute of Higher Learning (SSSIHL). For example, we partnered with RMRI to conduct research and collect community-level public health data that then informed state health policies [3]. This enterprise also introduced both U.S. and Indian participants to challenges that pertain to the

See **Leveraging Diversity** *on page 12*

[1] https://sdgs.un.org/goals
[2] https://www.cdc.gov/media/releases/2021/p0818-disease-forecasting-center.html
[3] https://www.mca2021.org/en/special-sessions/item/36-new-methods-and-emerging-applications-in-dynamics-networks-and-control
[4] https://sites.nationalacademies.org/PGA/PEER/PEERscience/PGA_174191
[5] https://sites.nationalacademies.org/PGA/PEER/PGA_365215

Data-driven Modeling of Dynamic Systems

By J. Nathan Kutz, Steven L. Brunton, Bingni W. Brunton, and Joshua L. Proctor

Dynamic Mode Decomposition: Data-driven Modeling of Complex Systems[1]—by J. Nathan Kutz, Steven L. Brunton, Bingni W. Brunton, and Joshua L. Proctor—was published by SIAM in 2016. It addresses the burgeoning field of data-driven dynamical systems and explores the dynamic mode decomposition (DMD).

DMD is a mathematical methodology that aims to distill interpretable and actionable insights from measurements of high-dimensional, complex systems. The mathematical, engineering, and scientific research communities have demonstrated the method's broad applicability to applications in areas such as control theory, computer science, and fluid dynamics. They have also identified a fundamental theoretical connection to the analysis of nonlinear dynamical systems. Perhaps the most exciting recent development, however, is the translation of this method from academic research to the lexicon of standard techniques for data scientists and machine learning practitioners in industry settings. DMD has proven to be a surprisingly efficient and simple yet powerful computational method that fills an important technical gap in the analysis of high-dimensional measurement data from *dynamically* evolving systems. Our book describes the theoretical foundations of DMD and demonstrates the technique on a variety of application areas.

The following text comes from chapter one of *Dynamic Mode Decomposition*, entitled "Dynamic Mode Decomposition: An Introduction," and has been modified slightly for clarity.

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The data-driven modeling and control of complex systems is a rapidly evolving field with great potential to transform the engineering, biological, and physical sciences. There is unprecedented availability of high-fidelity measurements from historical records, numerical simulations, and experimental data; but while data is abundant, models often remain elusive. Modern systems of interest—such as turbulent fluids, epidemiological systems, networks of neurons, financial markets, or the climate—may be characterized as high-dimensional, nonlinear dynamical systems that exhibit rich multiscale phenomena in both space and time. However complex, many of these systems evolve on a low-dimensional attractor that one may characterize by spatiotemporal coherent structures. Here we introduce the topic of this book—dynamic mode decomposition (DMD)—which is a powerful new technique for the discovery of dynamical systems from high-dimensional data.

FROM THE SIAM BOOKSHELF

The DMD method originated in the fluid dynamics community as a method to decompose complex flows into a simple representation based on spatiotemporal coherent structures. Peter Schmid and Jörn Sesterhenn [6, 7] first defined the DMD algorithm and demonstrated its ability to provide insights from high-dimensional fluids data. The growing success of DMD stems from the fact that it is an equation-free, data-driven method capable of providing an accurate decomposition of a complex system into spatiotemporal coherent structures, which one may use for short-time future-state prediction and control. More broadly, DMD has quickly gained popularity since several studies [2-5] showed that it is connected to the underlying nonlinear dynamics through Koopman operator theory [1] and is readily interpretable with standard dynamical systems techniques.

The development of DMD is timely due to the concurrent rise of data science, which encompasses a broad range of techniques from machine learning and statistical regression to computer vision and compressed sensing. Improved algorithms, abundant data, vastly expanded computational resources, and interconnectedness of data streams make this a fertile ground for rapid development.

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Enjoy this passage? Visit the SIAM Bookstore[2] *to learn more about Dynamic Mode Decomposition: Data-driven Modeling of Complex Systems*[3] *and browse other SIAM titles.*

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[1] https://my.siam.org/Store/Product/viewproduct/?ProductId=28216652

[2] https://my.siam.org/Store

[3] https://my.siam.org/Store/Product/viewproduct/?ProductId=28216652

Recapping the Inaugural 2021 SIAM Conference on Applied and Computational Discrete Algorithms

By Blair D. Sullivan and Bruce Hendrickson

Combinatorial algorithms are central to many complex problems that arise in the fields of applied and industrial mathematics. Researchers utilize them to model epidemics, optimize supply chains, understand biological systems, enhance computer performance, and make sense of communities, among other applications. Professionals in all of these areas have long been part of SIAM, but until recently they did not have a common home in which to share insights and facilitate collaborations.

To meet this need, the inaugural SIAM Conference on Applied and Computational Discrete Algorithms[1] (ACDA21) took place virtually in July, in conjunction with the 2021 SIAM Annual Meeting.[2] An outgrowth of the recently-founded SIAM Activity Group on ACDA,[3] ACDA21 sought to create a home for researchers who are engaged in the advancement of applied combinatorics across a diverse set of research communities. ACDA21 attendees and presenters showcased the breadth of related fields by exhibiting their research and expertise in discrete mathematics, theoretical computer science, algorithm engineering, operations research, computational biology, combinatorial scientific computing, and a variety of other modeling-based application areas. One of the Organizing Committee's primary challenges—other than the ongoing COVID-19 pandemic and necessity of a virtual platform—was designing an event that successfully navigated the disparate norms and expectations for conferences and publications across these many disciplines.

To that end, ACDA21 invited submissions in two formats: 10-page papers for competitive archival proceedings (similar to conferences in computer science and data mining) and two-page extended abstracts for presentation purposes only (which offered an easy way for journal-oriented researchers to participate). Both options were popular, and submissions comprised 56 proceedings papers and 22 extended abstracts. The Program Committee—which was chaired by Michael Bender and John Gilbert and included senior researchers from a wide variety of ACDA-related fields—assembled a program that highlighted exceptional applied combinatorics across all relevant disciplines. The committee accepted 21 papers and 11 abstracts, resulting in an acceptance rate of 37.5 percent for the proceedings.

To showcase the rich set of mathematical ideas and broad applications that are associated with discrete algorithms, the Organizing Committee invited six plenary speakers whose work spans the spectrum from foundational algorithm design to advances that are driven by application-related considerations. A particularly timely highlight was Madhav Marathe's (University of Virgina) talk about challenges in computational epidemiology, including several that arose during the COVID-19 pandemic. Andrew Goldberg (Amazon) delivered a presentation that addressed shortest path algorithms for road navigation, and a complementary Industrial Problems Session featured four researchers from industry: Michael Frumkin (NVIDIA), Edward Rothberg (Gurobi Optimization), Rob Johnson (VMware Research), and Vahab Mirrokni (Google, Inc.). These speakers described combinatorial problems that arise in their respective lines of work. ACDA21 also hosted two minitutorials—"An Introduction to Combinatorial Scientific Computing" and "Combinatorial Frontiers in Computational Biology"—to provide entry points into active research areas and encourage new collaborations.

While building community in a virtual setting is certainly challenging, several noteworthy design choices and elements improved attendee experience and overall engagement. For example, the organizers opted for a non-traditional presentation format that proved to effectively encourage discussion and interaction. The authors of each accepted paper or abstract recorded a 20-minute technical talk prior to ACDA21 that was available for asynchronous viewing before, during, or after the meeting. During the conference, presentations were grouped into thematically related sets of three or four short, live "lightning" talks; a collective discussion and question-and-answer session followed these talks. We believe that this format led to richer conversations and more dynamic experiences than traditional 15- or 20-minute talks with independent question periods, especially in the virtual setting. An ACDA21 Engagement Committee facilitated several additional activities, such as ice-breaker questions at the beginning of each session to encourage casual conversation, regular use of the ACDA Gather.town platform (including the koi pond and virtual coffee), and an "Introduction Blitz" session with slides and brief remarks from over 50 participants.

With the first meeting successfully behind us, the ACDA community is already planning the conference's next instantiation, which will take place in 2023 and be chaired by Uwe Naumann (RWTH Aachen University) and Lenore Cowen (Tufts University). We look forward to gathering in person for ACDA23 and continuing to build and promote this vibrant new community within the broader SIAM family.

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1 https://www.siam.org/conferences/cm/conference/acda21
2 https://www.siam.org/conferences/cm/conference/an21
3 https://www.siam.org/membership/activity-groups/detail/applied-and-computational-discrete-algorithms

Leveraging Diversity

Continued from page 10

collection of community information in real time under resource-limited conditions and even resulted in a Ph.D. dissertation [4]. In regards to the SSSIHL, research and training activities that are sponsored by the National Science Foundation (NSF)[6] are actively assembling a network of mathematicians that will benefit researchers and educators in both India and the U.S.

Global Initiatives in the U.S.

We are regularly carrying out multiple activities in the U.S. through Illinois State University's Intercollegiate Biomathematics Alliance,[7] a consortium that promotes research and education in biomathematics and unites global organizations. For example, we invited participants from our international collaborator institutions to partake in the annual Cross-Institutional Undergraduate Research Experience[8] program, International Symposium on Biomathematics and Ecology Education and Research,[9] and various webinars.[10] These multicultural and multinational programs have truly cultivated a community of mathematicians whose skills range from data science to survey data collection.

We are also currently building a unique multidisciplinary and cutting-edge science community via the Pandemic Research for Preparedness and Resilience[11] (PREPARE) initiative to help society address challenges that are related to global pandemics. Such challenges include virtual education, the spread of misinformation through social media, and efficient vaccination distribution. PREPARE aims to construct a roadmap of important research directions and breakthrough solutions for pandemic preparedness and resilience that will eventually provide a blueprint for researchers, funding agencies, and policymakers. NSF's Expeditions in Computing program[12] is creating another ambitious global initiative—Global Pervasive Computational Epidemiology[13]—to pursue fundamental research and training agendas that will define the future of computational epidemiology and infectious diseases. This program employs artificial intelligence and machine learning techniques to equip the next generation of public health champions with a variety of skills to tackle epidemics and forecast future disease burden in real time.

Other mathematics-based international programs in the U.S. have addressed global aspects (see Figure 1 for some examples). Moreover, the National Institutes of Health's Fogarty International Center, the Department of Defense, and the CDC have also undertaken a number of endeavors, though many of them focus primarily on clinical and public health fields. In contrast, most mathematics-based efforts have facilitated database sharing, distance education, and computer-mediated communication in order to access a large and diverse amount of data and conduct collaborative research.

Next, we plan to introduce carefully selected integrated tools that allow researchers to develop, access, and use models and data. Communities of mathematicians and interdisciplinary scientists—as well as ongoing regular communication channels via online platforms and hands-on field training—will encourage these actions. We therefore hope to create a tailored program that will boost cultural competence and awareness, move students toward higher levels of achievement and self-confidence, and ultimately increase representation in STEM fields.

Given the rapidly changing nature of global health knowledge, we must bring together and train the next generation of data scientists, expert disease modelers, public health emergency responders, and high-quality communicators to meet the needs of modern-day decision-makers. The aforementioned activities and programs have accelerated access to and use of data for public health officials who require local-to-global information to mitigate the social and economic effects of disease threats. Society will not be able to promptly and effectively face subsequent challenges until STEM researchers can proficiently model and forecast public health; address future ecological concerns; and share information in real time to activate governmental, private sector, and public actions in anticipation of both domestic and international threats. David Hilbert, one of the most influential mathematicians of the 19th and 20th centuries, once aptly said that *"Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country."* With this sentiment in mind, we recognize that the learning process can be tremendously efficient if we make it more active, engaging, and globally diverse.

Mathematical Program	Global Activities	Novelty
NSF-IRES: Population Dynamics and Complex Systems	Utilizing International Research Experiences for Students (IRES) to allow U.S. students to perform activities at Universidad de Los Andes in Bogotá, Colombia	Cultural immersion and ecological experiments in Colombia
DIMACS/MBI: US - African Biomathematics Initiative	Addressing challenges in biomathematical research and training in Africa through partnerships	Multiple joint workshops in the mathematics of conservation biology and ecology
NSF: Pandemic Research for Preparedness & Resilience (PREPARE)	Building a multidisciplinary community of global researchers to develop breakthrough solutions for pandemic preparedness and resilience	Use of COVID-19 to build a roadmap as a blueprint for researchers, funding agencies, and policymakers
NSF: Viral Infection Propagation Through Air-Travel (VIPRA)	Analyzing new strategies that reduce the risk of infection during global air travel by integrating global data sources with large computational infrastructure	The building and sharing of data sources and modeling frameworks

Figure 1. *Examples of several mathematics-based international programs in the U.S. that have different types of global compontents.*

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