Math Matters

Apply It.

The math behind... Predicting Traffic

Technical terms used:

Ordinary differential equation, partial differential equation, delay differential equations, cellular automata, Monte Carlo simulation, bifurcation analysis

Uses and applications:

Accurate traffic models can be used to pinpoint the causes of traffic and predict traffic levels. These models can be helpful for designing better road and highway systems and finding optimal driving routes from a starting location to a destination based on evolving traffic conditions.

How it works:

While traffic is a daily phenomenon, the causes of traffic are not always well understood. A variety of factors can influence the development of a traffic jam, such as the timing of traffic signals, the volume of cars on the road, the behavior of drivers, and the topology of the road network. Researchers who model traffic try to understand the causes of traffic jams by creating a model for a traffic situation and then observing how the predicted traffic levels vary as parameters in the model are adjusted.

One method of modeling traffic is to use equations developed in the study of fluid dynamics to model the traffic as a flow. These methods use partial differential equations to solve for the density of vehicles as functions of location and time. Traffic can also be modeled by means of cellular automata, as in the Nagel-Schreckenberg model, where the roadway is modeled as an array of cells that cars can occupy and the rules of the model dictate how cars change location and speed over time. Since these models usually involve elements of randomness, Monte Carlo simulations are used to solve for the traffic behavior over many trials. Alternatively, ordinary differential equations can be employed to model how the position of a single vehicle changes continuously in time based on the speed limit and the spacing between neighboring vehicles. These equations describe a dynamical system whose steady-state solutions can be analyzed using a combination of numerical simulation and analytical techniques, such as bifurcation analysis. Lastly, delay differential equations can also be used, where the reaction time of a driver is incorporated into the differential equation model.

Interesting fact:

Mathematical modeling has shown how a combination of congestion and the behavior of an individual driver can artificially create a traffic jam. If traffic is flowing smoothly on a roadway with a high volume of drivers and a single distracted driver aggressively brakes to slow down, then sudden braking can propagate backwards as a wave, bringing traffic to a standstill on the roadway several miles behind.

References:

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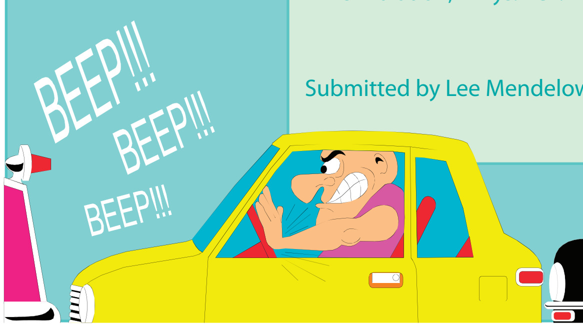

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