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(NWCS26)**

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3600 Market Street, 6th Floor

Philadelphia, PA 19104-2688 U.S.

Telephone: 800-447-7426 (U.S. & Canada) +1-215-382-9800 (Worldwide)

meetings@siam.org

IP1**Welcome Remarks/Surface Wave Effects on the Transport of Finite-sized Particles**

The ocean contains a wide range of particulate matter, including sediment, bubbles, microplastics, and plankton, whose transport is influenced by surface gravity waves. Due to their finite size, these particles are not constrained to act as fluid tracers. In this talk, I present work examining how waves can modify the transport of non-fluid particles. First, I show how waves can enhance non-neutrally buoyant particles vertical gravitational drift via a preferential sampling of the wave orbital motion. This effect arises at second order in wave steepness and is related to the waves Stokes drift. In addition, I present work on how waves can modify diffusive transport, effectively distorting the wave-averaged diffusivity. The results are illustrated through a combination of laboratory experiments, numerical simulations, and theoretical models, and highlight how surface waves can fundamentally alter particle transport in the ocean.

Michelle H. Dibenedetto
Princeton University
mdiben@princeton.edu

IP2**Long-time Dynamics and Stability of Coherent Structures in Dispersive Boussinesq Systems**

Dispersive Boussinesq systems arise in the modeling of free-surface inviscid fluids under the long-wave and shallow-water approximations. These equations have attracted considerable attention over the past decades, due to both their analytical richness and their relevance in describing realistic wave phenomena. In this talk, I will present recent advances in the understanding of their long-time dynamics, with a particular focus on nonlinear coherent structures such as solitary waves and solitons. Emphasis will be placed on recent results concerning the rigorous asymptotic stability of solitary waves, the analysis of their interactions and collisions in Boussinesq systems with variable bottom topography, and new developments combining rigorous analysis with deep-learning-based numerical approximations. The goal will be to provide a unified picture of how dispersion, nonlinearity, and variable media shape the persistence and evolution of coherent wave patterns in Boussinesq models.

Claudio Munoz
University Paris Sud-Orsay
cmunoz@dim.uchile.cl

IP3**Localised Multi-Dimensional Patterns**

I will present an overview of some of the latest work on developing a mathematical theory for the formation of spatially localised multi-dimensional patterns that occur in fluid mechanics (e.g. convectons in doubly-diffusive convection, sun hotspots, and spikes on the surface of a ferrofluid) to biology (fairy circle vegetation patches). The talk will be pedagogic in nature, taking the audience through some of the recent new ideas in the area and open problems that remain. Work in collaboration with Drs. Reinhard Richter (Bayreuth), Daniel Hill (Oxford), Jason Bramburger (Concordia), and Matthew Turner (Surrey). Reference: 1. J Bramburger, DJ Hill, DJB Lloyd, Localized Patterns, (2025), SIAM Review ac-

cepted. <https://arxiv.org/abs/2404.14987>

David Lloyd
University of Surrey
d.j.lloyd@surrey.ac.uk

IP4**Resonances as a Computational Tool**

A large toolbox of numerical schemes for dispersive equations has been established, based on different discretization techniques such as discretizing the variation-of-constants formula (e.g., exponential integrators) or splitting the full equation into a series of simpler subproblems (e.g., splitting methods). In many situations these classical schemes allow a precise and efficient approximation. This, however, drastically changes whenever non-smooth phenomena enter the scene such as for problems at low regularity and high oscillations. Classical schemes fail to capture the oscillatory nature of the solution, and this may lead to severe instabilities and loss of convergence. In this talk I present a new class of resonance based schemes. The key idea in the construction of the new schemes is to tackle and deeply embed the underlying nonlinear structure of resonances into the numerical discretization. As in the continuous case, these terms are central to structure preservation and offer the new schemes strong geometric properties at low regularity.

Katharina Schratz

University of Innsbruck
Department of Mathematics
katharina.schratz@sorbonne-universite.fr

IP5**Surfing Langmuir Waves**

In the Vlasov theory of excited electrons, Langmuir's waves or plasma oscillations arise due to the long-range interaction between electrons, leading to the oscillatory behavior of the electric field like a Klein-Gordon's dispersive wave. The classical Landau damping concerns decay of such oscillations, exponentially rapid for short-wave perturbations, however extremely slow or not available in the long-wave regime. This presentation shall provide an overview, plus recent advances, on this subject.

Toan Nguyen
Pennsylvania State University
nguyen@math.psu.edu

IP6**Instability of Waves Inside the Ocean**

The density of the ocean increases with depth due to changing temperature and salinity. Consequently, the ocean is said to be stratified, meaning that motion is predominantly horizontal, not vertical. However, the stratification also supports internal waves, which move due to buoyancy forces. These waves can become unstable through various mechanisms, one being triadic resonant instability (TRI), a generalization of parametric subharmonic instability. Through TRI a pair of sibling waves can resonantly interact with the parent wave allowing them to grow from small-scale noise. This mechanism has been well studied in the case of uniform stratification, for which the background density increases linearly with depth. But only recently has theory been developed for TRI in non-uniform stratification, which is more representative of the ocean. This

talk will describe a theory for the growth of sibling waves in near resonance due to a frequency mismatch between the forced frequency and natural frequency of the waves. The predictions are tested against fully nonlinear numerical simulations and laboratory experiments. All three show that near-resonance can occur if the parent wave frequency is sufficiently large. But negligible growth occurs if the parent has frequency representative of oceanic internal tides. The results thus bring into question whether TRI of internal tides occurs in reality, though limitations and advances of the theory will also be discussed.

Bruce Sutherland
University of Alberta
bsuther@ualberta.ca

SP1

T. Brooke Benjamin Prize Presentation: Soliton-Resolution and Long-Time Asymptotic Results for Integrable Dispersive Nonlinear Pdes

The soliton resolution conjecture informally states that for any dispersive nonlinear PDE which supports soliton solutions, the solution of the Cauchy problem given generic initial data will resolve at large times into a collection of solitons plus a decaying radiative component. Proofs of these types of results for particular PDEs and classes of initial data have been given by luminaries of the mathematics community. In this talk, I will describe how for integrable PDEs these questions can be answered in great detail using steepest descent methods in the long time limit.

Robert Jenkins
University of Central Florida
Robert.Jenkins@ucf.edu

CP1

Local Well-posedness and Boundary Behavior of Nonlinear Dirac Equation on the Half-line

In this talk, I will present recent results on the one-dimensional Dirac equation on the half-line with Dirichlet boundary conditions. Using the unified transform method, we obtain explicit solution representations for the linear inhomogeneous problem and examine boundary behavior, identifying the compatibility conditions required for smooth extensions to the spacetime boundary. We also establish Sobolev-space energy estimates for the solutions, including control of both spatial norms and time-regularity. These estimates provide the analytic framework needed to treat the nonlinear Dirac system with Thirring-type nonlinearity. Applying a contraction mapping argument in the appropriate function spaces yields local well-posedness for the nonlinear initial-boundary value problem.

Hassan Babaei, Jerry Bona
University of Illinois at Chicago
hbabae2@uic.edu, jbona@uic.edu

CP1

Evolution Pde on Semi-Infinite Intervals Revisited Via the Fokas Method: Non-Uniqueness, Long-Range Instabilities, and Long-Time Blow-Up Effects

In this talk we survey some of our recent findings concerning the rigorous analysis of fully non-homogeneous initial-boundary-value problems (IBVP) for several important evolution partial differential equations (PDE) formu-

lated in a spatiotemporal quarter-plane. Such PDE emerge in mathematical physics and the applied sciences as models of natural processes pertaining to water waves, continuum mechanics, heat-mass transfer, solid-fluid dynamics, electron physics, petroleum engineering, nanotechnology, etc. Our work is based on the synergy between: (i) the complex-analytic unified transform method (UTM) of Fokas, and (ii) a recent approach, developed by the speaker and a global network of many collaborators, aiming for qualitative analysis of solutions. New phenomena, uncovered during our investigations, include: break-down and long-time blow-up effects (see e.g. [J.L. Bona, A. Chatziafratis, H. Chen, S. Kamvissis, The linear BBM-equation on the half-line revisited, *Lett. Math. Phys.*, 2024]), long-range instabilities (e.g. [A. Chatziafratis, T. Ozawa, S.-F. Tian, Rigorous analysis of the unified transform method and long-range instabilities for the inhomogeneous time-dependent Schrödinger equation on the quarter-plane, *Math. Annalen*, 2024]), and solution non-uniqueness (e.g. [A. Chatziafratis, S. Kamvissis, Infinity of solutions to initial-boundary value problems for linear constant-coefficient evolution PDEs on semi-infinite intervals, *Bull. London Math. Soc.*, 2025]).

Andreas T. Chatziafratis
National and Kapodistrian University of Athens
chatziafrati@math.uoa.gr

CP1

On Ill-Posedness for Dispersion-Managed Nonlinear Schrödinger Equations

The dispersion-managed nonlinear Schrödinger equation models the propagation of pulses through long-haul optical fibers, where the dispersion profile and hence the focusing or defocusing nature varies periodically. When the dispersion oscillates rapidly, this leads to the Gbitov-Turitsyn equation, a nonlocal nonlinear Schrödinger equation obtained by evolving the nonlinearity under the linear flow and averaging in time. Despite substantial attention in physics and numerics, rigorous results for this model are rare and the sharp well-posedness theory has remained largely unclear. In this talk, we present recent results that identify a threshold for well-posedness, below which contraction mapping arguments fail, and a second, distinct threshold below which norm inflation can be shown under suitable restrictions.

Matthew Kowalski
University of California, Los Angeles
mattkowalski@math.ucla.edu

CP1

Small-Time Asymptotics for the KdV Equation

Solutions of the KdV equation on the real line exhibit dispersive waves that travel in the negative x direction. For initial conditions that are analytic functions of x , the amplitude of these waves is exponentially small as $t \rightarrow 0^+$, and therefore the wave cannot be captured by a naive algebraic expansion in powers of t . Instead, these dispersive waves can be described by considering exponentially small corrections to the algebraic series and employing Stokes phenomenon in complex x plane, as shall be explained.

Scott McCue
School of Mathematical Sciences
Queensland University of Technology

scott.mccue@qut.edu.au

andreas.schiffer@ku.ac.ae

CP2

Cauchy Problems for Novikov-Type Equations with Quadratic Nonlinearities

The partial differential equations in the following form has attracted attention in partial differential equations theory:

$$(1 - D_x^2)u_t = F(u, u_x, u_{xx}, \dots). \quad (1)$$

The equations which have quadratic nonlinearities and proved to be integrable will appear in this talk. The most prominent examples are Camassa-Holm and Degasperis-Procesi equations. Novikov [3] proved that there are more integrable equations in the class (1). Initial value problems corresponding to these equations either on the line or on the circle will be presented and in particular the periodic problem for

$$(1 - D_x^2)u_t = D_x(2 - D_x)(1 + D_x)u^2 \quad (2)$$

will be discussed. The recent results obtained both for local well-posedness and global well-posedness (2) will be provided followed by blow-up criterion [1,2]. References: [1] N. Duruk Mutlubas and I. L. Freire, Existence and uniqueness of periodic pseudospherical surfaces emanating from Cauchy problems, Proc. R. Soc. A., vol. 480, paper 20230670, (2024). [2] N. Duruk Mutlubas and I. L. Freire, Global solutions for a non-local integrable equation with applications to geometry, <https://doi.org/10.48550/arXiv.2505.12232>, (2025). [3] V. Novikov, Generalizations of the Camassa-Holm equation, J. Phys. A: Math. Theor., vol. 42, paper 342002, (2009).

Nilay Duruk Mutlubas
Sabanci University
nilay.duruk@sabanciuniv.edu

CP2

Periodic Shooting of Traveling Waves in the Sine-Gordon Equation with Step-Function

In this talk, we examine the sine-Gordon equation modified by a step-function potential within the sine term, focusing on the resulting periodic shooting of traveling waves. We begin by reviewing the spatial solution structure relevant to this setup. We then present numerical simulations demonstrating the periodic emission of traveling waves and describe the procedure used to compute the corresponding shooting period. These numerical findings are compared with an approximate prediction obtained using a perturbative approach proposed initially by Aranson et al. (1999). Both the numerical and approximate results capture the overall trend of the shooting period, with the approximation showing a consistent undershoot. This study provides further insight into wave-generation mechanisms and may help illuminate related phenomena such as supratransmission.

Muhammmad Ismail Yunus
Khalifa University of Science and Technology
100063760@ku.ac.ae

Hadi Susanto
Khalifa University
hadi.susanto@ku.ac.ae

Andreas Schiffer
Khalifa University of Science and Technology

CP3

Exponential Asymptotics for Discrete Quantum Droplets and Bubbles

In this talk, I will present the formation and stability of localized states, known as quantum droplets and bubbles, in the quadratic-cubic discrete nonlinear Schrödinger equation. Around a Maxwell point, these states emerge from two fronts connecting the bistable uniform equilibria. By varying a control parameter, a "pinning region" exists around the Maxwell point, whereby multiple localized states coexist and are interconnected through homoclinic snaking. Using exponential asymptotics, the width of the pinning region and its dependence on the coupling strength are determined, revealing an exponentially small relationship between them as the coupling strength increases. Additionally, the stability of these localized states is determined via an eigenvalue counting argument and analysis of the critical eigenvalue of the corresponding linearized operators of the fronts that form them. The analysis shows that onsite fronts unstable and intersite fronts stable. The theoretical results are validated through numerical simulations, which show excellent agreement with our analytical predictions.

Farrell T. Adriano, Hadi Susanto, Tae Yeon Kim
Khalifa University
100065157@ku.ac.ae, hadi.susanto@ku.ac.ae,
taeyeon.kim@ku.ac.ae

CP3

Supratransmission in Lattices with Purely Nonlinear Coupling

Supratransmission, the nonlinear transmission of energy beyond forbidden frequency ranges, has traditionally been studied in systems with a linear passband. In this talk, we explore a new frontier: supratransmission in lattices without any linear spectrum, where energy transport arises purely from nonlinear interactions. Starting from the governing dynamics, we employ asymptotic analysis to derive a Discrete p-Schrödinger (DpS) equation that captures the essential behavior of standing and traveling waves. Analytical results reveal how the critical driving amplitude depends on the excitation frequency and the nonlinearity power p , approaching a constant for large p . Numerical simulations confirm these findings in the weak coupling regime and identify where the DpS approximation breaks down under stronger coupling. Bifurcation analysis reveals a saddle-node structure marking the onset of supratransmission, while the nonlinear coupling itself enhances energy transport and produces rich, irregular propagation patterns that are absent in linearly coupled lattices. Together, these results demonstrate, for the first time, the existence and mechanisms of supratransmission in systems without a linear spectrum, thereby extending the phenomenon beyond the realm of dispersive media.

Defri Ahmad
Khalifa University of Science and Technology
100064480@ku.ac.ae

Hadi Susanto, Tae Yeon Kim
Khalifa University
hadi.susanto@ku.ac.ae, taeyeon.kim@ku.ac.ae

Andreas Schiffer

Khalifa University of Science and Technology
andreas.schiffer@ku.ac.ae

Jinkyu Yang
Seoul National University
jkyang11@snu.ac.kr

CP3

Stability and Dynamics of Localized Patterns in the FitzHugh-Nagumo Model

The FitzHugh-Nagumo model, originally introduced to study neural dynamics, has since found applications across a variety of fields which include biology and even cardiology. Although this model has been broadly studied theoretically, the origin and bifurcation structure of spatially localized patterns remains underexplored. In this work, we present a detailed bifurcation analysis of such localized structures in one spatial dimension in the FitzHugh-Nagumo model. We show that these localized states undergo a smooth transition between standard and collapsed homoclinic snaking as the system shifts from pattern-uniform to uniform-uniform bistability. This transition occurs due to the simultaneous coexistence of uniform-pattern-uniform states within, what we call, a tristable regime. Moreover, we explore the oscillatory dynamics exhibited by these states when varying the time-scale separation and diffusion coefficient. This investigation reveals the emergence of Turing-Hopf localized patterns, where the background of the states oscillates, and the presence of breathers. Through a combination of analytical and numerical techniques, our study uncovers the stability and dynamic regimes of spatially localized structures, offering new insights into the mechanisms controlling spatial localization in this system.

Pedro Parra Rivas
Universidad de Almería
University of Almería
pedro.parra-rivas@ual.es

CP4

Precipitation in Nonlinear Mountain Waves with Temperature-Dependent Enthalpy

Mountains play a crucial role in shaping regional climates. For example, precipitation may occur when a moist air mass is forced over a mountain range, where the associated cooling as it rises results in the air becoming saturated. In this talk, we will consider an exact solution to the nonlinear governing equations for mountain waves, in the Lagrangian framework. The waves propagate in a moist atmosphere, and we incorporate a temperature dependence in the enthalpy of condensation of water vapour. We show that if a streamline in the atmospheric flow at a given temperature is saturated, then the explicit specification of the dynamics of the flow is sufficient to deduce temperature and vapour pressure profiles for any other saturated streamline in the flow. We further deduce a restriction on the domain for which precipitation is admissible, enforced by the second-law of thermodynamics. Based on joint work with Dr. Tony Lyons (South East Technological University, Ireland), *J. Math. Fluid Mech.* **27**, 41 (2025).

Jordan Mccarney
University College Cork, Ireland
jordanmccarney@umail.ucc.ie

Tony Lyons

South East Technological University, Ireland
tony.lyons@setu.ie

CP4

Near-Inertial Pollard Waves Modeling the Arctic Halocline

The presence of the halocline, i.e. a stratum of fresh and cold water, between the warm and salty water at the bottom and the fresh and cold water at the top, is an essential feature of the Arctic Ocean. One of its primary role is to work as a shield that prevent the warm water from melting the ice cover of the Arctic, from below. As a consequence, it is very important for the World's climate. In this lecture we will present an explicit and exact solution to the governing equations (in the f -plane approximation) describing the vertical structure of the Arctic Ocean region centered around the North Pole. The solution describes a stratified water column with three constant-density regions: a motionless bottom layer, a middle layer - the halocline - described by nonhydrostatic, near-inertial Pollard waves, and an upper layer presenting a mean current and a wave motion associated with the one in the halocline layer. The dispersion relation relates the characteristic of the water column with the wave-speed. The analysis also highlights the importance of the nonlinear terms in the governing equations by proving that the linearised problem is not suitable for the model under consideration. Moreover, the nonlinear solution obtained is compatible with some characteristics of the halocline observed from oceanographic measurements.

Christian Puntini
University of Vienna
christian.puntini@univie.ac.at

CP4

On Large-Scale Oceanic Wind-Drift Currents

Wind-generated currents play a crucial role in the near-surface oceanic circulation. They are characterised by a three-way balance between the horizontal pressure gradient, the Coriolis force, and the horizontal stress that arises through turbulent motion due to the overlying wind. Historically, the first model that successfully captured the main qualitative properties of wind-drift currents was proposed in 1905 by V. W. Ekman, as an attempt to explain seemingly counter-intuitive observations made by Fridtjof Nansen during his celebrated *Fram* expedition of 1893–1896. In this talk, I will first provide an overview of the physical principles driving wind-drift currents and discuss the key aspects of the derivation of a more general and consistent model for these flows, beyond Ekman's classical model. Then, I will discuss some results about the properties of solutions of this model and give a brief outlook on possible further directions of research. This is joint work with Christian Puntini (Universitt Wien).

Luigi Roberti
Universität Wien
roberti@ifam.uni-hannover.de

Christian Puntini
University of Vienna

christian.puntini@univie.ac.at

CP4

Mountain Waves: Linear Theory

In this talk, I will discuss the linear theory of mountain waves in two dimensions. After an introduction to the physical relevance and the typically observed wave patterns, we turn to the underlying boundary value problem for the compressible Euler equations coupled to the ideal gas law and the first law of thermodynamics. In particular, we are interested in their linearization at a background state, corresponding to an incoming horizontal wind profile. We show how the linearized equations can be reduced to a Helmholtz-like equation (the Scorer equation, well-known in the applied literature) for the vertical velocity on the upper-half plane. We then present a solution theory for the corresponding boundary value problem. Here, we have to pay special attention to a careful implementation of a physically correct radiation condition that is fundamentally different to typical radiation conditions la Sommerfeld, which are relevant in the context of electromagnetic and acoustic waves, but physically incorrect for mountain waves. The talk is based on joint work with Adrian Constantin (U Vienna).

Jörg Weber
University of Vienna
joerg.weber@univie.ac.at

CP5

Modulational Instability of Viscous Water Waves

The modulational, or Benjamin-Feir, instability refers to the breakup of a periodic surface water wave induced by long wave perturbations. This process results in the formation of large-amplitude surface waves, and has been extensively studied both experimentally and through weakly-nonlinear theory. In weakly nonlinear theory, the growth rate of the modulational instability can be obtained by studying the nonlinear Schrödinger (NLS) equation, which captures the modulation of a periodic wave over large length and time scales. However, while viscosity is unavoidable in experimental results, such weakly-nonlinear approaches have either been applied: (i) directly to the Euler equations without viscosity; (ii) on a model NLS equation with artificial viscous damping; or (iii) on a system in which viscosity enters at second-order in the weakly nonlinear expansion and so has no effect on the wave train. In this talk, we present the first derivation of an NLS equation directly from the Navier-Stokes equations, in which viscosity both contributes to the decay in time of the dominant wave train and the modulational instability. This work is joint with J. Shelton (St Andrews).

Murray Kiernan
University of St. Andrews
mk373@st-andrews.ac.uk

CP5

Stochastic Nonlinear Fractional Wave Equation

We consider the stochastic time and time-space fractional wave equations with Caputo time fractional derivative of order $1 < \alpha < 2$, with space variable coefficients and on an unbounded domain. The space derivatives that appear in the equations are of integer or fractional order (left or right Liouville fractional derivative, Riesz fractional derivative).

To solve the problem we use generalized uniformly continuous solution operators. We obtain the unique solution within a certain Colombeau generalized stochastic process space. In our solving procedure, instead of the originate problem we solve a certain approximate problem, where operators of the original and the approximate problem are L^2 -associated.

Danijela Rajter-Ciric
Department of Mathematics and informatics,
Faculty of Sciences, University of Novi Sad
rajter@dmi.uns.ac.rs

CP5

Reflection of Waves in Klein-Gordon Time-Nonlocal Thermo-Viscoelastic Porous Medium under Non-Simple MooreGibsonThompson Memory Dependent Derivative Thermo-Elasticity.

Abstract: The governing equations for a viscoelastic porous medium are formulated under non simple MooreGibsonThompson memory dependent derivative thermoelasticity. The time-nonlocality model is developed based on the Klein-Gordon nonlocality stress theory to predict the porous behaviour of the material. Mathematical expression for wave speed is derived. Velocity equation is solved to show the existence of three dilatation and one shear waves propagating in the medium. Expression for amplitude ratio is derived. Effect of nonlocality and porosity on the speed of waves and reflection coefficient are studied. The numerical results for a particular material are illustrated graphically to observe these effects.

Anand Kumar Yadav
Shishu Niketan Model senior secondary school, sector
22-D, c
yadavanand977@gmail.com

CP6

Covariant Formulation of Shock Waves Propagation for Traffic Flow on Curved Roads

Traffic jams and stop-and-go waves on highways are manifestations of shock waves in vehicular flow, yet existing traffic models typically neglect the geometric properties of real road networks, curves, hills, and ramps. We present a covariant formulation for shock wave propagation along spatial curves embedded in \mathbb{R}^3 , where road geometry curvature $\kappa(s)$ and torsion $\tau(s)$ couples naturally to the flow dynamics. This framework extends classical scalar conservation laws to curved geometries and applies to traffic flow. We solve the model numerically using a Godunov finite volume scheme that preserves mass conservation to machine precision. We validate the formulation on four test geometries: straight lines ($\kappa = 0, \tau = 0$), circles ($\kappa = \text{const}, \tau = 0$), helices ($\kappa = \text{const}, \tau = \text{const}$), and modulated circles with spatially-varying geometry. For constant-geometry cases, theoretical predictions for shock velocity match numerical simulations with zero error. This framework provides a rigorous foundation for studying nonlinear waves on curved geometries and suggests future work on agent-based validation, experimental verification with traffic data, and extension to active matter systems.

Hector J. Medel
Tecnologico de Monterrey

hmedel@tec.mx

CP6

Full Euler Equations for Waves Generated by Horizontal Seabed Displacements

We numerically investigate the generation and propagation of water waves induced by seabed displacements. For this purpose, we employ the classical hydrodynamic model based on the full Euler equations. Our study focuses on identifying the parameters that trigger nonlinear effects in waves generated by a rigid block sliding along the bottom of a channel according to a prescribed velocity. The numerical simulations combine a time-dependent conformal mapping, which captures the evolving geometry of both the seabed and the free surface, with Hermite interpolation to reproduce geometrical configurations used in laboratory experiments. As a benchmark, we reproduce experimental results on wave generation and explore variations in topographic and kinematic parameters to identify configurations where the nonlinear wave dynamics deviate significantly from the corresponding linear solutions.

Roberto Ribeiro, João Vitor P. Poletto
Federal University of Paraná
robertoribeiro@ufpr.br, jvppoletto@gmail.com

David Andrade
Universidad del Rosario
daveidu.andrade@urosario.edu.co

Marcelo V. Flamarion
Pontificia Universidad Católica del Perú,
mvellosflamarionvasconcellos@pucp.edu.pe

CP6

Subharmonic Instability of a Periodic Surface Wave with a Submerged Row of Point Vortices.

In an unbounded fluid, a row of equally spaced point vortices that lies in equilibrium is linearly unstable with respect to subharmonic perturbations. This is known as the pairing instability, in which the dominant growth rate has wavelength twice that of the vortex separation and so results in the row of vortices splitting into a staggered vortex street. Similarly, nonlinear gravity waves (travelling surface waves of a free-surface fluid under the action of gravity) are also subharmonically unstable, and the well-known Benjamin-Feir instability occurs on long-wavelength perturbations. Many recent works have calculated steady solutions to the surface-wave problem with submerged point vortices, both numerically and via exact methods, and studying the stability of these is the objective of this talk. Our spectral numerical method allows for the study of subharmonic instabilities of these steady solutions in which Floquet theory is used to study perturbations to both the free surface and the vortex row that are subharmonic. This work is joint with W. Choi (NJIT) and D. Dritschel (St Andrews).

Josh Shelton
University of St. Andrews
josh.shelton@st-andrews.ac.uk

CP6

Field Observation of Soliton Gases in the Deep

Open Ocean

Soliton gases are large ensembles of random solitons with distinct characteristics arising from integrable system dynamics. They have been widely studied in theory and experiments, and were observed in natural lagoons. However, it remained an open question whether they occur naturally in the open ocean. In this talk, I report on the first soliton gases that have been identified in real-world measurements from the deep open ocean. The talk is based on the preprint arXiv:2510.04662v1 [nlin.PS] by Yu-Chen Lee and Sander Wahls.

Sander Wahls
Karlsruhe Institute of Technology
sander.wahls@kit.edu

MS1

Proximal Integrable Methods for Modeling Coherent Structures

Nonlinear wave equations often exhibit a hidden structure encoded in a Lax pair: a pair of operators whose compatibility governs the dynamics and yields conserved spectral invariants. Although exact integrability is rare in applications, many physical systems lie near integrable limits, and their coherent structures are controlled by slowly varying spectral data. This talk presents a new approach for reduced modeling that exploits these ideas directly. The method learns an approximate Lax representation from data, filters it to the nearest compatible pair through a proximal optimization step, and then evolves the system in the resulting spectral coordinates. The learned operators recover the key invariants that anchor solitons, dispersive shocks, and radiative tails, while the reduced evolution produces accurate long-time dynamics with orders-of-magnitude lower computational cost. Examples from perturbed Korteweg–de Vries and nonlinear Schrödinger dynamics illustrate how spectral coordinates provide a natural setting for model reduction, capturing both coherent structure motion and weak nonintegrable effects. The framework shows that integrability serves not only as a theoretical limit but also as a practical organizing principle for scientific machine learning.

Jimmie Adriaola
Arizona State University
jimmie.adriaola@asu.edu

MS1

From Betti to Sindy: Data-Driven Topological Diagnostic Evolution for Nonlinear Wave Dynamics

We aim to develop a topological classification of stationary vortex type solutions of the three-dimensional cubic nonlinear Schrödinger equation (NLS) with harmonic trapping by employing tools from Topological Data Analysis (TDA), including Morse reconstruction and homological persistence. The first goal is to extract topological information such as critical point structure, homology generators and persistence barcodes starting from the vorticity associated with these vortex states. Our intention is then to investigate how these topological quantities evolve under time dynamics by perturbing these stationary states. Ultimately, we seek to integrate these topological descriptors into data driven model discovery frameworks such as Sparse Identification Nonlinear Dynamics (SINDy), with the purpose of identifying models that capture both the topology and the dynamics of vortex solutions in nonlinear

Schrödinger systems. This work is currently in progress.

Marco Calabrese

University of Massachusetts, Amherst
mcalabrese@umass.edu

MS1

Intrinsic Neural Tangent Flows for Quantum Fluids

Quantum fluid models exhibit rich structure: Hamiltonian form, symplectic geometry, nonlocal dispersion, and delicate phase information that governs long-time coherence. Standard neural surrogates trained on trajectory data often violate this structure and produce models that drift, damp, or artificially regularize the dynamics. This talk introduces *intrinsic neural tangent flows*, a structure-aware learning framework that identifies the generator of the dynamics rather than a discrete time-stepping map. The method parameterizes a skew-adjoint (or metriplectic) operator and learns its action directly on the state through an intrinsic tangent-space representation. By constraining the generator—not the trajectories—the learned model preserves energy, phase, and other invariants exactly at each step of the evolution. Applications to one- and two-dimensional quantum hydrodynamic systems demonstrate that the method captures coherent structures, dispersive shocks, and long-horizon interference patterns with fidelity comparable to high-resolution numerical solvers. The results highlight a broader theme: learning the geometry of the flow yields models that are both interpretable and faithful to the underlying physics.

Kimball Johnston

Arizona State University
kjohn313@asu.edu

MS2

On the Soliton Resolution Conjecture for the Benjamin-Ono Equation

I shall explain why solutions of the Benjamin-Ono equation, with sufficiently decaying and regular initial data, decouple in the long time asymptotics as a finite sum of solitons added to a radiative term. The proof is based on an explicit representation formula of the solutions, and on a spectral analysis of the associated Lax operator. This talk is based on a jointwork with Louise Gassot and Peter Miller.

Patrick Gérard

Université Paris-Saclay
patrick.gerard@universite-paris-saclay.fr

MS3

Towards a Kinetic Theory of Two-dimensional Soliton Gases

This talk aims at presenting some recent results about the description of two-dimensional soliton gases. Specifically, we study a non-stationary two-dimensional soliton gas in the framework of the Kadomtsev-Petviashvili II (KPII) equation and write down the governing kinetic equations. We apply the kinetic equation to study two problems: (i) line soliton refraction by a stationary soliton gas; (ii) oblique interference of two stationary soliton gases. We verify the analytical predictions of the kinetic theory in these two problems by numerically building various 2D soliton gases via exact KPII N -soliton solutions with large N

and appropriately chosen random distributions for the soliton parameters.

Gino Biondini

State University of New York at Buffalo
Department of Mathematics
biondini@buffalo.edu

MS3

Dynamical Reduction for Solitonic Filaments

In this talk we describe techniques for the dynamical reduction of soliton stripes in nonlinear spatio-temporal systems. The central idea is to cast reductions to accurately describe these structures with lower-dimensional models that are more easily tackled, both mathematically and computationally. In turn, the reduced models allow for an unprecedented description of the statics, stability, dynamics, and interactions of these structures.

Ricardo Carretero

Department of Mathematics and Statistics
San Diego State University
rcarretero@sdsu.edu

MS3

(In)stability of Relaxation Oscillations and Phase Waves in Reaction Diffusion Systems

Motivated by the appearance of periodic patterns in the wake of invasion fronts into unstable states in the FitzHugh-Nagumo system, we explore the stability and instability of spatially periodic traveling phase waves and spatially homogeneous relaxation oscillations in two-component singularly perturbed reaction diffusion systems. We consider systems whose reaction kinetics admit relaxation oscillations which, using geometric singular perturbation theory, can be constructed by concatenating fast layers and portions of a slow manifold which exhibit fold points. The associated stability problem in the full PDE inherits the slow-fast structure of the existence analysis, resulting in an eigenvalue problem whose behavior is characterized by the nonhyperbolic dynamics at the folds, and which can be solved by employing geometric desingularization techniques. We describe stability criteria for the resulting waves, and we find that spatially homogeneous oscillations can exhibit finite wavelength instabilities leading to oscillatory Turing patterns.

Paul Carter

UC Irvine
pacarter@uci.edu

MS3

IST-Based Perturbation Theory for Dark Solitons

Perturbation theory for bright soliton solutions of integrable systems based on a complete set of squared Jost eigenfunctions has been well understood for decades. Over the years, several attempts have been made to apply a similar method to dark solitons on a nonzero background. In this talk, we revisit the problem of dark soliton perturbation theory for the defocusing NLS equation, addressing several aspects that had not been properly accounted for in previous works. We derive a completeness relation for the squared eigenfunctions and use it to obtain slow-time evolution equations for all soliton parameters, as well as to compute the first order term in the perturbation expansion. Furthermore, we show that the first order correction

can be used to effectively describe the propagating shelf that develops around the perturbed soliton, which results from the singularities in the scattering data at the branch points of the continuous spectrum. We apply the method to several physically relevant examples and compare our results to numerical simulations.

Nicholas Ossi

State University of New York at Buffalo
nossi@buffalo.edu

Barbara Prinari
University at Buffalo
bprinari@buffalo.edu

Jianke Yang
Department of Mathematics and Statistics
University of Vermont
jxyang@uvm.edu

MS4

Heterogeneities in Cardiac Models

We study the influence of spatial heterogeneities on the evolution of front and pulse (two-front) solutions in the FitzHugh-Nagumo system, one of the simplest models of cardiac electrophysiology. In particular, we demonstrate how the propagation of action potentials is affected by heterogeneities that are neither small in amplitude nor endowed with additional structure (such as spatial localization or periodicity). Even though our analysis employs perturbation techniques involving a small parameter, the novelty lies in the fact that this parameter does not measure the magnitude of the spatial heterogeneity. Instead, it reflects a separation of scales or proximity to a critical regime, allowing order-one heterogeneities while preserving analytical tractability. The analytical results are complemented by a comprehensive numerical investigation of several types of heterogeneities that arise in cardiac applications. This is joint work with Tim De Coster (Max Planck Institute for Dynamics and Self-organisation, Goettingen, Germany).

Martina Chirilus-Bruckner

University of Leiden
Mathematical Institute
m.chirilus-bruckner@math.leidenuniv.nl

MS4

Spatial Pattern Formation and the Evolution of Cooperative Behavior

Social dilemmas featuring tension between the individual incentive to cheat and a collective goal to maintain cooperative behavior arise across a range of natural and social systems, from the origins of multicellular life to the sustainable management of shared natural resources. Evolutionary game theory provides a helpful analytical framework for describing this conflict between individual and collective interests, exploring mechanisms that can help the emergence of cooperative behaviors. In this talk, we discuss several PDE models for evolutionary games featuring diffusion of individuals and directed motion towards either increasing payoff or improved environmental quality. We show that biased motion of cooperators can promote the formation of spatial patterns featuring regions with greater population density and increased average payoffs and environmental quality in regions in which cooperators have aggregated. However, by measuring the average payoff of the popula-

tion or the average level of environmental quality across the population, we see that these pattern-forming mechanisms can actually decrease the overall success of the population, relative to the equilibrium outcome in the absence of spatial motion. This suggests that payoff-driven and environmental-driven motion can produce a kind of spatial social dilemma, in which biased motions towards more beneficial regions can produce emergent patterns featuring a worse overall environment for the population.

Tianyong Yao
University of Michigan
yuzutyao@umich.edu

Chenning Xu
California Institute of Technology
cxu7@caltech.edu

Daniel B. Cooney
University of Illinois Urbana-Champaign
danielbcooney@gmail.com

MS4

Traveling Pulses and the Effects of Virulence in An Evo-Epidemiological Model

We consider an extension of an evolutionary-epidemiological model originally proposed by Lin et al. in 2003 for modeling Influenza A drift. This model takes the form of an SIR-type system consisting of one-dimensional integro-differential equations, where non-local terms couple the infected and recovered populations through an immunity kernel. Using a combination of asymptotic techniques we establish distinguished parameter regimes for which solutions to algebraic equations can be used to construct approximations to traveling pulse solutions of the evo-epidemiological model. Using these asymptotic approximations we perform a detailed exploration of parameter space and in particular investigate infection prevalence as a function of virulence and other epidemiological and population-level parameters.

Daniel Gomez
University of New Mexico
danielgomez@unm.edu

Yoichiro Mori, Joshua Plotkin
University of Pennsylvania
y1mori@math.upenn.edu, jplotkin@sas.upenn.edu

MS4

Spatial Heterogeneity of Extracellular Serotonin Induced by Firing Dynamics of Neural Fiber Meshworks

All vertebrate brains, from fish to humans, contain dense meshworks of axons (fibers) that release and reuptake serotonin (a key signaling molecule) through discretely located varicosities along the fibers. The role of this massive system is poorly understood. To investigate how the geometry of the spatial arrangement of varicosities and the timing of release shape serotonin concentrations in microscopic 2-D slices of brain volumes, each varicosity is modeled as a small disk where the kinetics of serotonin release and uptake are incorporated. The disks interact with the surrounding diffusive space through an infinitely permeable boundary. This compartmental-reaction diffusion system can be rigorously reduced to an integro-ordinary-differential system that can be numerically solved efficiently. Periodic

firing dynamics are incorporated through external forcing terms in the intra-compartmental kinetics. Period-averaging enables us to compute estimates on the period-averaged steady-state. Our system highlights precise coupling terms across varicosities that capture the diffusive memory dependence and global coupling and can be solved with arbitrary serotonin reaction kinetics at the varicosities. Using biologically realistic parameters, we provide, starting with the case of five neighboring parallel fibers, tight bounds on the large spatial serotonin variation near the varicosities and on the reservoir concentration further away that depends on the surrounding varicosity density.

Merlin Pelz

University of Minnesota, Twin Cities
mpelz@umn.edu

Skirmantas Janušonis

University of California, Santa Barbara
janusonis@ucsb.edu jjanusonis@ucsb.edu

Gregory Handy

University of Minnesota
ghandy@umn.edu

MS5

Coupling Between Melting Solids and Convective Flows

The melting of ice has shaped the terrain, climate, and life of our planet, and its coupling to fluid flows is also a focus of modern fluid dynamics. In this talk, we will introduce several recent studies conducted at the Applied Math Lab of NYU and NYU Shanghai, where the coupling between melting ice and convective flows is investigated through laboratory-scale experiments. Among them, we will address questions such as if natural convective flow can move a floating iceberg and what may serve as an equilibrium between melting and freezing. These investigations, geophysically inspired, may provide new insights into the fluid-structure interactions behind melting dynamics.

Mac Huang

NYU Shanghai
machuang@nyu.edu

MS5

Accumulation of Suspended Particles on Lateral Walls Through Stratification-induced Suction

Fluids with gravitationally stable density stratification caused by diffusing solutes are ubiquitous in nature and may spontaneously generate flows in the destabilizing presence of immersed bodies. Here we document the discovery of a counterintuitive phenomenon: objects in such fluids may self-induce suction forces producing near constant accelerations towards nearby walls, in the absence of any external forces. Here we present an experimental, computational, and theoretical study to fully explore this new phenomenon. First, experiments exhibiting wall collapse are presented. Next, flow and density structures are measured and compared quantitatively to computational simulations with spheres and cylinders, both in free space and near symmetry-disrupting vertical walls. Further computations reveal a competition between the pressure and viscous stress forces that enable a “lubrication screening,” overcoming the resistance of a thin lubricating layer. In particular, a low pressure region in the gap develops and the particle spontaneously moves to fill the vacuum by being

pushed along by ensuing flows. The resulting unexpected motion in a viscous dominated flow propels the particle almost all the way to the wall within a distance scale set by the stratified fluid properties, ultimately decelerating with a soft-landing. Lastly, extensions of these new phenomena to thin and porous geometries are discussed with theoretical and computational predictions showing how the wall-induced motion can be reversed by porosity, pushing the body away from the wall.

Richard McLaughlin

University of North Carolina at Chapel Hill
rmm@email.unc.edu

MS5

Cardio-Respiratory Variability and Turbulence Transitions in the Equine Aorta and Airway

Horses operate at extreme physiological limits, generating some of the highest Reynolds numbers observed in biological systems. Yet we know little about how natural variability in cardiac and respiratory rhythms shapes transitions to turbulence in the aorta and upper airway. In this talk, I combine new experimental measurements of heart-rate variability (HRV) and respiratory variability indices (RVI) with numerical simulations of pulsatile flow to examine how beat-to-beat and breath-to-breath modulation affects flow separation, shear, and turbulence onset. This work highlights new opportunities for integrating mathematics, physiology, and experimental biomechanics.

Laura A. Miller

University of Arizona
Department of Mathematics
lauram9@math.arizona.edu

MS5

Anomalous Waves Induced by Abrupt Depth Change: Laboratory Experiments and KdV Statistical Mechanics

I will discuss both laboratory experiments and theory to describe randomized surface waves propagating over variable bathymetry. The experiments show that an abrupt depth change can qualitatively alter wave statistics, transforming an initially Gaussian wave field into a highly skewed one. In our experiments, the probability of a rogue wave can increase by a factor of 50 when compared to Gaussian. A theoretical framework, based on dynamical and statistical analysis of the truncated KdV equations, accurately captures key features of the experiments, including the skewed wave distributions that emerge downstream and the associated excitation of higher frequencies in the spectrum. More recently obtained rigorous results lead to a highly-efficient and parallel algorithm to sample waves from the KdV Gibbs measure without the need to simulate the underlying dynamics.

Nick Moore

Colgate University
nickmoore83@gmail.com

Matthew N. Moore

Florida State University
nickmoore83@gmail.com

MS6

Orbital stability of periodic waves in Hamiltonian

systems against localized perturbations

We investigate the stability and long-term behavior of spatially periodic solutions in Hamiltonian systems against localized disturbances. Such periodic waves often correspond to robust structures that arise in a multitude of physical settings, with notable examples including water waves, periodic light pulses in nonlinear optical fibers, or wave trains in Bose-Einstein condensates. To date, nonlinear stability results for periodic waves in Hamiltonian systems have primarily addressed co-periodic or subharmonic perturbations. Their stability with respect to localized perturbations - a natural setting in many physical applications - remains a longstanding open problem, as such perturbations render the wave neither localized nor periodic, placing its stability analysis outside the scope of the classical orbital stability framework for Hamiltonian systems developed by Grillakis, Shatah, and Strauss. We present an alternative approach that combines variational techniques, leveraging conserved quantities tailored to the perturbation equation, with Duhamel-based methods and a modulational ansatz - tools known from the stability analysis of periodic waves in dissipative systems and reaction diffusion models. Using this approach, we establish first orbital stability results in key Hamiltonian models, such as the Klein-Gordon, Korteweg de Vries and nonlinear Schrödinger equation with respect to localized perturbations. This is joint work with Bjrn de Rijk (KIT).

Emile Bukieda

Karlsruhe Institute of Technology (KIT)
emile.bukieda@kit.edu

MS6

Shocks of Hyperbolic Partial Differential Equations and Applications to Inertial Marangoni Flow

Shocks due to hyperbolic PDEs appear in a wide range of applications. A famous example is shock formation of the inviscid Burgers equation $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0$. Previous studies have shown that when shocks form for the inviscid Burgers equation, the dynamics are locally self-similar and universal for positions and times close to the shock. We show that, in fact, shock formation is self-similar and universal for general first-order strictly hyperbolic PDEs in one spatial dimension, where we give analytical expressions. The self-similarity is shown to be like that of the inviscid Burgers equation. Furthermore, we apply these ideas to understand the thinning of air-liquid-air sheets due to Marangoni effects, which is an example of Marangoni flow where inertia often appears in the leading order physics.

Jun Eshima

Princeton University, USA
jeshima@princeton.edu

Howard Stone

Princeton University
hastone@princeton.edu

Luc Deike

Mechanical and aerospace engineering
Princeton University, USA
ldeike@princeton.edu

MS6

Strongly Nonlinear Long Wave Approximation of

Internal Fronts and Gravity Currents

Internal waves generated at the interface between two homogeneous fluids of different densities may take the form of internal fronts of which the interface is asymptotically flat at both ends of upstream and downstream with different depths. This work considers internal fronts in a two-fluid system bounded above and below by horizontal rigid walls. As the interface of internal front shifts vertically upwards, this flow approaches a gravity current of the light fluid propagating along the top wall into the heavy fluid. This limit behavior is investigated using the strongly nonlinear long wave approximation for this two-fluid system. In particular, we try to evaluate the validity of some mathematical models for local flow near the separation point of gravity current at which the interface is detached from the top wall.

Sunao Murashige

Department of Mathematics and Informatics
Ibaraki University
sunao.murashige.sci@vc.ibaraki.ac.jp

MS6

Numerical Results on the Vanishing Viscosity Limit for (possibly Overturning) Water Waves

In this presentation, we would like to discuss a novel, finite elements-based, numerical method to approximate the solution of the single-fluid incompressible Navier-Stokes equations with a free surface advected in a Lagrangian manner. The entire mesh is moved using the Arbitrary Lagrangian-Eulerian (ALE) method, allowing to capture the breaking phenomenon. Simulations with different values of the Reynolds number are carried out and compared with the inviscid irrotational solution of Euler's free-surface system, computed numerically using the a different method. We discuss in which cases the vanishing viscosity limit is expected to hold and when it does not, through analysis of the boundary layers stability.

Alan Riquier

École Normale Supérieure
alan.riquier@normalesup.org

MS7

From soliton dynamics to the Schrödinger equation in a non-standard pattern-forming system

We show that quantum dynamics can emerge from the collective behavior of solitons in a classical nonlinear field theory. Considering a Galilean-invariant complex field supporting stable localized structures, solitons behave as classical particles on a zero background, obeying Newton's law. When embedded in a spatio-temporally chaotic background generated by intrinsic instabilities of the field, however, their motion develops fluctuations arising from deterministic interactions with the small-scale turbulence. We derive an exact uncertainty relation linking position and momentum fluctuations of solitons and demonstrate that the Schrödinger equation governs the ensemble dynamics of solitons' deterministic trajectories. In this framework, the effective Planck constant is set by the amplitude of background fluctuations, providing a direct connection between microscopic pattern dynamics and emergent quantum behavior. Numerical simulations of solitons interacting with potential barriers show quantitative agreement between ensemble statistics and solutions of the Schrödinger equation, reproducing tunneling as an emergent phenomenon. The

results illustrate how non-standard pattern-forming systems with conserved quantities and chaotic backgrounds can generate ensemble dynamics fundamentally different from other classical counterparts, suggesting a new route linking nonlinear waves, soliton dynamics, and quantum-like statistical descriptions.

Damia Gomila
IFISC (CSIC-UIB)
damia@ifisc.uib-csic.es

MS7

Cylindrical Lipid Bilayer Bifurcation and Stability

We consider the L^2 gradient flow of the Helfrich model for lipid bilayers. The model incorporates constraints on membrane inextensibility (area constraint) and the absence of osmotic exchange (volume constraint). These constraints give rise to Lagrange multipliers, which appear as non-local terms. The lipid bilayer serves as a simplified model for the shape of red blood cells as well as other self-organizing cellular structures in biology. We studied bifurcation of closed vesicles using `pde2path`. We extend this to cylindrical topology. Well-known phenomena are the pearling instability of the cylindrical shape and transitions to coiled structures solution. We find them and other bifurcations using center manifold analysis and numerical continuation. Due to the presence of Lagrange multipliers, we take a non-standard approach to derive amplitude equations for each bifurcation scenario. Within this framework, we analyze the stability of the bifurcation branches. To provide a broader perspective on the shape transitions of cylindrical structures, we employ numerical continuation, similar to our approach for closed vesicles. However the analysis shows some discrepancies with experiments.

Alexander Meiners
Department of Mathematics
University of Oldenburg
alexander.meiners@uni-oldenburg.de

MS7

Oscillons and Jumping Oscillons in a Three-Variable Reaction-Diffusion System on a Line and a Disk

We investigate the self-organization of spatiotemporal patterns in a three-variable reaction-diffusion system that exhibits a finite wavenumber Hopf (wave) instability. By employing numerical continuation and direct numerical simulations, we analyze the properties and organization of spatially localized structures in both one-dimensional (1D) domains and two-dimensional (2D) disks. In 1D, we demonstrate that localized standing waves (oscillons) organize in a non-collapsing homoclinic snaking structure. We further show that traveling pulses organize via a complex snaking mechanism involving wavelength-doubling bifurcations, which may be related to a codimension-two Shilnikov-Hopf bifurcation. These numerical results are complemented by an analysis of coupled complex Ginzburg-Landau amplitude equations in 1D derived near the instability onset. In 2D, we explore the interaction between wall-attached states and bulk dynamics on a disk. We identify 2D analogs to the 1D states, including rotating and oscillating spots and "jumping oscillons," and demonstrate that while wall-attached states retain similarities to their 1D counterparts, the coupling with the bulk mode leads to significantly richer spatiotemporal dynamics, including the formation of spirals, source-sink pairs, and ir-

regular rotational reversals.

Edgar Knobloch
University of California at Berkeley
Dept of Physics
knobloch@berkeley.edu

Saar Modai
Department of Physics
Ben-Gurion University
modais@post.bgu.ac.il

Hannes Uecker
Department of Mathematics
University of Oldenburg
hannes.uecker@uni-oldenburg.de

Arik Yochelis
Swiss Institute for Dryland Environmental & Energy
Research
Ben-Gurion University
yochelis@bgu.ac.il

MS8

Localized Time-Periodic Patterns in a 1D Lattice

Motivated by numerical continuation studies of coupled mechanical oscillators, this talk present recent findings on branches of localized time-periodic solutions of one-dimensional chains of coupled oscillators. We focus on Ginzburg-Landau equations with nonlinearities of Lambda-Omega type and establish the existence of localized synchrony patterns in the case of weak coupling and weak-amplitude dependence of the oscillator periods. Depending on the coupling, localized synchrony patterns lie on a discrete stack of isola branches or on a single connected snaking branch.

Jason J. Bramburger
Concordia University, Canada
jason.bramburger@concordia.ca

MS8

Robust impact mitigation enabled by nonlinear wave inverse design

In mechanics, the interplay between nonlinearity in elastic constitutive response and dispersion due to discreteness in a system can be a useful mechanism for impact mitigation. Using a variant of the famed Fermi-Pasta-Ulam-Tsingou (FPUT) system with uniform masses and linear dampers, we computationally study discrete systems of two families: (1) a lattice with a homogeneous nonlinearity, and (2) a heterogeneous lattice, with four unique allowable nonlinearities. We identify optimal solutions that minimize the transmission of kinetic energy to a protected boundary and compare solution sensitivity to varied impactor conditions and subtle changes in nonlinearity.

Maya Brandy
University of California San Diego
U.S.
mbrandy@ucsd.edu

MS8

Computing the Spectrum of Traveling Waves in

Networks and Lattices by Continuation

Characterizing the stability of a periodic traveling waves in large ODE networks is a difficult problem when dealing with a large number of nodes. We present a framework that overcomes this challenge for Z_n -equivariant rings and lattices. By deriving a delayadvance differential equation, we formulate a two-point boundary value problem (2PBVP) whose numerical continuation in a suitable parameter yields a master stability curve that captures the spectrum of a traveling wave independent of the network size or wavenumber. In the lattice case, this curve coincides with the essential spectrum of the wave. We further extend the 2PBVP to compute the curvature of the master stability curve, enabling the characterization of the Eckhaus stability boundaries as functions of system parameters. We illustrate this framework on two problems: dissipatively coupled FitzHugh-Nagumo rings and networks of coupled Duffing oscillators. In this way, we demonstrate the robustness of our framework for both excitable and mechanical lattice systems.

Andrus Giraldo

Korea Institute for Advanced Study
agiraldo@kias.re.kr

Stefan Ruschel

School of Mathematics
University of Leeds, United Kingdom
stefan.ruschel@nottingham.ac.uk

Yousefzadeh Behrooz

Concordia University
behrooz.yousefzadeh@concordia.edu

MS8

Experimental Realization of Extreme Nonlinear Wave Phenomena in Mechanical Metamaterials

We present a series of experimental platforms that realize extreme nonlinear wave phenomena in mechanical metamaterials. First, we introduce asymmetric linkage systems that produce topological boundary modes and analytically tractable mode shapes, providing a mechanical realization of zero-modes governed by geometric constraints. Second, we demonstrate highly versatile linkage lattices with tunable strain-softening and strain-hardening responses, enabling the controlled generation of large-amplitude localizations, supersonic compression pulses, and wave-mode conversions in strongly nonlinear chains. Third, we report the experimental formation of mechanically generated rogue-wave-like bursts in a nonlinear spring-mass lattice with prescribed initial conditions, and validate the observed focusing behavior using reduced FPUT and nonlinear Schrodinger-type models. Together, these results establish an integrated experimental framework for studying nonlinear wave propagation, topological modes, and extreme energy localization in structured materials, offering opportunities for theory-experiment cross-fertilization in nonlinear dynamics.

Myeonggyun Joo, Jae-Kyeong Kim, Dongwook Kwak,
Joohyung Lee, Yasuhiro Miyazawa, Jinkyu Yang
Seoul National University
joomk1201@snu.ac.kr,
ume1838@snu.ac.kr,

coorla@snu.ac.kr,
pooh7578@snu.ac.kr,

y Miyazawa@snu.ac.kr, jkyang11@snu.ac.kr

MS9

Structure Preserving and Data-Driven Modeling of Nonlinear Waves

We present a new class of data driven bracket dynamics from which we are able to build stochastic models for hyperbolic systems.

Nat Trask

University of Pennsylvania, U.S.
ntrask@seas.upenn.edu

MS9

Principled Operator Learning for Pattern Forming Systems

Many nonlinear parabolic systems relax toward low-dimensional attractor manifolds, producing spatial patterns such as spots, stripes, and labyrinths. While the forward dynamics are well understood, the map taking an initial condition to its eventual steady state is both highly nonlinear and implicitly defined through long-time evolution of the PDE. We propose a data-driven framework for learning this attractor operator directly from simulation data. Training pairs are generated by integrating initial conditions to steady state, and the operator is approximated using neural operator architectures including Fourier Neural Operators and DeepONets. We validate the approach on two examples of increasing complexity: a one-dimensional fixed-point map arising from Newton's method, and the two-component Gray-Scott reaction-diffusion system in the pattern-forming regime. The learned operator partitions input space into basins of attraction corresponding to distinct spatial patterns, enabling instantaneous prediction of final states without time-marching. Regularization terms enforce approximate equivariance under translation and reflection symmetries. This framework provides a path toward understanding basin geometry through learned feature embeddings and extends naturally to parameterized PDEs, where variation of the attractor operator across parameter space encodes information about bifurcations and stability transitions.

Erli Wind-Andersen

Icahn School of Medicine, Mount Sinai
erliwind@gmail.com

MS9

Learning Flow-Boxes in Phase Space Via Neural Deflation

In this talk, we present the neural deflation framework for discovering conservation laws and assessing system integrability. The method iteratively trains neural networks to identify local or global conserved quantities while enforcing their functional independence. It also allows us to detect singularities in conservation laws and determine the maximal domain over which a local conservation law exists. We illustrate its effectiveness on a range of Hamiltonian systems, including harmonic oscillators, the double pendulum, the Hnon-Heiles system, and discrete lattice equations.

Wei Zhu

Georgia Institute of Technology

weizhu@gatech.edu

MS10

Coherent energy cascades in (semi-)random Hamiltonian systems

The problem of Sobolev norm growth, namely, the search for mechanisms of energy transfer from low to arbitrarily high modes has been intensively studied in Hamiltonian systems with an organized, deterministic structure of mode interactions, such as the cubic nonlinear Schrödinger equation. In this talk, I will present an extension of this problem to Hamiltonian systems dominated by random nonlinear interactions. I will introduce analytic solutions describing three types of energy cascades that lead either to unbounded growth or to finite-time blow-up of Sobolev norms. I will then present numerical simulations demonstrating the emergence of these dynamics from incoherent initial conditions. Taken together, these results demonstrate that coherent (phase-organized) energy cascades can be robust mechanisms of energy transfer even in systems with a random structure.

Anxo Biasi

University of Santiago de Compostela
anxo.biasi@gmail.com

MS10

Random Solitons, Solitons Gasses and All That

The concept of a soliton gas was originally introduced by Zakharov in the Seventies and it can be loosely described as a class of solutions of nonlinear integrable PDEs that displays an infinite number of -possibly random- solitons. I will present a collection of recent results from a collaborative effort to rigorously understand various aspects of several soliton gas models: long-time asymptotics, random statistics, and interaction dynamics. In particular, we consider a gas of random N solitons for the KdV equation and we study the limit as N goes to infinity. We derive the limiting solution and we prove that its fluctuations are Gaussian random variables. The starting point is the formulation of a multi-soliton solution and of a regular, dense soliton gas solution in terms of Riemann-Hilbert problems. Tools from asymptotic analysis and probability will be used to provide a detailed description of interesting phenomena.

Manuela Girotti

Emory University
manuela.girotti@emory.edu

Tamara Grava

SISSA, via Beirut, 2-4, 34014, Trieste, Italy
grava@sissa.it or tamara.grava@bristol.ac.uk

Ken McLaughlin

Tulane University
kmclaughlin@tulane.edu

Joseph Najnudel

University of Bristol
joseph.najnudel@bristol.ac.uk

MS11

Generation and Evolution of Singularities in Hy-

drodynamic Models of Wave Propagation

Interesting phenomena in fluid dynamics, both from a mathematical and a physical perspective, often stem from the interplay between fluids and their boundaries. Singularities can form in finite time when material surfaces are in smooth contact with boundaries of a fluid under gravity, and, conversely, certain regularity can be regained in the course of the evolution. These effects can be analytically and numerically predicted by simple mathematical models and observed in simple experimental setups.

Roberto Camassa

University of North Carolina at Chapel Hill
camassa@amath.unc.edu

MS11

Fully nonlinear dynamics of fluid free surface on Riemann sheets

A fully nonlinear dynamics for potential flow of ideal incompressible fluid with a free surface is considered in two dimensional geometry. It is shown that a general free surface dynamics involves a motion of infinite number of branch points located in infinite number of sheets of the Riemann surface. An efficient description of motion of these branch points is developed.

Pavel M. Lushnikov

University of New Mexico, U.S.
plushnik@unm.edu

MS11

Well-Posedness of the KdV-KS Equation on a Finite Interval

The Korteweg-de Vries Kuramoto-Sivashinsky (KdV-KS) equation is a fourth-order nonlinear evolution equation with a KdV-type nonlinearity. The traditional KS equation serves to model a variety of phenomena, such as disturbances in laminar flame fronts and widths of liquid films as they run down a surface. The KdV-KS equation includes a linear third derivative term in addition to the second and fourth found in KS, with a relative strength that acts as a parameter of the problem. In the liquid film example, this parameter allows the modeler to account for inclination in the surface, such as in the case of rain running down a smooth roof. In this talk, we consider the initial-boundary value problem for the one-dimensional KdV-KS on the finite interval. We establish the local well-posedness of this problem in the sense of Hadamard (existence and uniqueness of the solution as well as its continuous dependence on the data) for initial data in the Sobolev space H^s and boundary data in suitable Sobolev spaces determined by the regularity of the initial data and the KdV-KS equation. A foundational element of our proof is the linear solution operator derived through the unified transform of Fokas, from which we extract a wide variety of linear estimates. Combining these with certain nonlinear estimates, we manage to show well-posedness all the way down to negative-order Sobolev spaces.

Chris Mayo

University of Kansas
cmayo@ku.edu

MS11

Shallow-Water Modelling Via a Variational Ap-

proach: Derivation, Properties, and Numerics

We present a variational framework for deriving a family of shallow-water wave models with increasing order of accuracy that bridges the classical asymptotic long-wave theory and nonlocal Hamiltonian formulations. Starting from Lukes principle, we approximate the velocity potential with vertical polynomials motivated by the shallow-water structure of the Dirichlet-to-Neumann operator. This leads to a hierarchy of models written directly in canonical variables, whose evolution equations retain the original problems canonical, nonlocal Hamiltonian structure. In uniform water depth cases, we discuss the equivalence of this formulation with the IsobeKakinuma model introduced in the mid-1990s, and clarify how this class of models relates to classical and modern asymptotic models. We analyze its linear dispersion and stability properties from a theoretical perspective, and then demonstrate, through numerical simulations, accurate solitary-wave propagation and interactions. We also outline the extension to variable bathymetry and present numerical results illustrating nonlinear wave-wave and wavebottom interactions.

Christos Papoutsellis

École Nationale des Ponts et Chaussées
christos.papoutsellis@enpc.fr

MS12**Landscape-Scale Self-Organized Vegetation Patterns in Drylands**

A stunning example of spontaneous pattern formation occurs in certain drylands around the globe. Regularly-spaced bands of vegetation alternate with bands of bare soil on a landscape-scale. The vegetation bands are oriented transverse to gentle elevation grades, and, interestingly, can slowly migrate uphill as an approximately periodic traveling wave pattern. Natural questions to ask are what sets the spacing of these bands and what sets the upslope migration speed? We use a conceptual consumer-resource modeling framework to investigate these questions, where vegetation is the consumer and soil water is the resource. Key nonlinear feedbacks between these components lead to pattern formation under aridity stress. A challenging feature of drylands is that the water inputs to the system are not continuous; they are the result of rare storm events, which are unpredictable and highly variable. Our model captures the disparity of timescales, and the stochasticity, by treating rain events as random impulses to the system. The soil water and vegetation then evolve together, between storms, on the slow ecosystem timescale via simple deterministic reaction-diffusion PDEs. This modeling framework allows us to explore how changes to storm characteristics, such as their frequency and intensity, might impact the ecosystem. New pattern formation questions arise as we probe the resilience of these remarkable ecosystems under changing rainfall patterns.

Mary Silber

University of Chicago
Committee on Computational & Applied Math+Statistics
Dept.
msilber@uchicago.edu

Punit Gandhi
Virginia Commonwealth University

gandhipr@vcu.edu

MS12**Emergent Spatial Structures in Heterogeneous Cancer Cell Populations**

The coexistence within the same tumour of cancer cells that express different phenotypic characteristics poses a major obstacle to successful anti-cancer therapy and management of disease relapse. Mathematical models can help shed light on the emergence of coherent spatial structures following the selection of cells with different phenotypic characteristics in different points in physical space, during different stages of cancer progression. The dynamics of space- and phenotype-structured cancer cell populations can be described by models formulated as nonlinear integro-differential equations, the solutions of which may develop biologically-relevant singularities or travelling wave solutions with sharp fronts and composite shapes. In this talk we will explore how their qualitative properties may be investigated by means of formal asymptotic and travelling wave analysis, along with numerical simulations, and the open analytical and numerical challenges of this field.

Chiara Villa

Laboratoire MAP5
CNRS & Université Paris Cité
Chiara.Villa@math.cnrs.fr

MS12**Maxwell Points: The Origin of Homoclinic Snaking and Localised Patterns Beyond All Asymptotic Orders**

Coherent structures are ubiquitous in several areas of science. My presentation is related to localised structures that are formed through homoclinic snaking, close to codimension-two Turing bifurcations. Specifically, close to these bifurcation points where there is a change of criticality, one can formally prove, generically, the existence of Maxwell points that produce the formation of a front connecting the homogeneous steady state and a periodic orbit. The context I have introduced so far has been studied intensively in the literature. However, what remained to be studied is the width of the snake. That is, the range to which you can move a parameter of the system from the Maxwell point while still preserving these localised structures in the system. I will give the general result for patterning in reaction-diffusion equations and also introduce further generalizations I am currently working on, to do with more general classes of equations, and localised spatiotemporal patterns.

Edgardo Villar-Sepúlveda

University of Bristol
edgardo.villar-sepulveda@bristol.ac.uk

MS13**Pilot-Wave Hydrodynamics**

Pilot-wave hydrodynamics represents a platform for analogizing quantum systems on a macroscopic scale. We consider here one of the most beguiling hydrodynamic quantum analogs, that of the quantum corral, in which walking droplets explore a closed domain. Experiments demonstrate that despite complex, chaotic trajectories, robust, coherent wave-like statistics emerge. We here rationalize

this emergent statistics in terms of stochastic droplet dynamics over a wave-induced ponderomotive potential. Our study forges new links with Bohmian mechanics, a dynamic reinterpretation of quantum mechanics. Specifically, we show that the ponderomotive potential in the hydrodynamic system plays the role of the quantum corral in Bohmian mechanics.

John W. Bush
Massachusetts Institute of Technology
bush@math.mit.edu

MS13

New Regimes for Shallow Water Waves with Towed Topography

We revisit the seminal experiment of wave generation by flow over topography (Lee, Yates and Wu, 1989, Lee, 1985), whereby a shallow, steady current over a localized bottom bump can act as a periodic source of upstream running long waves. We explore new parametric regimes in the two-parameter space of incoming stream velocity (Froude number) and topography amplitude (ratio of bump height and undisturbed fluid depth) in our 27m wave tank with towed topography, and compare our data with numerical simulations and theoretical predictions. New experiments will be presented pushing further into the nonlinear/breaking wave regime and companion modeling explored. Time permitting, investigations with towed objects and their companion circulations will be discussed. NSF: DMS-2308063 ONR: N00014-23-1-2478 Sarah Steele Danhoff Undergraduate Research Fund administered by Honors Carolina

Joshua Carlson
UNC
jjcarl2@email.unc.edu

MS13

Wave Scattering on Quantum Graphs

Quantum Graphs is a research topic that has evolved to account for a broad class of problems, beyond quantum applications. It concerns metric graphs equipped with a differential operator along its edges. A central theme is solution-properties due to different graph topology, as well as different choices of compatibility conditions at the vertices. We consider water waves on channel-networks which allows for studying a wide range of quantum-graph wave scattering problems. For water waves it is natural to consider hyperbolic PDEs, as well as dispersive PDEs. Also, linear and nonlinear problems, where one can inquire about the persistence of coherent travelling-waves as they scatter, or not, at a vertex. We will briefly report on the only article we know carrying water-wave laboratory experiments in forked channels.

Andre Nachbin
Worcester Polytechnic Institute
anachbin@wpi.edu

MS13

Galloping Bubbles

In this talk, we introduce a new symmetry-breaking mechanism that enables bubbles to gallop" along horizontal surfaces in a vertically vibrated fluid chamber, propelling themselves without any net external forcing along their direction of motion. The resulting active bubbles exhibit a

rich variety of trajectory regimes including rectilinear, orbital, and run-and-tumble dynamics that can be tuned by adjusting the forcing parameters. We demonstrate how nonlinear coupling between shape oscillation modes leads to nonreciprocal body deformations that, together with inertial effects, generate self-propulsion. Proof-of-concept experiments illustrate the potential of this galloping mechanism for applications in bubble manipulation, transport, and sorting, as well as navigation through complex fluid networks and surface cleaning. We conclude by discussing prospects for collective behaviors in ensembles of galloping bubbles and the emergence of active foams.

Pedro Saenz
University of North Carolina at Chapel Hill
saenz@unc.edu

MS14

Evolution of Nonlinear Waves in Sea Ice

We investigate the attenuation of nonlinear ocean waves as they propagate into the Marginal Ice Zone (MIZ) using weakly nonlinear theoretical models, including dissipative nonlinear Schrödinger (NLS), Dysthe and Zakharov-type equations. We examine the evolution of random wave fields, modulational instability, breather dynamics, and related nonlinear phenomena in the presence of heterogeneous damping and forcing. This work is joint with Alberto Alberello (UEA).

Emilian I. Parau
University of East Anglia
e.parau@uea.ac.uk

MS14

Wave Solutions in the Viscous and Plastic Regimes in Continuous Viscous-Plastic Granular Sea Ice Models

This talk is about wave solutions in the viscous and plastic regimes in continuous viscous-plastic granular sea ice models and implication on their mode of fracture and well-posedness.

Bruno Tremblay
McGill University
bruno.tremblay@mcgill.ca

MS14

Computing Flexural-Gravity Waves with Varying Bathymetry

In this talk, we present three-dimensional nonlinear flexural-gravity waves propagating over localized bottom topography. The problem is reformulated using a desingularised boundary-integral approach in terms of surface variables and bathymetry modelled as either a bump or a crater. The resulting nonlinear system is discretised with finite differences and solved using a preconditioned Newton-Krylov method. Finally, results in several physically distinct parameter regimes are presented.

Olga Trichtchenko
Physics and Astronomy
The University of Western Ontario

otrlichtc@uwo.ca

MS14**Nonlinear Waves over Variable Bathymetry**

In this talk, we present a numerical method for studying water waves over variable bathymetry. This method utilizes the conformal mapping formulation of the equations of motion. We discuss two applications of the method: a periodic water wave propagating over a periodic bathymetry whose period is irrationally related to that of the wave, and a steady solitary wave over a bathymetry with a localized bump.

Xinyu ZhaoNew Jersey Institute of Technology
xinyu.zhao@njit.edu**MS15****Rolls and Homoclinic Snaking in a Swift-Hohenberg equation with Continuous Non-smooth Nonlinearity**

We study pattern formation in the one-dimensional Swift-Hohenberg equation, a classical model used to describe spatial patterns in many physical and biological systems. In the standard model the nonlinear terms are smooth. However, several models in applications contain nonlinearities that are continuous but not smooth. To capture this situation, we replace the usual quadratic-cubic nonlinearity with $\nu|u|^\alpha - |u|^\beta u$, where $\alpha \geq 1$ and $\beta \geq 0$. Because these terms are not smooth near $u = 0$, many standard analytical tools such as Taylor expansions and classical bifurcation theory cannot be applied directly. We investigate how this change affects the formation of spatial patterns. In particular, we study periodic roll patterns and spatially localized states organized by homoclinic snaking. Our numerical results show that the presence of non-smooth nonlinearities can significantly change how patterned solutions appear and evolve. In particular, we observe that the snaking structure of localized states can emerge discontinuously at $\alpha = 1$ (for example when $\beta = 2$ and $\nu > 0$).

Lutfiye Masur, Jens RademacherUniversity of Hamburg
luetfiye.masur@uni-hamburg.de, jens.rademacher@uni-hamburg.de**MS15****Amplitude Equations for a Turing-Hopf Instability with Conservation Laws and Advection**

Travelling waves occurring in MIN-D protein concentration of *E. coli* have been known to play a vital role in cell center localization during cell division. Building on the work of S. Meindlhummer et al. we investigate a simplified reaction-diffusion-advection model for reaction kinetics of the MIN proteins. Of particular interest are two conservation laws that lead to an infinite family of equilibrium states as well as some analytic and numerical difficulties. We numerically identify Turing-Hopf- and Eckhaus-type instabilities as well as travelling waves arising from them and derive an amplitude equation approximating the solutions close to the Turing-Hopf instability.

Sebastian Suckau

University of Hamburg

sebastian.suckau@uni-hamburg.de

MS15**Coarsening and wavelength selection in mass-conserving reaction-diffusion systems**

: Intracellular processes must be precisely organized in space and time. A paradigmatic example is the symmetric division of bacteria, which, in *Escherichia coli*, is spatially controlled by the ATP-driven oscillation of Min proteins between the cell poles. I will present the variety of reaction-diffusion patterns formed by such intracellular protein systems in vivo and in vitro. We will then discuss conceptual models and see how mass conservation of the protein species can be used to construct fully nonlinear patterns and predict their generic long-time dynamics independent of the specific mathematical form of the reaction term. We thereby uncover similarities of the reaction-diffusion patterns with phase-separating liquid mixtures and develop the concept of wavelength selection by interrupted coarsening.

Henrik WeyerUC Santa Barbara
U.S.
@tbd**MS16****Nonlinear Scalar Field Equations with a Critical Hardy Potential**

We study the existence of solutions for the nonlinear scalar field equation

$$-\Delta u - \frac{(N-2)^2}{4|x|^2} u = g(u), \quad \text{in } \mathbb{R}^N \setminus \{0\},$$

where the potential $-\frac{(N-2)^2}{4|x|^2}$ is the critical Hardy potential and $N \geq 3$. The nonlinearity g is continuous and satisfies general subcritical growth assumptions of the Berestycki-Lions type. The problem is approached using variational methods within a non-standard functional setting. The natural energy functional associated with the equation is defined on the space $X^1(\mathbb{R}^N)$, which is the completion of $H^1(\mathbb{R}^N)$ with respect to the norm induced by the quadratic part of the functional. We establish the existence of a non-trivial solution $u_0 \in X^1(\mathbb{R}^N)$ that satisfies the Pohožaev constraint \mathcal{M} and minimizes the energy functional on \mathcal{M} . Furthermore, assuming g is odd, we prove the existence of at least one non-radial solution. This is a joint work with D. Strzelecki. B. Bieganowski and D. Strzelecki: Nonlinear scalar field equations with a critical Hardy potential, arXiv:2511.15668

Bartosz BieganowskiUniversity of Warsaw
bartoszb@mimuw.edu.pl**MS16****Polychromatic Localized Waves in Nonlinear Maxwell Equations with Time Delayed Polarization**

We study the Maxwell equations with a cubically nonlinear and nonlocal in time dependence of the polarization on the electric field. In an essentially one-dimensional wave-guide geometry, we deduce a method to obtain so called polychromatic solutions with complex frequencies for the generally non-selfadjoint problem. These solutions are in the

form of a Fourier series of travelling waves along the waveguide, combined with a series over arbitrarily large decay rates in time. The leading frequency is given as a complex eigenvalue of the corresponding linear operator pencil and the leading order terms of the solution are given by a corresponding eigenfunction. Our method allows us to construct such polychromatic solutions by repeatedly solving *linear* ODEs under some assumptions on the spectrum and resolvent estimates of the linear operator. We also present specific, physical examples of material functions for which these assumptions are fulfilled.

Maximilian Hanisch

Martin Luther University Halle-Wittenberg
maximilian.hanisch@mathematik.uni-halle.de

Tomas Dohnal

Martin Luther University Halle-Wittenberg
Department of Mathematics
tomas.dohnal@mathematik.uni-halle.de

Runan He

Instituto de Ciencias Matemáticas
Madrid
runan.he@icmat.es

MS16

Time-periodic solutions to nonlinear Maxwell equations with retarded material laws

We consider Maxwell's equations in \mathbb{R}^3 without charges and currents. For material relations

$$\mathbf{B} = \mu_0 \mathbf{H}, \quad \mathbf{D} = \epsilon_0 (\mathbf{E} + \mathbf{P}(\mathbf{x}, \mathbf{E})),$$

modeling Kerr-type optical materials, with carefully chosen bounded inhomogeneous material coefficients, we show existence of breather solutions using variational methods. These are time-periodic, traveling, localized, real-valued, TE-polarized functions. We focus on the nonlinearity

$$\mathbf{P}(\mathbf{x}, \mathbf{E}) = g(x_1, t) *_t \mathbf{E} + h(x_1) (\nu(t) *_t \mathbf{E})^3$$

with $\mathbf{F}^3 |\mathbf{F}|^2 \mathbf{F}$, and find breathers as saddle points of the energy functional of a dual problem using the mountain pass method. This talk is based on the paper <https://doi.org/10.1007/s00030-026-01197-0> (Sebastian Ohrem; NoDEA 2026).

Sebastian Ohrem

TBD
sebastian.ohrem@kit.edu

MS16

Time-Periodic Waves for Maxwells Equations with Nonlinear Polarization

Maxwell's equations govern the propagation of electromagnetic waves in matter. For a class of materials (e.g. silica crystals) the refractive index changes in the presence of a sufficiently strong electric field E . In this talk I will consider a model for a class of materials with nonlinear polarization properties. In special wave-guide geometries we prove the existence of propagating time-periodic electromagnetic waves which are localized in directions orthogonal to the propagation direction. This problem leads to a quasilinear hyperbolic nonlinear partial differential equation

$$\nabla \times \nabla \times E + \epsilon_0 \mu_0 (E + P(E))_{tt} = 0$$

for the electric field E and the polarization $P(E)$, which includes linear and nonlinear parts. Solutions with the above properties (localized, time-periodic, propagating) are found by a variational principle. This is joint work with Sebastian Ohrem (KIT). References: Sebastian Ohrem and Wolfgang Reichel, Journal of Nonlinear Waves, 2025 Sebastian Ohrem and Wolfgang Reichel: <https://arxiv.org/abs/2407.18729>, to appear Calc. Var. PDEs. Sebastian Ohrem: <https://arxiv.org/abs/2508.20938>

Wolfgang Reichel

KIT, Department of Mathematics
wolfgang.reichel@kit.edu

Sebastian Ohrem

Karlsruhe Institute of Technology, Department of Mathematics
sebastian.ohrem@kit.edu

MS17

On the Well-posedness and Solitary Waves in Higher-order KdV-type Equations

We study the Cauchy problem for the fifth-order generalized Korteweg de Vries (5gKdV) equation with power-type nonlinearities, including the sublinear regime ($0 < \alpha < 1$). We prove local well-posedness for initial data in suitable weighted Sobolev spaces that encode spatial decay, and also show how the Kato smoothing-type estimates influence the choice of nonlinearities. These models arise in the study of weakly nonlinear dispersive waves, including plasma waves, gravity-capillary and capillary waves, and internal waves in stratified fluids. In addition, we perform numerical simulations to show solutions dynamics in this model.

Chandler Haight

Florida International University
chaig004@fiu.edu

MS17

Dispersive Regularization of Talanov Focusing in the AKNS Hierarchy

It was shown in the 1960s by Talanov that there exist solutions of the dispersionless focusing nonlinear Schrödinger equation with a parabolic amplitude profile that collapse and blow up in finite time. Accounting for the effect of small dispersion, Suleimanov conjectured in 2017 that the corresponding solution of the nonlinear Schrödinger equation should be regularized with a particular wave profile that was later characterized as a certain infinite-order limit of fundamental rogue waves. We describe a recent proof of a version of Suleimanov's conjecture and explain how it generalizes to the full AKNS hierarchy. This is joint work with Robert Buckingham and Robert Jenkins.

Peter D. Miller

University of Michigan
millerpd@umich.edu

MS18

The "good" Boussinesq Equation on the half-line with Robin boundary conditions

The "good" Boussinesq equation is a classical dispersive partial differential equation that arises in the modelling

of nonlinear water waves. In this talk, we consider this nonlinear model on the half-line equipped with Robin boundary conditions. Our objective is to establish local well-posedness of the corresponding initial-boundary value problem in the sense of Hadamard, i.e., to demonstrate existence, uniqueness, and continuous dependence on the initial data. We begin by deriving an explicit solution representation for the linearized problem by means of the unified transform (also known as the Fokas method) and then use this formula to establish rigorous estimates of Sobolev as well as Strichartz type for data belonging to suitable Sobolev spaces. These estimates are key to proving our local Hadamard well-posedness result through a contraction mapping argument.

Shivani Agarwal
University of Kansas
shivaniagarwal@ku.edu

MS18

Global Analytic Solutions of a Pseudospherical Novikov Equation

In this talk we consider a Novikov equation to extend some recent results of regularity of its solutions. By making use of the global well-posedness in Sobolev spaces, for analytic initial data in Gevrey spaces we prove some new estimates for the solution in order to apply the Kato-Masuda Theorem and obtain a lower bound for the radius of spatial analyticity. After that, we use embeddings between spaces to then conclude that the unique solution is, in fact, globally analytic in both variables.

Priscila L. Da Silva
Universidade Federal de São Carlos
pri.leal.silva@gmail.com

MS18

Discrete Nonlinear Schrödinger Versus Ablowitz-Ladik: Existence and Dynamics of Generalized NLS-Type Lattices over a Nonzero Background

The question of whether features and behaviors that are characteristic to completely integrable systems persist in the transition to non-integrable settings is a central one in the study of nonlinear evolution equations. This issue is closely related to the broader problem of the stability of evolution equations. Another fundamental question concerns the lifespan of solutions: whether it is infinite or finite distinguishes between global-in-time existence and instability phenomena, the latter manifested as blow-up in finite time. We examine these questions in the context of the Nonlinear Schrödinger Equation (NLS) and NLS-type lattices, supplemented with nonzero boundary conditions at infinity. Numerical investigations, based on high-accuracy schemes highlight the relevance of the accompanying mathematical analysis and yield numerical results in excellent agreement with theoretical predictions.

Dirk Hennig, Nikos I. Karachalios
Department of Mathematics
University of Thessaly
hennigd@physik.hu-berlin.de, karan@uth.gr

Dionyssis Mantzavinos
University of Kansas
mantzavinos@ku.edu

Dimitrios Mitsotakis

School of Mathematics and Statistics
Victoria University of Wellington
dimitrios.mitsotakis@vuw.ac.nz

MS18

A new approach for the analysis of evolution partial differential equations on a finite interval

We show that, for certain evolution partial differential equations, the solution on a finite interval $(0, \ell)$ can be reconstructed as a superposition of restrictions to $(0, \ell)$ of solutions to two associated partial differential equations posed on the half-lines $(0, \infty)$ and $(-\infty, \ell)$. Determining the appropriate data for these half-line problems amounts to solving an inverse problem, which we formulate via the unified transform of Fokas and address via a fixed point argument in L^2 -based Sobolev spaces, including fractional ones through interpolation techniques. We illustrate our approach through two canonical examples, the heat equation and the Korteweg-de Vries (KdV) equation, and provide numerical simulations for the former example. An important consequence of our approach is that spatial and temporal regularity estimates for problems on a finite interval can be directly derived from the corresponding estimates on the half-line. These results can, in turn, be used to establish local well-posedness for related nonlinear problems, as the essential ingredients are the linear estimates within nonlinear frameworks.

Turker Ozsari
Izmir Institute of Technology
tozsari@gmail.com

Dionyssis Mantzavinos
University of Kansas
mantzavinos@ku.edu

Konstantinos Kalimeris
Academy of Athens
kkalimeris@academyofathens.gr

MS19

How to determine the speed and amplitude of the leading edge of a dispersive shock wave

The aim of my talk is to analytically describe the solitary wave of largest amplitude in the dispersive shock that appears in the solution of the Riemann problem for dispersive equations describing long nonlinear dispersive waves, in particular, for the Benjamin-Bona-Mahony equation and the Serre-Green-Naghdi equations. Such a large-amplitude solitary wave is the leading wave of the corresponding dispersive shock. Its speed and amplitude are defined analytically through the solitary limit of the corresponding Whitham modulation equations. The numerical results are in accordance with the analytical prediction. **References** 1. 2022 S. Gavriluk and K.-M. Shyue, *Nonlinearity* 35 (2022) 388410 2. 2025 T. Congy, G. El, S. Gavriluk, M. Hofer and K.-M. Shyue, *Solitary wave-mean flow interaction in strongly nonlinear dispersive shallow water waves*, *J. Nonlinear Waves*, v. 1.

Sergey Gavriluk
University Aix-Marseille, UMR CNRS 7343 IUSTI

sergey.gavrilyuk@univ-amu.fr

MS19

On modulations of periodic wave trains

In this talk, we will discuss recent progress in describing the modulational stability and instability of periodic wave trains in nonlinear dispersive equations. Specifically, we are interested in connecting formal predictions from Whitham's theory of modulations to the rigorous spectral stability of the underlying wave. This is joint work with Jeffrey Oregero.

Mat Johnson

University of Kansas
matjohn@ku.edu

MS19

Numerical Computation of the Whitham Modulation Equations for General Dispersive Hydrodynamic Models

Unidirectional dispersive hydrodynamic models typically consist of a conservation law modified by a conservative integro-differential operator. Key features of nonlinear wave dynamics can be understood using Whitham modulation theory, which yields conservation laws that describe slow modulations of a periodic wavetrain. The Whitham modulation equations involve averages of periodic solutions that are generally not available in closed form. To make the modulation equations tractable, we employ two complementary approaches: a Stokes expansion to approximate small-amplitude periodic waves and numerical computation to approximate large-amplitude waves. These approximations allow us to investigate the modulational instability of periodic wavetrains. This approach will be used to identify MI in multiple dispersive hydrodynamic models: the Kawahara equation, the Whitham equation, and a semi-discrete conservation law.

Patrick Sprenger

Department of Applied Mathematics
University of California Merced
sprenger@ucmerced.edu

MS19

Modulation theory for a continuum dispersive approximation of FPU dynamics

We consider a dispersive quasicontinuum approximation of the Fermi-Pasta-Ulam problem governed by the modified regularized Boussinesq equation. We focus on the case of general cubic nonlinearity, which allows us to obtain explicit periodic traveling wave solutions and their solitary-wave and kink limits. We derive Whitham modulation equations and analyze their convexity, analytically in the solitary-wave and harmonic limits and numerically in the general case. In particular, we identify the parameter regions where the modulation system loses strict hyperbolicity, which leads to modulational instability of periodic traveling waves. The onset of modulational instability is verified by numerical computations of the linear spectra and initial value problems that also reveal additional short-wave instabilities.

Mark A. Hoefer

University of Colorado, Boulder
U.S.
hoefer@colorado.edu

Anna Vainchtein

Department of Mathematics
University of Pittsburgh
aav4@pitt.edu

MS20

Viscous Core-Annular Flows: A Laboratory Playground for Dispersive Hydrodynamics

Dispersive Hydrodynamics is a mathematical framework to describe multiscale nonlinear wave phenomena in dispersive media, including both dynamic and stochastic aspects of wave propagation. It is a vibrant and continuously developing field, with a variety of physical applications. This talk will present experiments at CU Boulder with a special core-annular flow of two viscous, miscible fluids with high viscosity contrast, one rising buoyantly within the other in order to highlight a rich variety of dispersive hydrodynamic phenomena including solitons, dispersive shock waves, breathers, and soliton gases. Along the way, Whitham modulation theory will be used to help describe the experimental results.

Mark A. Hoefer

University of Colorado, Boulder
U.S.
hoefer@colorado.edu

MS20

Embedded Solitary Waves in the stratified Euler equations: theory and experiments

The ocean and atmosphere are density stratified fluids. A wide variety of gravity waves propagate in their interior, redistributing energy and mixing the fluid, affecting global climate. Stratified fluids with narrow regions of rapid density variation (pycnoclines) support horizontally propagating internal gravity waves. We focus on mode-2 waves (typically) occur within the linear spectrum of mode-1 waves (i.e. the range of speeds of mode-2 waves is a subset of the range for mode-1 waves), and thus mode-2 solitary waves are generically not believed to exist due to a resonance with a mode-1 wave, resulting in non-decaying oscillatory behavior in the tails. We will show that these tail oscillations can be found to have zero amplitude, thus resulting in families of localized solutions (so-called embedded solitary waves) in the Euler equations. This is the first example we know of embedded solitary waves in the Euler equations. We will also show recent experimental evidence for these waves.

Paul Milewski

Penn State University
Dept of Mathematics
ppm5454@psu.edu

MS21

On the Long Time Well-Posedness of a Weakly Dispersive Green-Naghdi Model

Fully dispersive Whitham-Green-Naghdi systems describe the propagation of surface gravity waves in the shallow water regime. These models are nonlocal quasilinear hyperbolic systems that preserve the exact linear dispersion relation of the water waves equations while extending classical Green-Naghdi theory to weakly dispersive settings. We discuss the full justification of these systems as asymptotic models.

otic models of the water waves equations. In particular, we give rigorous consistency results at a nontrivial order of precision with respect to the shallowness, nonlinearity and bathymetry parameters. We also establish long-time well-posedness and convergence of solutions to those of the full water waves equations for a specific Whitham-Green-Naghdi model. These results provide new theoretical evidence that accurately capturing dispersive effects is essential for modeling wave propagation over uneven bathymetry.

Louis Emerald

Mathematics

Nazarbayev University, Kazakhstan

louisemerald76@gmail.com

MS21

Numerical Simulations of the Hamiltonian Dysthe Equation

We present numerical simulations of the Hamiltonian Dysthe equation for surface gravity waves on deep water in two and three dimensions. Special attention is paid to the time evolution of modulated Stokes waves and their Benjamin-Feir instability under sideband perturbations. Comparison with direct simulations of the Euler equations is shown. Extension to the 2D case with constant vorticity is also considered. In all these cases, an excellent agreement is found. This is joint work with A. Kairzhan and C. Sulem.

Philippe Guyenne

University of Delaware

guyenne@udel.edu

MS21

Hamiltonian Transformation Theory and Dispersive PDEs

In this talk, we present the derivation of a Hamiltonian Dysthe equation for the slowly varying envelope of modulated wavetrains based on a Hamiltonian formulation of the water wave problem and by applying techniques from Hamiltonian theory. The models we consider include 2d and 3d surface gravity waves and 2d water waves with constant vorticity. Our method provides a procedure to reconstruct the surface elevation from the wave envelope, based on the Birkhoff normal form transformation to eliminate all non-resonant triads. The talk is based on a series of works with Catherine Sulem (University of Toronto) and Philippe Guyenne (University of Delaware).

Adilbek Kairzhan

Nazarbayev University

akairzhan@nu.edu.kz

MS21

Complexity in Beyond-All-Orders Analysis of Water Waves: from Hybrid Numerical-Asymptotics to Exotic Stokes Lines

Here, I will discuss the efforts of those in the exponential-asymptotics or beyond-all-orders community to study those specialised problems in water waves where exponentially-small effects govern crucial phenomena in some singular limit. Classically, these have involved the study of generalised gravity-capillary solitary waves and wave-structure interactions at low Froude or Bond numbers. However, complexity can be introduced through extensions to full water-wave equations, three-dimensionality,

and through a variety of problem geometries. The introduction of physical complexity can often result in strikingly complex asymptotic structure as well; although exponential smallness might be expected in a variety of problems, the lack of mathematical techniques for resolving the analysis remains a bottleneck. I discuss recent work on water waves past submerged curved structures, and parasitic capillary ripples on steep gravity waves.

Philippe Trinh

University of Bath

p.trinh@bath.ac.uk

MS22

On Population Dynamics Models Including Various Diffusion Strategies in Heterogeneous Environments: Influence on Average Population Levels and Competition Outcomes

In 2006, Lou proved that, once the intrinsic growth rate r in the logistic model is proportional to the spatially heterogeneous carrying capacity K ($r = K^1$), the total population under the regular diffusion exceeds the total of the carrying capacity. He also conjectured that the dependency of the total population on the diffusion coefficient is unimodal, increasing to its maximum and then decreasing to the asymptote which is the total of the carrying capacity. DeAngelis et al (2016) argued that the prevalence of the population over the carrying capacity is only observed when the growth rate and the carrying capacity are positively correlated, at least for slow dispersal. Guo et al (2020) justified that, once r is constant ($r = K^0$), the total population is less than the cumulative carrying capacity. Our paper fills up the gap for when $r = K^\lambda$ for any real λ . We define a diffusion strategy as the tendency to have a distribution proportional to a certain positive prescribed function, once a diffusion coefficient grows infinitely, and explore the interplay of harvesting and dispersal strategies and their influence on the outcome of the competition for two resource-sharing species. While achieving extinction by excessive culling of the undesired species is simple and efficient, keeping biodiversity is a more complicated task. Proposing such heterogeneous harvesting that the two populations become an ideal free pair allows to guarantee coexistence.

Elena Braverman

Department of Mathematics & Statistics

University of Calgary

maelena@math.ucalgary.ca

Jennifer Lawson, André Rickes

University of Calgary

jennifer.lawson@ucalgary.ca, andre.rickes@ucalgary.ca

MS22

Animal Movement Ecology: "Surfing" Seasonal Green Waves

Seasonal changes in vegetation greenness are real-world examples of wave-like phenomena that play out over vast portions of the earth's surface every year, across both latitudes and elevation gradients. Many consumer species track such spatial translation of transient resources, moving so as to take advantage of pulses of resource quality that are only available temporarily in any one location. Species (or individuals) that consistently match their spatial location to optimal feeding conditions are said to 'surf the green wave', whereas others will either jump ahead of or lag behind the

green wave. The degree of (mis)match of consumers to the resource wave is of interest to biologists, who seek to understand the contributions of such factors as individual and collective experience, mobility constraints, stochastic external forcing (e.g., precipitation), and landscape complexity. The underlying wave-like dynamics play out over a range of spatial and temporal scales, and examples with diverse topologies are known, including closed loops. I will discuss efforts to model such dynamics from a mathematical perspective, and demonstrate the key role that empirical animal movement data plays in efforts to decode the biological and environmental drivers of green waves and green wave surfing.

Bill Fagan
University of Maryland College Park
bfagan@umd.edu

MS22

Emergence and Bistability of Traveling Waves and Wave-Pinning States in a Mass-Conserved Reaction-Diffusion System

The transition from random walk to directional motion (and back) is one of the intriguing phenomena observed in motile eukaryotic cells. However, typically, theoretical studies distinguish between the two phenomena and focus either on dissipative models or models obeying gradient flows, respectively. Using a three-variable dissipative reaction-diffusion system with mass conservation, we show how pulses, waves, fronts, and (stationary) mesas generically organize about high codimension bifurcations. Specifically, we demonstrate the novelty of mass conservation, which enters via a long-wavenumber bifurcation of a large-scale mode. Lastly, following the biological interest, we address the bistability between traveling wave and wave-pinning solution branches, which emerge from a codimension-2 bifurcation to a finite wavenumber Hopf and a conserved large-scale mode.

Jack Hughes
University of British Columbia
jack.hughes@mail.concordia.ca

Saar Modai
Department of Physics
Ben-Gurion University
modais@post.bgu.ac.il

Leah Edelstein-Keshet
University of British Columbia
Department of Mathematics
keshet@math.ubc.ca

Arik Yochelis
Swiss Institute for Dryland Environmental & Energy Research
Ben-Gurion University
yochelis@bgu.ac.il

MS22

Rogue-like waves in an enzymatic activator-inhibitor-substrate system: Stochastic output from deterministic dynamics

Rogue waves are large excitations that appear intermittently and unpredictably, arising across different scales, ranging from ocean waves through optics to Bose-Einstein condensates. Motivated by spatiotemporal dynamics in a

model for branching, I will describe the emergence of rogue wave-like dynamics in an enzymatic activator-inhibitor-substrate system that arise as a result of a subcritical Turing instability and in the absence of an oscillatory instability. This state is present in the regime where all time-independent states are unstable, and consists of intermittent excitation of spatially localized spikes, followed by collapse to an unstable state and subsequent regrowth. Characterization of the spatiotemporal organization of spikes shows that in sufficiently large domains, the dynamics are consistent with a memoryless process. Since the Turing instability is a generic pattern-forming instability of reaction-diffusion models, the results reveal a generic mechanism that sheds fresh light on time-dependent patterns in physicochemical and biological applications.

Arik Yochelis
Swiss Institute for Dryland Environmental & Energy Research
Ben-Gurion University
yochelis@bgu.ac.il

Edgar Knobloch
University of California at Berkeley
Dept of Physics
knobloch@berkeley.edu

MS23

Wave-Packets at a Material Interface in Nonlinear Maxwell Equations

We consider the propagation of asymptotically small wave-packets in Maxwell equations along the interface of two nonlinear materials. A rigorous justification of amplitude asymptotics is provided. In the first part we study the case of non-dispersive media (instantaneous response) with x -dependent Kerr nonlinear materials in each half space in two spatial dimensions. The solution of the second order equation for the electric field is found in $\cap_{j=0}^3 C^j([0, T], H^{3-j}(\mathbb{R}_+^2) \oplus H^{3-j}(\mathbb{R}_-^2))$. The wave-packet is approximated by the classical nonlinear Schrödinger asymptotics. In the second part we study dispersive (delayed response) media in a cylindrical geometry $\Omega \subset \mathbb{R}^3$. Here Maxwell equations include a memory term. The wave-packet asymptotics are approximated by the complex Ginzburg-Landau equation. Employing the abstract method of evolution equations, we study the Maxwell solution in $L_\rho^2(\mathbb{R}, L^2(\Omega)^6)$, where ρ denotes an exponential weight in time. This work is in collaboration with Roland Schnaubelt (KIT, Germany), Daniel P. Tietz (OIST, Japan), Mathias I. Tira (independent, Germany), and Markus Waurick (TU Freiberg, Germany).

Tomas Dohnal
Martin Luther University Halle-Wittenberg
Department of Mathematics
tomas.dohnal@mathematik.uni-halle.de

Roland Schnaubelt
Department of Mathematics
Karlsruhe Institute of Technology
schnaubelt@kit.edu

Daniel Paul Tietz
Okinawa Institute of Science and Technology
daniel.tietz@oist.jp

Mathias Ionescu Tira
independent researcher

mathias.tira@mail.de

Markus Waurick
TU Freiberg
marcus.waurick@math.tu-freiberg.de

MS23

Modified Scattering for the Three Dimensional Maxwell-Dirac System

In this talk, we discuss global well-posedness and modified scattering for the massive Maxwell-Dirac system in the Lorenz gauge in $(1+3)$ -dimensional spacetime for small, sufficiently smooth and decaying initial data. Our approach both exploits the close connection of the massive Maxwell-Dirac system with the wave-Klein-Gordon equations and specific structural properties of the Dirac equation. The modified scattering result follows from a precise description of the asymptotic behavior of the solution inside the light cone, which we derive via the method of testing with wave packets of Ifrim-Tataru.

Sebastian Herr
Bielefeld University
herr@math.uni-bielefeld.de

Mihaela Ifrim
University of Wisconsin - Madison
ifrim@wisc.edu

Martin Spitz
Bielefeld University
mspitz@math.uni-bielefeld.de

MS24

The Riemann-Hilbert Problem for the Semiclassical Defocusing Nonlinear Schrödinger Equation with Nonzero Boundary Conditions

We study solutions to the nonlinear defocusing Schrödinger equation with nonzero boundary conditions in the semiclassical limit by analyzing the associated Riemann-Hilbert problem. This builds on the work of [Jin, S.; Levermore C. D.; McLaughlin, D. W. The behavior of solutions of the NLS equation in the semiclassical limit, 1999], who studied the problem by the Lax-Levermore method. We discuss recent developments in strengthening the analysis in order to capture the so-called microstructure of the solutions.

Joanne Dong
University of Michigan
jtdong@umich.edu

MS24

The small-dispersion limit of the KdV equation with sech^2 initial data: genus one

The Korteweg-de Vries equation is perhaps one of the most studied integrable nonlinear PDE since the discovery of its solution by inverse scattering transform in 1967. In the time since, several results have been obtained regarding the small-dispersion limit, $\epsilon \rightarrow 0^+$. Examples include Lax and Levermore's convergence in a weak- L^2 sense for semiclassical soliton ensembles or Claeys and Grava's confirmation, in the case with no solitons, of Dubrovin's conjecture regarding universal behavior at the point of gradient catastrophe. The current work seeks to close a gap in the literature by considering an initial condition which contains

an increasing number of solitons in the small-dispersion limit. We use Riemann-Hilbert analysis and the method of nonlinear steepest descent to construct asymptotic formulae for the solution which are uniformly valid as $\epsilon \rightarrow 0^+$ on compact sets of spacetime. This talk will focus on the analysis in the genus one region.

Robert J. Buckingham
Dept. of Mathematical Sciences
The University of Cincinnati
buckinrt@uc.edu

Kurt Schmidt, Matthew Mitchell
University of Central Florida
kurt.schmidt@ucf.edu, matt.mit@ucf.edu

MS24

Small Dispersion KdV with Positive Initial Data in the Genus Zero Case

n/a

Kurt Schmidt, Matt Mitchell
University of Central Florida
kurt.schmidt@ucf.edu, matt.mit@ucf.edu

Robert Buckingham
University of Cincinnati
buckinrt@ucmail.uc.edu

MS25

Stability of Largely Modulated Reaction-Diffusion Wave Trains Against Cub(R) -Perturbations

We study reaction-diffusion systems on the extended real line admitting a periodic traveling-wave solution ϕ_0 which is diffusively spectrally stable. Having a solution u and a phase modulation ψ at hand, we focus on the long-standing question of how the modulated perturbation $u(\cdot, t) - \phi_0(\cdot + \psi(\cdot, t))$ with respect to suitable norms and initial conditions evolves in time. In my presentation, I will give an overview of existing answers and present our new result, outlining the main aspects and challenges of its proof. The essential extension consists of lifting any localization requirement on the phase modulation, the initial perturbation and their derivatives, as well as removing the smallness assumption on $\|\psi(\cdot, t)\|_{L^8}$. Our method is robust and we expect that it can be applied to other semilinear (non-parabolic) systems.

Joannis Alexopoulos
Karlsruhe Institute of Technology
USA
joannis.alexopoulos@kit.edu

MS25

A General Approach for Proving the Symmetry of Localized Patterns in a Class of Pdes Posed on Unbounded Domains

In this talk, we will show a general method for constructively proving the existence of localized patterns in a class of semilinear autonomous partial differential equations (PDEs) posed on unbounded domains along with any symmetry they may possess. We will summarize our main approach for proving solutions on unbounded domains. We use a Newton-Kantorovich argument involving quantities to estimate partially by hand and partially on the com-

puter. This makes our approach computer assisted. Following this, we will discuss previous methods that use computer assisted proofs (CAPs) for proving certain symmetries of solutions using Fourier series. Combining these methods can lead to proofs of some symmetries of localized patterns, but not all possible symmetries. The goal of this talk is to bridge this gap and demonstrate our general approach for proving any symmetry. This will include the construction of the approximate solution, approximate inverse, and the necessary Newton-Kantorovich argument. We will use dihedral symmetries in the 2D Swift Hohenberg PDE as our example. To conclude, we will discuss future improvements we would like to investigate with regards to the method.

Dominic Blanco

McGill University

dominic.blanco@mail.mcgill.ca

MS25

Exponential Dichotomies and Their Application to the Construction of Localized Non-Symmetric Patterns in Higher Dimensions

Exponential dichotomies, when they exist, provide powerful information about the structure of bounded solutions even in the case of an ill-posed evolutionary equation. The method of spatial dynamics, in which one views a spatial variable as a time-like evolutionary variable, allows for the use of classical dynamical systems techniques, such as exponential dichotomies, in broader contexts. This has been utilized to study stationary solutions of PDEs on spatial domains with a distinguished unbounded direction (e.g. the real line or a channel of the form $\mathbb{R} \times \Omega$). Recent work has shown how to extend the spatial dynamics framework to elliptic PDEs posed on general multi-dimensional spatial domains. In this talk we show that, in the same context, exponential dichotomies do exist, thus allowing for their use in future analyses of coherent structures, such as spatial patterns in reaction-diffusion equations on more general domains.

Alanna Haslam-Hyde

Department of Mathematics and Statistics

Boston University

ajhaslam@bu.edu

Margaret Beck

Boston University, U.S.

mabeck@bu.edu

Ryan Goh

Boston University

Dept. of Mathematics and Statistics

rgoh@bu.edu

MS25

The Dynamics of Travelling Fronts Between Spatially Heterogeneous Background States

We study travelling fronts in a two-component reaction-diffusion system with a slow-fast structure and spatially varying coefficients appearing in the slow equation. In particular, we do not assume these spatial variations to be small. We show that the system exhibits bi-stability in the form of two stable stationary heterogeneous background states, which are connected by stationary and travelling front solutions. The front interface moves with a non-uniform speed. Therefore, unlike classical travelling waves,

these fronts are not stationary in any co-moving frame. We develop a non-compact extension of Fenichel theory to construct both the background states and stationary fronts; in addition, we establish the existence of travelling front solutions, and derive a leading-order expression for the dynamic position of the moving interface through a non-autonomous spatial dynamics approach. This dynamic interface equation takes the form of a delay-differential equation, and its accuracy is validated through numerical simulations. Our derivation of the dynamic interface equation circumvents the traditional reliance on spectral analysis, enabling us to describe front dynamics beyond bifurcations from stationary fronts. This approach has the potential to be extended to other settings in which spectral properties at onset preclude conventional reduction techniques.

Frits Veerman, Lara van Vianen

University of Leiden

The Netherlands

f.w.j.veerman@math.leidenuniv.nl,

l.a.van.vianen@math.leidenuniv.nl

Martina Chirilus-Bruckner

University of Leiden

Mathematical Institute

m.chirilus-bruckner@math.leidenuniv.nl

MS26

Stochastic inverse scattering transform and the probability density function in semiclassical integrable turbulence

We present a general analytical approach for determining the probability distribution of random wave fields governed by the focusing nonlinear Schrödinger equation (fNLSE) in regimes where typical realisations are dominated by the solitonic spectral component, enabling a soliton gas description. We introduce a stochastic analogue of the inverse scattering transform by establishing a direct link between the spectral density of states of the underlying bound-state composite soliton gas and the probability density function (PDF) of the turbulent wave-field intensity. The resulting explicit integral representation for the PDF is shown to be in excellent agreement with direct numerical simulations across several representative examples of fNLSE integrable turbulence. It also naturally incorporates recent theoretical results on kurtosis doubling during the evolution of semiclassical random fields. The obtained results have broad applications in water waves, nonlinear optics, and superfluids.

Thibault Congy

School of Engineering, Physics and Mathematics

Northumbria University

thibault.congy@northumbria.ac.uk

Gennady El

Northumbria University

gennady.el@northumbria.ac.uk

MS26

Perturbed Nonlinear Evolution of Soliton Gases: Recent Experiments in Optics, Hydrodynamics and Electrical Lines

The study of nonlinear random wave fields in physical systems described at leading order by integrable equations is addressed within the framework of 'integrable turbulence,' a concept introduced by Zakharov in 2009 [1]. A central

element of integrable turbulence is the concept of soliton gases, which refers to large ensembles of solitons with random characteristics. In this presentation, I will discuss recent experimental findings related to integrable turbulence, with a specific focus on the topic of soliton gases propagating in nonlinear electrical transmission lines [2] and in optical fibers [3]. [1] V. E. Zakharov, *Stud. Appl. Math.* **122**, 219 (2009)
 [2] L. Fache et al, *Phys. Rev. Lett.* **134**, 147201 (2024)
 [3] L. Fache et al, *Phys. Rev. Lett.* **135**, 157201 (2024)

Stephane Randoux, Loic Fache, Hervé Damart
 Université de Lille
 stephane.randoux@univ-lille.fr,
 loic-joseph.fache@u-pariscite.fr, herve.damart@univ-lille.fr

Felicien Bonnefoy, Guillaume Ducrozet
 Ecole Centrale de Nantes, France
 felicien.bonnefoy@ec-nantes.fr,
 guillaume.ducrozet@ec-nantes.fr

Filip Novkoski, Guillaume Ricard
 Université Paris Cité, France
 filip.novkoski@uliege.be, g.ricard@tudelft.nl

Eric Falcon
 MSC CNRS University Paris Diderot
 eric.falcon@univ-paris-diderot.fr

Giacomo Roberti
 Northumbria University
 g.roberti@northumbria.ac.uk

Thibault Congy
 School of Engineering, Physics and Mathematics
 Northumbria University
 thibault.congy@northumbria.ac.uk

Thibault Bonnemain
 Université Cergy-Paris
 thibault.bonnemain@cyu.fr

Francois Copie
 Université de Lille, France
 francois.copie@univ-lille.fr

Gennady El
 Northumbria University
 gennady.el@northumbria.ac.uk

Pierre Suret
 Université de Lille, France
 pierre.suret@univ-lille.fr

MS26

New Results in 2D Soliton Gas Theory

We study two-dimensional stationary and non-stationary soliton gases in the framework of the Kadomtsev-Petviashvili II (KP-II) equation. For the stationary case, we exploit the reduction to the integrable 'good' Boussinesq equation to construct kinetic equations invoking recent results on bidirectional soliton gases and generalised hydrodynamics. The kinetic theory is then extended to the non-stationary non-resonant case. Both frameworks are applied to describe key 2D soliton gas interactions: refraction of a line soliton by a soliton gas, and oblique interference of two soliton gases. All analytical predictions are

verified numerically via exact N-soliton solutions, with N large, of KP-II. These results apply to a variety of physical systems from shallow water waves to Bose-Einstein condensates.

Giacomo Roberti
 Northumbria University
 g.roberti@northumbria.ac.uk

Thibault Bonnemain
 Université Cergy-Paris
 thibault.bonnemain@cyu.fr

Gennady El
 Northumbria University
 gennady.el@northumbria.ac.uk

Benjamin Doyon
 King's College London, UK
 benjamin.doyon@kcl.ac.uk

Gino Biondini
 State University of New York at Buffalo
 Department of Mathematics
 biondini@buffalo.edu

MS26

Soliton Gas Computations and Orthogonal Polynomials

A condensate formed from a large number of solitons poses interesting numerical challenges. The pure soliton computation, with a large number of solitons, can be tamed up to a threshold, after which ill-conditioning and complexity scaling presents serious computational barriers. The Riemann–Hilbert formulation can push this threshold further, but it eventually suffers from the same fate as other methods. Motivated by the work of Dyachenko, Zakharov and Zakharov (2016) on primitive potentials, and that of Girotti, Grava, Jenkins, McLaughlin and Minakov (2021, 2023), we present a numerical method to evaluate soliton gas primitive potential. This method is then extended to evaluate the long-time evolution efficiently, without the need for high-precision arithmetic. The numerical method presented has strong connections to the theory of orthogonal polynomials.

Thomas Trogdon
 University of Washington
 Department of Applied Mathematics
 trogdon@uw.edu

Cade Ballew
 University of Washington
 ballew@uw.edu

Deniz Bilman
 University of Cincinnati
 bilman@uc.edu

MS27

Data-Driven Modeling of Sargassum Motion

Since 2011, rafts of floating Sargassum seaweed have obstructed coasts in the Intra-Americas Seas. Their motion is modeled by a high-dimensional nonlinear dynamical system, the eBOMB model, which extends the Maxey-Riley equation by incorporating interactions between clumps of

Sargassum within a raft. In practice, only the motion of raft centers of mass is needed, but the corresponding closed-form law remains unknown, motivating the use of machine learning to derive a low-dimensional surrogate. We explore and compare Long Short-Term Memory (LSTM) Recurrent Neural Networks and Sparse Identification of Nonlinear Dynamics (SINDy), both used as physics-informed closure models grounded in eBOMB. The LSTM learns a mapping from selected eBOMB variables to the difference between the raft center-of-mass velocity and the surrounding ocean velocity. The SINDy approach employs a library of candidate functions built from eBOMB variables, augmented with windowed velocity terms to represent far-field flow effects. Both approaches achieve higher accuracy for tightly connected rafts and degrade in performance for more complex configurations involving wind forcing and loosely connected rafts. LSTMs work well with relatively simple architectures, needing few neurons and layers. The use of windowed velocity terms in SINDy, which explicitly identifies functional dependencies, improves the representation of nonlocal interactions with sparsely connected rafts. Joint work with Gage Bonner.

Francisco J. Beron-Vera
RSMAS, University of Miami
fberon@miami.edu

MS27

Particle Trajectories in Nonlinear Schrödinger Models

The nonlinear Schrödinger equation (NLS) is well known as a universal equation in the study of wave motion. In the context of wave motion at the free surface of an incompressible fluid, NLS accurately predicts the evolution of modulated wave trains with low to moderate wave steepness. In this talk, we reconstruct the velocity potential and surface displacement from NLS coordinates in order to compute particle trajectories in physical coordinates. We use these particle trajectories to compute the mean transport properties of modulated wave trains. Additionally, we present particle trajectories and mean transport properties for the Dysthe equation and two dissipative generalizations of NLS.

John Carter
Seattle University
Mathematics Department
carterj1@seattleu.edu

MS27

Particle Kinematics in Deep-Water Finite Amplitude Waves with and Without Vorticity

We present particle kinematics in steep and overturning deep-water surface gravity waves using theoretical and numerical analysis. Specifically, exact solutions are presented that capture not only the geometry of the "barrel" of the overturning wave, but also the kinematics in this highly dynamic region. The theoretical predictions are then compared with numerical simulations of the fully nonlinear potential free surface flow equations, and agreement is shown. Generalizations of these results to waves with vorticity are then discussed.

Nicholas Pizzo
Graduate School of Oceanography
University of Rhode Island

nicholas.pizzo@uri.edu

MS27

Settling of Porous Objects: a Model of Marine Snow

Settling of Porous Objects: a Model of Marine Snow Marine snow refers to biological detritus that gradually settles from the upper ocean to depth, constituting a crucial branch of the carbon cycle - providing nutrients to deep-sea organisms and permanently removing carbon from the climate system. More so than snow flakes in the atmosphere, marine snow has a wide variety of complex shapes and sizes, making prediction of their settling speed a challenge. Heuristic attempts have used the Stokes settling speed, which approximates the objects as spheres with some estimated density and radius. However, marine snow is far from spherical: it is porous and composed of thread- and plate-like elements with varying density. As a step toward improving models of the settling of porous objects, we perform laboratory experiments of settling meshes and spiky objects. These show that even with relatively low porosity, ambient fluid passes predominantly through, rather than around, the objects. To develop better insight, we perform numerical simulations of the experiments, modelling the objects by a descending mask within which the ambient flow speed adjusts to the speed of the mask on a specified relaxation time scale. Thus a longer time scale in effect makes the mask more porous. Simulations show that this approach can reliably capture the observed settling dynamics.

Bruce Sutherland
University of Alberta
bsuther@ualberta.ca

MS28

Traveling Waves Solutions of An Epidemiological Model

In a diffusive epidemiological model with saturating treatment, we demonstrate the existence of traveling waves. When the diffusion rate of the infected population is significantly larger than the susceptible population, we show that the underlying dynamics of these patterns is governed by a modified version of the Burgers-Huxley equation.

Vahagn Manukian
Miami University Hamilton
manukive@miamioh.edu

MS29

Shilnikov-Type Outbreaks in Two-Strain Epidemic Models

The interplay between co-circulating pathogen strains with partial cross-immunity is a known driver of recurrent epidemic waves, yet the nonlinear mechanisms generating large-scale periodic outbreaks remain opaque. This talk presents a novel dynamical framework for understanding these oscillations in two-strain systems. By analyzing a tractable family of models where secondary infection with one strain is excluded, we derive explicit stability thresholds and reveal that instability arises when the recovery rate from secondary infections falls below a critical value. Crucially, we identify Shilnikov saddle-focus loops as a robust mechanism driving short, intense outbreaks. We prove that single-strain endemic equilibria become saddle-foci

satisfying the Shilnikov condition for chaos. The systems trajectory is characterized by a slow spiral into a single-strain equilibrium, followed by a rapid ejection along an unstable manifold that triggers a sharp outbreak of the competing strain. These findings persist in extended models incorporating waning immunity and quarantine, shifting the analytical focus from coexistence equilibria to the global dynamics near single-strain states

Nir Gavish

Faculty of Mathematics
Technion IIT
ngavish@technion.ac.il

MS29

Wave Propagation in Dense Cell Cultures: Expansion of a Monolayer

In this talk, we consider an expansion of a dense monolayer of cells: a collective multicellular phenomenon, where cells divide, grow, and maintain contacts with their neighbors. During migration, cells display complex behavior, adjusting both their division rate and their growth after division to the local mechanical environment. Experimental observations show that cells near the edge of the expanding monolayer are larger and move faster than cells deep inside the colony. To explain these observations and describe cell migration patterns, we formulate a spatiotemporal theoretical model for multicellular dynamics in terms of the cell area distribution [E. Khain, J. Straetmans, Dynamics of an expanding cell monolayer, *J. Stat. Phys.* 184:20 (2021)], the model includes cell growth after division and effective pressure. Numerical simulations of the model predict both the speed of invasion and the width of the outer proliferative rim. Theoretical analysis yields the equation for density of cells and reveals a novel type of propagating front with compact support. The velocity of front propagation (monolayer expansion) is obtained analytically and its dependence on all relevant parameters is determined. The theoretical and numerical results are in a good agreement with experimental observations of expanding monolayers [E. Gauquelin, S. Tlili, C. Gay et al., Influence of proliferation on the motions of epithelial monolayers invading adherent strips, *Soft Matter* 15:2798 (2019)].

Evgeniy Khain

Department of Physics, Oakland University
khain@oakland.edu

MS29

Dynamics of Phenotypic Transitions and Cueing Fronts in Water-Stressed Vegetation

Plants exhibit a remarkable ability alter their phenotype to adapt to fluctuating environmental stresses. Understanding the dynamics of these phenotypic changes is crucial for assessing plant resilience to severe droughts and predicting the risk of ecosystem collapse. We introduce a general model for phenotypic transitions in water-stressed vegetation, motivated by analyzing an example of phenotypic stomatal closure. The model's bifurcation structure resembles those found in spatial patterning contexts and consequently shares similar behaviors concerning resilience and tipping to dysfunctional states. We present phase diagrams mapping these behaviors across parameters defining environmental stress (intensity, time scales of appearance and duration) and human intervention and discuss the implications for managing ecosystems at risk of collapse. In bistability ranges of stress-tolerant and stress-

intolerant phenotypes, the model predicts the existence of cueing fronts, where information about impending stress is propagated in space and elicits phenotypic changes in nearby unstressed vegetation. We discuss our model's relevance to recent experimental findings on information propagation across plant populations and conclude with a discussion of multilevel responses involving both plant-level phenotypic changes and population-level spatial patterning.

Ehud Meron

Ben-Gurion University of the Negev
ehud@bgu.ac.il

MS29

Breaking Synchrony in Strongly Nonlinear Ecological Oscillations

We investigate cyclic dominance models and their extensions to both network systems and reaction-diffusion frameworks. Through linear stability analysis, we establish the relationship between the stability of synchronized states in networked systems and the response of homogeneous solutions subjected to spatially periodic perturbations. Furthermore, we explore the mathematical properties of networks characterized by strong nonlinear oscillations in an ecological context. Finally, we present numerical results for the master stability function of a competitive three-species Lotka-Volterra model, highlighting its role in understanding the dynamics of cyclic competition.

Idan Sorin, Alexander Nepomnyashchy

Technion- Israel Institute of Technology
idansorin@campus.technion.ac.il, nepom@technion.ac.il

MS30

Analysis of Models of the Motion of Flame Fronts

The Kuramoto-Sivashinsky equation is a model of the motion of flame fronts, and is well-known for exhibiting chaotic dynamics. It can be derived as a weakly nonlinear asymptotic model of a coordinate-free model of flame fronts, with the coordinate-free model being closer to physical principles. We will describe analytical results for the two-dimensional Kuramoto-Sivashinsky equation and for the one-dimensional and two-dimensional coordinate-free models. This includes global existence results for Kuramoto-Sivashinsky. For the coordinate-free models, this includes validation of Kuramoto-Sivashinsky as an asymptotic model, existence of traveling waves, well-posedness of the initial value problem, and a non-stiff numerical method for the initial value problem. A number of ideas from fluid dynamics, including the non-stiff numerical method of Hou, Lowengrub, and Shelley for vortex sheets with surface tension, are applied to the coordinate-free model. This includes joint work with Sultan Aitghan, Benjamin Akers, Fazel Hadadifard, Shunlian Liu, Anna Mazzucato, and J. Douglas Wright.

David Ambrose

Drexel University
dma68@drexel.edu

MS30

Integral Asymptotics, Coalescing Saddles, and Multiple-Scales Analysis of a Generalised Swift-Hohenberg Equation

Modelling interfaces in nonequilibrium thermodynamics

typically relies on either sharp or diffuse interface descriptions. Both assume local equilibrium in the bulk, but diffuse models additionally impose a “superlocal equilibrium” condition within the interface itself. Instead of validating these approaches against molecular simulations, this talk examines their mutual compatibility: can diffuse models truly be viewed as a more detailed counterpart of sharp ones? Using a diffuse model based on van der Waals entropy and Cahn-Hilliard energy, together with a corresponding sharp-interface formulation with interfacial state variables, we test their consistency under nonequilibrium conditions. A key prediction of diffuse models—equality of several interfacial temperatures—fails when diffuse-model data are interpreted through sharp-interface theory, in contrast with molecular simulations. This inconsistency points to the breakdown of the superlocal equilibrium assumption, most clearly reflected in the accessibility of the entropy density profile. Recent evidence suggests that sharp interface models align more reliably with molecular data. In this sense, sharp interfaces are found to be superior to diffuse interfaces in their general ability to model physical systems with interfaces. The talk is based on Klika, V., & Öttinger, H. C. (2024). On the Compatibility of Sharp and Diffuse Interfaces Out of Equilibrium. *Multiscale Modeling & Simulation*, 22(1), 369-405.

Václav Klika

Czech technical university of Prague
vaclav.klika@cvut.cz

MS30

Interfaces As Dimensional Reduction, An Application to Sharp Fronts in Reaction Diffusion Systems

It is a notoriously hard problem to classify all behaviors of a partial differential equation, nevertheless given certain regimes or conditions it might be that the dimensionality of the problem is way smaller than what the full PDE would suggest. For instance we shall discuss how in a class of singular bistable systems of reaction diffusion equations there is a regime described by interface interaction equations, which are in one spatial dimension typically ordinary differential equations. The main focus will converge to the techniques involved in the dimensional reduction from the infinite dimensional partial differential equation to the finite dimensional interface interaction equations followed by a brief discussion of validity arguments for such equations.

Tommaso Lamma

Leiden University
t.lamma@math.leidenuniv.nl

MS30

Relativistic Effective Actions for Domain Walls and Strings

We will explain a systematic method to derive the effective action for domain walls and local strings directly from the field theory that gives rise to these solitonic solutions. The effective action for the Goldstone modes, which characterize the solitons position, is shown to consist of the Nambu-Goto action supplemented by a finite set of higher-order curvature invariants associated to the worldvolume metric. Additionally, we will show how the method can be extended to include bound modes as scalar fields on the worldvolume, and to derive their couplings to the geometric invariants of the worldvolume geometry. A significant consequence of this coupling is the emergence of a parametric instability, driven by interactions between the bound states and the

Goldstone mode.

José M. Queiruga
University of Salamanca
xose.queiruga@usal.es

Alberto G. Martin Caro
University of Vigo
agmcaro@gmail.com

MS31

Transfer operator methods for learning persistent and transient structural information from dynamic data

I will describe recent advances in transfer operators and machine learning to extract actionable information from dynamical systems and dynamic data sources. I will draw on recent work concerning (i) transient almost-invariant sets and coherent sets, which allow deviation from purely Lagrangian motion to detect dynamic regime changes and coherent structure merging and breakup, (ii) data-driven methods for optimising dynamical system perturbations based on optimal linear response theory with applications to climate cycles, and (iii) an ML approach to learning efficient function bases for transfer and Koopman operator representation. These topics are drawn from collaborations with Jason Atnip, Aleks Badza, Dimitris Giannakis, Peter Koltai, Kevin Kuhl, Nicholas Peters, Roshan Samuel, and Joerg Schumacher.

Gary Froyland
UNSW Sydney
g.froyland@unsw.edu.au

MS31

Data-driven classification and continuation of patterned structures

I will give an overview of two complementary approaches to classifying patterned structures. Examples of such patterns are spot and stripe clusters in agent-based models and source defects that mediate between different wave trains in PDEs. The first (supervised) approach focuses on identifying patterns using pattern statistics, which are probability measures that capture the distribution of certain features, such as the number of connected components or their areas, across samples. This approach can be used with arc-length predictor-corrector continuation to trace out transition and bifurcation curves in parameter space by maximizing the Wasserstein distance of the pattern statistics. The second approach consists of clustering features such as persistence diagrams obtained from topological data analysis in an unsupervised or semi-supervised fashion. This approach can be used to classify structures within and across models. I will discuss applications of these frameworks to source defects in the Brusselator model and to clustering of heterogeneous cell populations in models of zebrafish pigment patterns.

Dhananjay Bhaskar
Yale University
dhananjay.bhaskar@yale.edu

Samuel Maffa
Broad Institute
samuel_maffa@alumni.brown.edu

Bjorn Sandstede

Division of Applied Mathematics
Brown University
bjorn_sandstede@brown.edu

Ezra Seidel
Brown University
ezra_seidel@brown.edu

Alexandria Volkening
Purdue University
avolkening@purdue.edu

Ian Wong
Brown University
ian_wong@brown.edu

Wenjun Zhao
Wake Forest
zhaow@wfu.edu

MS32

Degeneracy Problems in Zero-dispersion Asymptotics for the Benjamin-Ono equation with rational initial data

We study the zero-dispersion limit of the Benjamin-Ono equation with rational initial data, following the recent work of Blackstone, Gassot, Grard, and Miller. In this setting, the solution admits an explicit determinantal representation involving contour integrals with a phase function $h(z)$, whose critical points correspond to the characteristic roots of the inviscid Burgers equation. The asymptotic analysis as the dispersion parameter $\epsilon \rightarrow 0$ is therefore reduced to a steepest descent problem for oscillatory integrals. In this talk, we emphasize the geometric and analytic role of the phase function $h(z)$ and its critical points. In particular, we discuss how the asymptotic description can be extended to situations where $h(z)$ admits multiple or degenerate complex critical points. By adapting the steepest descent method and carefully analyzing the associated level curves and contour deformations, one can still obtain uniform asymptotic expansions, even in the presence of coalescing saddles. Furthermore, we also modify this explicit determinantal representation in the case of initial data with higher order poles, where the phase function exhibits essential poles.

Zijian CHENG
University of Michigan
USA
zjcheng@umich.edu

MS32

Coherent Structures in the Benjamin-Ono Soliton Gas

We study the large- N limit of N -soliton solutions to the Benjamin-Ono equation, where soliton amplitudes and phases arise from discretizations of admissible functions. This provides a first step toward a rigorous soliton gas theory for the equation. Our approach uses Generalized Locally Toeplitz sequences together with a generalized weak Szegő theorem framework to establish weak convergence of the rescaled solutions. We also show that, under suitable conditions, the strong limit vanishes outside a compact interval. We then outline a program toward strong asymptotics in the bulk. This requires precise control of individual eigenvalue locations and spacing; in particular, certain

regimes appear to necessitate a Riemann-Hilbert problem analysis.

Yutong Qing
University of Michigan
tqing@umich.edu

MS33

Hydrodynamics of Manakov-Type Solitons and Breathers

Rogue waves pose a significant threat to marine vessels and offshore infrastructure, making a clear understanding of their dynamics and kinematics essential. While numerical simulations and laboratory experiments play a central role in investigating these phenomena, an ongoing debate within the research community concerns the relative importance of nonlinearity and breather wave groups in the formation of realistic directional rogue waves. This talk addresses this issue by highlighting the pivotal role of weakly nonlinear wave theory, specifically the coupled nonlinear Schrödinger equation and the Manakov system, in the generation and control of directional rogue waves, both in the open ocean and in controlled water wave basin experiments.

Amin Chabchoub

Okinawa Institute of Science and Technology
The University of Melbourne
amin.chabchoub@oist.jp

MS33

Dam-Break Experiments in An Elongated Bose-Einstein Condensate

Bose-Einstein condensates offer a rich and versatile platform for studying a variety of hydrodynamic phenomena, including solitons, vortices, shock waves, and turbulence. Much of the previous work in this field has focused on weak perturbations and small-amplitude excitations to generate these features. However, generating an extended region of superfluid flow can reveal intricate dynamics analogous to other complex physical systems. In this work, we experimentally simulate a dam-break scenario in a weak harmonic trap, resulting in a nontrivial flow across the system. We first characterize the rarefaction flow using an experimental technique that results in a measurement of the Riemann invariants along the flow profile. We then introduce additional dynamics by allowing the resulting rarefaction wave to collide with a barrier, generating a complex, turbulent region that propagates through the system. The status and future scope of this work will be discussed.

Maren E. Mossman

University of San Diego
Washington State University
mmossman@sandiego.edu

MS33

Topological Analysis of Two-Dimensional Structures in Dispersive Quantum Fluids

I will present combined experimental and theoretical investigations of vortex leapfrogging in a two-dimensional superfluid. By generating configurations in which leapfrogging breaks down, we observe two distinct dissipation mechanisms. The first involves discrete phase-slip events. I will introduce a new topological analysis that has recently

enabled the direct experimental visualization of Anderson phase slips in this context. The second dissipation mechanism occurs when the injection of multiply charged vortices produces a quasideimensional dispersive shock wave that acts as a continuous source of phase slippage.

Nicolas Pavloff

Laboratoire de Physique Théorique et Modèles
Statistiques
Université Paris-Saclay
nicolas.pavloff@universite-paris-saclay.fr

MS33

Breaking a Superfluid Harmonic Dam: Rarefaction Dynamics of a Trapped Condensate

We consider the vacuum dambreak problem for the Gross-Pitaevskii (GP) / nonlinear Schrödinger (NLS) equation in the presence of a harmonic trap, which models the dynamics of confined Bose-Einstein condensates (BECs). In this setting, an initial density jump evolves into a rarefaction wave that expands into the channel while being influenced by the external confinement. In the absence of an external potential, the defocusing one-dimensional (1D) NLS equation admits simple self-similar rarefaction wave solutions, characterised by one constant dispersionless Riemann invariant. In the presence of a potential, however, such simple wave solutions do not exist. As a result, the classical rarefaction wave description no longer captures the trap induced dynamics, which motivates the development of a theory that explicitly incorporates the effects of harmonic confinement. Using a perturbative framework, we derive exact closed-form solutions of the 1D NLS equation with harmonic potential describing such *harmonic* rarefaction waves. These solutions are then used to construct solutions of the vacuum dambreak Riemann problem in a harmonic trap. We also obtain approximate solutions corresponding to ground state initial data, and compare them with experiments on quasi-1D BECs and experimentally realistic 3D numerical simulations of the NLS equation, finding excellent agreement.

Shashwat Sharan

University of California Merced
ssharan2@ucmerced.edu

MS34

Bright and Dark Pulses in the Lugiato-Lefever Equation with Periodic Forcing

We consider a damped and periodically driven nonlinear Schrödinger (NLS) equation,

$$iu_t = -du_{xx} + icu_x + (\zeta - i)u - |u|^2u + if(x)$$

which is a variant of the Lugiato-Lefever equation. This model arises in the description of optical Kerr frequency combs that form in a nonlinear microresonator driven by multiple laser inputs. In the focusing case $d > 0$, we use the Lyapunov-Schmidt reduction method to show that periodic forcing leads to stable, localized 1-pulse solutions with oscillatory tails that bifurcate from the bright NLS soliton. Adopting a spatial dynamics approach, we then construct stable multi-pulse solutions that resemble well-separated copies of the 1-pulses. For the defocusing case $d < 0$, we employ exponential dichotomies to prove the first existence result of dark pulse solutions, which we obtain by pasting together a front and a back solution. Numerical simulations with `pde2path` corroborate our analytical findings.

This is based on a joint project with Björn de Rijk (KIT).

Lukas Bengel, Björn de Rijk

Karlsruhe Institute of Technology
lukas.bengel@kit.edu, bjoern.rijck@kit.edu

MS34

An Asymptotic Analysis of Spike Self-Replication and Spike Nucleation of Reaction-Diffusion Patterns on Growing 1-D Domains

Pattern formation on growing domains is one of the key issues in developmental biology, where domain growth has been shown to generate robust patterns under Turing instability. In this work, we investigate the mechanisms responsible for generating new spikes on a growing domain within the semi-strong interaction regime, focusing on three classical reaction-diffusion models: the Schnakenberg, Brusselator, and Gierer-Meinhardt (GM) systems. Our analysis identifies two distinct mechanisms of spike generation as the domain length increases. The first mechanism is spike self-replication, in which individual spikes split into two, effectively doubling the number of spikes. The second mechanism is spike nucleation, where new spikes emerge from a quiescent background via a saddle-node bifurcation of spike equilibria. Critical stability thresholds for these processes are derived, and global bifurcation diagrams are computed using the bifurcation software `pde2path`. These results yield a phase diagram in parameter space, outlining the distinct dynamical behaviors that can occur.

Chunyi Gai

University of Northern British Columbia
Mathematics and Statistics
chunyi.gai@unbc.ca

MS34

The Brusselator Also Possesses Spatially Periodic Canard Solutions

The Brusselator PDE is a canonical example of 2-component reaction-diffusion system which produces spatially periodic solutions. In the singular limit as one diffusion rate goes to 0, the behavior of periodic solutions departs from the typical sinusoidal structure. Near this singular Turing bifurcation, the 1D spatial dynamics can be expressed as a 2-fast 2-slow system with reversibility symmetry. The bifurcation point itself corresponds to a RFSN-II point in the spatial dynamics. This structure leads to periodic orbits with long, slowly varying segments separated by fast jumps in activator concentration. In fact, the Brusselator contains the second known example of spatially periodic canard solutions, some of which appear to be stable equilibria of the full PDE. We will discuss the construction of these solutions and various key features of the slow-fast system which characterize these solutions. Many of these features appear to be generic in the context of singular Turing bifurcations and suggest these canards may play an important role in the analysis of subcritical Turing bifurcations more broadly.

Robert Jencks

Boston University
U.S.

@tbd

MS34

Existence of Asymmetric Grain Boundaries

Grain boundaries both asymmetric and symmetric are commonly observed in nature, experiments, and numerical simulations as fundamental defects in spatially periodic patterns. In this work, we establish the existence of asymmetric grain boundaries in the Swift-Hohenberg equation posed on the full Euclidean plane. We further show that the wavenumbers of the far-field roll patterns are selected by the periodicity along the grain boundary together with the rotational angles of the roll patterns in the far field. As in the symmetric case, we employ spatial dynamics and center-manifold reduction to recast the existence problem for these solutions in the stationary Swift-Hohenberg equation a fourth-order elliptic PDE into the problem of finding heteroclinic orbits in a reduced ODE system. The construction of these heteroclinic orbits, however, differs substantially from the symmetric setting and constitutes the central contribution of this work. Specifically, we first establish the existence of heteroclinic orbits in the normal form up to cubic order using techniques from the calculus of variations. We then demonstrate the persistence of these orbits in the full reduced ODE system via a nontrivial Lyapunov-Schmidt reduction argument."

Qiliang Wu
Ohio University
wuq@ohio.edu

MS35

Nonlinear interaction of flexural wave in finite depth on flow of constant vorticity

The interaction of ocean waves with floating ice sheets plays a critical role in polar environments, where ice stability is affected by waves, storms, and large-scale climate variability. While linear theories of flexural-gravity waves provide valuable insights, they fail to capture the nonlinear deformation observed in field conditions, especially under large-amplitude waves and strong compressive stresses. This study develops a third-order asymptotic solution for nonlinear flexural-gravity waves beneath a thin elastic ice sheet in finite depth on flow of constant vorticity. The governing equations are derived within the nonlinear Euler-Bernoulli framework, incorporating inertia, rigidity, and compressive stress. A perturbation series solution for steady hydroelastic gravity waves, accurate up to the third order, is derived using a classical Stokes expansion procedure, which allows us to include the effects of compressive stress and shear current effects in the analysis of wave-current interactions in the presence of constant vorticity. The analytical results are then compared with numerical computations with the full equations. The buckling and wave blocking thresholds of the ice sheets are also discussed.

Hung-Chu Hsu
Tainan Hydraulics Laboratory
National Cheng Kung University, Taiwan
hungchuhsu@gmail.com

Hung-Yu Huang
Department of Marine Environment and Engineering
National Sun Yat-Sen University
a0912132598@gmail.com

Malek Abid, Christian Kharif

Aix-Marseille Université, Institut de Recherche sur les Phénomenes
malek.abid@univ-amu.fr, christian.kharif@irphe.univ-mrs.fr

MS35

Lagrangian Dynamics in Ocean Waves

This presentation will give an overview over recent work in particle dynamics in nearshore ocean waves. The goals of the work are three-fold. First, we report on a field campaign aimed at understanding wave-by-wave dynamics and Lagrangian properties of waves in the surf zone. Secondly, we show that breaking waves can be identified using an acceleration criterion due to Longuet-Higgins. Thirdly, we look at larger-scale circulation patterns in the near-shore zone.

Henrik Kalisch
University of Bergen
henrik.kalisch@math.uib.no

MS35

Kinematics of nonlinear water waves traveling along a laboratory flume

The advancement of our knowledge on water wave mechanics and related wave processes started from early development of theoretical solutions to two-dimensional boundary-value problems in periodic domains. Linear (Airy waves) and weakly-nonlinear (Stokes waves) potential flow approximations were derived to give first insights into underlying physics. In case of a weakly-nonlinear solution, a phase-averaged Stokes drift was observed due to not closing of orbital paths of water particles leading to wave-induced mass transport of water. More exact nonlinear potential flow solutions gave some corrections to the weakly-nonlinear value of drift velocity, while still being valid only for periodic domains. In order to measure particle trajectories and phase-averaged velocity profiles under waves, an appropriate experimental procedure involving a closed wave flume should be introduced. However, physical modeling in wave laboratories requires basic knowledge and awareness of inevitable limitations and side effects, e.g. excitation of evanescent modes, presence of spurious free waves and return current action. These processes affect the characteristics of wave-induced mass transport and may trigger false conclusions. To avoid misinterpretation, the analysis of results should be based on a nonlinear wavemaker theory, which will be presented and used to discuss intrinsic kinematics of mechanically generated nonlinear waves and their relation to theoretical periodic oscillatory flow.

Maciej Paprota
Department of Wave Mechanics and Structural Dynamics
Institute of Hydro-Engineering, Polish Academy of Sciences
mapap@ibwpan.gda.pl

MS35

Transport of finite-size particles by nearshore gravity waves

In this talk, I will discuss the transport of finite-size particles induced by nearshore gravity waves. To capture the essential dynamics, we employ governing equations for inertial spherical particles subject to nonlinear drag, suitable for the Reynolds-number range and the low Stokes num-

bers characteristic of small particles in the marine environment. The modulated second-order wave group used in our study reproduces key features of asymmetric and evolving wave fields commonly observed in nearshore regions over gently sloping beaches. Numerical simulations reveal that inertial particles follow downward-spiraling trajectories, exhibiting a back-and-forth motion synchronized with the low-frequency bound wave. These findings highlight the dominant influence of low-frequency wave components on inertial particle transport and offer new insight into wave-driven mass transport of impurities in coastal environments. The accuracy of this approach is assessed through comparison with field measurements collected during a 2019 campaign on the island of Sylt, showing excellent qualitative and quantitative agreement. I will also discuss a simplified model for neutrally buoyant small particles and present results describing their transport under cnoidal and solitary waves. This work is in collaboration with Henrik Kalisch.

Rosa Maria Vargas Magana
University of Bergen
Norway
rosa.vargas@uib.no

MS37

Pattern Formation in a 2D Thermocapillary Thin-Film Equation

It is experimentally known that thin films of viscous fluids on heated plates develop polygonal, spatially periodic patterns. This is due to a self-sustaining thermocapillary effect causing an instability of the trivial constant state. Building upon a previous work on the one-dimensional case, we consider a two-dimensional thin-film equation. It can be formally derived from the Bénard–Marangoni problem via a long-wave approximation. We consider the stationary problem, which we are able to reduce to a second-order equation amenable to analytic bifurcation theory. The constant solution destabilizes via a (conserved) long-wave instability and we prove existence of a global bifurcation branch of stationary solutions of fixed mass, which are symmetric and periodic with respect to a fixed square or hexagonal lattice. We finally analyse qualitative aspects of the solutions on the branch both analytically and numerically. Most importantly, we prove that solutions exhibit film rupture, that is, their minimal height tends to zero, under the condition that the Marangoni number is uniformly bounded on the bifurcation branch. This conditional result is substantiated by numerical experiments.

Stefano Böhmer
Lund University
stefano.bohmer@math.lu.se

Jonas Jansen
Fraunhofer SCAI
jonasjansen@posteo.de

Bastian Hilder
Technical University of Munich
bastian.hilder@tum.de

MS37

Dynamics of a 3D rimming-flow equation

A rimming flow is the flow of a fluid partially filling a rotating cylinder. We assume that the fluid forms a capillary-driven thin film coating the inner wall. We formally derive

a closed fourth-order degenerate-parabolic evolution equation for the height of the liquid film h :

$$h_t + h_\theta + \gamma \operatorname{div} (h^3 \nabla (\Delta h + h)) = \delta (h^3 \cos(\theta))_\theta$$

with time $t > 0$, cylindrical coordinates $(\theta, z) \in \mathbb{T} \times (0, L)$ and Neumann-type boundary conditions on the cylinder covers. Competing effects relevant to this equation are the cylinder rotation, surface tension $\gamma > 0$, viscosity and gravity $\delta \geq 0$. We investigate the stability of positive steady states. For small gravity $0 < \delta \ll 1$ and short cylinders ($L < \pi$) we show that certain steady states are stable and while they are unstable for long cylinders ($L > \pi$). For cylinders of critical length $L = \pi$ we investigate solutions of the form

$$h(t, \theta, z) = m + a_1 (\delta^2 t) e^{i(\theta-t)} + a_{-1} (\delta^2 t) e^{-i(\theta-t)} + b (\delta^2 t) \cos(z)$$

and formally derive an ODE for the (slow) coefficients $a_1, a_{-1} \in \mathbb{C}, b \in \mathbb{R}$, thus describing the dynamics of such solutions on the slow time scale $\tau = \delta^2 t$. This talk is based on joint work with Janne Laudien (Stuttgart), Christina Lienstromberg (Stuttgart) and Juan J. L. Velazquez (Bonn).

Juri Joussen
Stuttgart University
juri.joussen@mathematik.uni-stuttgart.de

Janne Laudien, Christina Lienstromberg
University of Stuttgart, Germany
st179362@stud.uni-stuttgart.de,
christina.lienstromberg@iadm.uni-stuttgart.de

Juan Velazquez
Institute of Applied Mathematics
University of Bonn
velazquez@iam.uni-bonn.de

MS37

Nonlocal Effects in Biological Pattern Formation

Biological pattern formation has been extensively studied using reaction-diffusion and agent-based models. In this talk we will discuss nonlocal pattern forming mechanisms in the context of bacterial colony formation with an emphasis on arrested fronts. This will lead to a novel nonlocal framework to understand the interfacial motion in biological systems. We will then discuss data-driven model discovery in the setting of nonlocal models.

Scott McCalla
Montana State University
scott.mccalla@montana.edu

MS37

Crystals, Bubbles, and Fissures: Reversible Clustering and Sorting in Interacting Particle Systems

Imagine agents moving according to simple rules that disfavor close crowding but favor a mutual intermediate range proximity. Systems of this type appear across the sciences, from molecular dynamics to microbiology and ecology. The competition between the underlying short-range repulsion and intermediate-range attraction can lead to a phase transition, where the preferred state changes from a "crystalline" equidistribution to the formation of clusters (or colonies) separated by vacuum regions. I will describe recent work that analyzes this transition (or bifurcation) in some detail, emphasizing a curious aspect that makes

this transition "non-hysteretic" or "reversible" in an infinite system-size limit, thus allowing for easy switching from crystal to cluster – and back. Results include predictions for sizes and shapes of vacuum regions, corrections due to noise, and expansions for finite system size corrections. This is based on joint work with Angela Stevens and on a summer REU project mentored by Olivia Clifton.

Arnd Scheel

University of Minnesota, Minneapolis
School of Mathematics
scheel@math.umn.edu

MS38

Stability and Bifurcation in Gradient Flows of Patterns and Surfaces

We present a comprehensive framework for the development of gradient flows of parameterization independent surface energies naturally expressed in terms of intrinsic quantities (curvature and metric). To this mix we add cartesian distance which allows adhesion-repulsion energies that guide folding flows of cellular organelles, and surface diffusion of embedded agents (eg proteins). Via a penalty method on membrane density, we derive a mechanism to generate locally incompressible flows of fluidic membranes. In space dimension two, we show that stability analysis of surface patterns can be converted to an analysis of the second variation of the surface energy subject to the non-linear constraints imposed on the first and second fundamental forms. We outline derivation of quasi steady dynamics, making application to adhesion-repulsion energies that guide folding flows of cellular organelles.

Keith Promislow

Michigan State University
kpromisl@math.msu.edu

Qiliang Wu

Ohio University
Department of mathematics
wuqwohio.edu

Truong Vu

Univ. Illinois Chicago
Dept of Mathematics
tvu25@uic.edu

Brian R. Wetton

University of British Columbia
Department of Mathematics
wetton@math.ubc.ca

MS38

Intrinsic Motion of Fronts in Some Slow-Fast Reaction Diffusion System

Here we consider motion of front interfaces in spatially one-dimensional in isotropic reaction-diffusion systems. It has been found at least since the late 80s that coupled systems, in particular those with bistable nonlinearity, can exhibit non-trivial front motion. In a more recent series of articles such phenomena were studied for single fronts in certain singularly perturbed multi-component reaction-diffusion systems. Here bifurcations of a stationary front yields non-trivial motion due to a persistent nilpotent structure relating to the translation symmetry. Generalising this, we prove in particular that chaotic motion can occur in the sense of an unfolded Shil'nikov homoclinic orbit.

We also comment on ongoing work concerning more general systems that admits other nilpotency structure. This is joint work with Martina Chirilus-Bruckner (Leiden) and Peter van Heijster (Wageningen).

Martina Chirilus-Bruckner

University of Leiden
Mathematical Institute
m.chirilus-bruckner@math.leidenuniv.nl

Peter van Heijster

Biometris
Wageningen University & Research
Peter.vanheijster@wur.nl

Jens Rademacher

University of Hamburg
jens.rademacher@uni-hamburg.de

MS38

Stress Tensor Methods for the Dynamics of Singularities in Dissipative and Driven Systems

We show how the equations of motion for point defects, domain walls, disclination lines, and other singularities in translationally invariant systems with dissipation can be derived directly from the properties of the stress tensor. This approach eliminates the need to analyze the short-distance core structure near the singularity, greatly simplifying the derivation of its dynamics. We then extend our results to systems with weak translational-symmetry breaking and external energy injection, focusing on interface dynamics in phase-separating media. This framework enables us to describe the motion of droplets in systems with generic spatial and temporal modulations of material properties. In particular, we show how modulations of surface tension, droplet density and chemical potential combine with the interface geometry to determine its motion. Due to their flexibility and generality, our results offer a unified perspective on the behavior of interfaces and singularities in a broad class of dissipative and driven systems.

Jacopo Romano

SISSA
jromano@sissa.it

MS39

Power spectra in 2D inertial Poiseuille and Couette flow due to the van der Waals effect

We numerically simulate the two-dimensional inertial flow with the van der Waals effect in a straight periodic channel near the Poiseuille and Couette stationary states. Even though the flow technically remains "laminar" (that is, the tracers do not mix on a macroscopic scale), we observe the chaotic dynamics and power spectra in the small fluctuations of the density, velocity divergence and vorticity near their respective stationary background states. Remarkably, setting the vorticity to its background state, and leaving only the density and velocity divergence as the variables, results in a qualitatively similar chaotic dynamics and power spectra to those of the full system. This suggests that the mechanics of the power spectra reside primarily in the density and velocity divergence variables, and are not directly related to the vorticity of the flow.

Rafail Abramov

Department of Mathematics, Statistics and Computer

Science
University of Illinois at Chicago
abramov@uic.edu

MS39

Collective Dynamics in Integrable Systems

This talk describes phenomena at scales much larger than the width of an individual soliton. Since, numerically, periodic boundary conditions are typically used, we here briefly describe the corresponding inverse-spectral transform using the Riemann-Hilbert problem for the focusing nonlinear Schrödinger equation (NLS). We then describe the numerical results revealing an effective dispersion relation for NLS waves and the paradoxical fact that the larger their amplitude, the closer these waves are to linear in a statistical sense. Finally, we describe analytically and numerically a dispersive regularization of a shock wave as a fan of solitons in the small-dispersion limit of the derivative NLS equation.

Gregor Kovacic
Rensselaer Polytechnic Inst
Dept of Mathematical Sciences
kovacg@rpi.edu

Zachery Wolski
Rensselaer Polytechnic Institute
Department of Mathematical Sciences
wolskz@rpi.edu

Jeffrey W. Banks
Rensselaer Polytechnic Institute
banksj3@rpi.edu

Zechuan Zhang
Scuola Internazionale Superiore di Studi Avanzati
Trieste, Italy
zezhang@sissa.it

Gino Biondini
State University of New York at Buffalo
Department of Mathematics
biondini@buffalo.edu

Alexander Tovbis
University of Central Florida
Department of Mathematics
alexander.tovbis@ucf.edu

Katelyn J. Leisman
University of Illinois Urbana-Champaign
katelyn.leisman@northwestern.edu

Douglas Zhou
Shanghai Jiao Tong University
zdz@sjtu.edu.cn

David Cai
Courant Institute for Mathematical Sciences, NYU
Shanghai Jiao-Tong University
cai@cims.nyu.edu

MS39

Metastability in Stochastic Partial Differential

Equations with Spatially-Correlated Noise

Stochastic partial differential equations (SPDEs) are continuum models of spatially-extended noisy physical systems. Metastability is a key feature to understand in which the system spends long periods of time in localized regions of phase space with fast transitions between localized regions. In prior work, I showed how metastability can arise in a nonlinear wave equation with stochastic initial conditions, with an identical mean transition time to that of the overdamped noisy Langevin counterpart. Here, I derive transition times for systems with spatially-correlated noise, a necessary consideration in physical systems with finite correlation length scales violating the assumption of independent (white) noise in SPDE models. This is based on a Freidlin-Wentzell-like action minimization problem, leading to transition paths that depend on the correlated noise structure that preserves a Gibbs equilibrium distribution.

Katherine Newhall
Dept. of Mathematics
University of North Carolina at Chapel Hill
knewhall@unc.edu

MS39

Generalized Constantin-Lax-Majda Equation with Dissipation

We consider the generalized Constantin-Lax-Majda equation with dissipation $\omega_t = -au\omega_x + \omega\mathcal{H}\omega - \nu\Lambda^\sigma(\omega)$, $u_x = \mathcal{H}\omega$, where $\widehat{\Lambda^\sigma} = |k|^\sigma$, both for the problem on the circle $x \in [-\pi, \pi]$ and the real line, and summarize known and new results for global-in-time existence of the solutions and collapsing solutions. We also point out the gaps and parameter regions that are worthy of further investigation.

Denis Silantyev
UCCS
dsilanty@uccs.edu

Pavel M. Lushnikov
University of New Mexico, U.S.
plushnik@unm.edu

Michael Siegel
New Jersey Institute of Technology
michael.s.siegel@njit.edu

David Ambrose
Drexel University
dma68@drexel.edu

MS40

Coherent spatiotemporal pattern extraction using symmetry-aware kernel integral operators

A hallmark feature of coherent structures generated by nonlinear spatiotemporal dynamics is intermittency in both space and time. Oftentimes, this is the outcome of symmetries or approximate symmetries of the equations of motion under transformations such as translations or rotations in space. Conventional snapshot-based feature extraction methods are agnostic of such symmetries; as a result, the extracted spatial patterns are oftentimes pure symmetry modes (e.g., Fourier modes in a periodic domain), with minimal dynamical significance and interpretability. In this talk, we present a kernel

eigendecomposition technique for feature extraction that treats spatiotemporal data as function-valued observables of a dynamical system. Using delay-coordinate maps defined pointwise in the spatial domain, we build a kernel integral operator whose eigenspaces are automatically invariant under the action of dynamical symmetry groups, providing efficient representation of non-separable intermittent coherent structures. The method has an equivalent interpretation as an operator-valued kernel technique. We present applications in prototypical PDE models with complex spatiotemporal dynamics such as the Kuramoto-Sivashinsky model. We also discuss low-rank approximation techniques for efficient computation of eigenvalues and eigenvectors of large kernel matrices associated with spatiotemporal datasets.

Dimitrios Giannakis
Dartmouth College
Dimitrios.Giannakis@dartmouth.edu

Joanna M. Slawinska
Department of Mathematics
Dartmouth College
joanna.m.slawinska@dartmouth.edu

Chris Vales
Dartmouth College
chris.vales@dartmouth.edu

MS43

Spatio-Temporal Heterogeneity and Rate-Induced Bifurcations in Pattern-Forming Pdes

Biological systems use spatio-temporally varying input signals to self-organise into macroscale structures (patterns) that are vital for many physiological processes. As observed by Turing in 1952, these patterns are in a state of continual development, and are usually transitioning from one pattern into another. How can this self-organising process be robust in the presence of confounding effects caused by unpredictable or heterogeneous environments? By generating a framework for the multiscale analysis of rate-induced bifurcations in PDEs, I present a general theory of pattern formation in the presence of spatio-temporal input variations, and show how biological systems can generate non-standard dynamic robustness for 'free' over physiologically relevant timescales. I apply this theory to paradigmatic pattern-forming systems, and predict that they are robust with respect to non-physiological variations in input signal. More broadly, I show how the dynamics of pattern-forming systems with spatio-temporally varying parameters can be classified based on the bifurcations in their governing equations.

Mohit P. Dalwadi
University of Oxford
mohit.dalwadi@maths.ox.ac.uk

MS43

Rate-Induced Tipping in a Moving Habitat

This paper investigates rate-induced tipping in a model of a moving habitat due to environmental change. We study a scalar reaction-diffusion equation with a non-autonomous reaction term representing a spatially localized habitat moving from one asymptotic location to another. The movement is characterized by displacement d and rate parameter r . We consider a system that admits three steady states: a stable extinction state $u_0^* = 0$, an unstable pulse

$u_1^*(x) > 0$ (edge state), and a stable pulse $u_2^*(x) > u_1^*(x)$ (base state). These exist in both asymptotic habitat locations. We identify a critical displacement d^* and, for $d > d^*$, demonstrate the existence of a critical rate $r_c(d)$ at which rate-induced tipping occurs: for $r > r_c$ an initially thriving population becomes extinct due to too rapid habitat change. We provide rigorous results for two limiting cases. For $r \ll 1$, solutions track the moving base state with error $O(r)$. For $r \gg 1$, solutions converge to extinction when $d > d^*$. For $d < d^*$, no tipping occurs regardless of r . Numerical simulations complement these analytical results.

Blake Barker
Brigham Young University
blake@math.byu.edu

Emmanuel Fleurantin
George Mason University
efleuran@gmu.edu

Matt Holzer
Department of Mathematics
George Mason University
mholzer@gmu.edu

Christopher Jones
George Mason University
U.S.
ckrtj@renci.org

Sebastian M. Wicczorek
University College Cork
Department of Applied Mathematics
sebastian.wicczorek@ucc.ie

MS43

Towards Geometric Singular Perturbation Theory of PDE Patterns

Geometric singular perturbation theory has been very successfully used in the context of spatial dynamics to understand patterns arising in partial differential equations (PDEs). Yet, for many pattern-forming systems, one cannot rely on spatial dynamics and has to work more directly with the underlying partial differential equation. In this talk I am going to explain some recent progress trying to lift slow manifold and blow-up methods directly to PDEs. In particular, I shall explain recent extensions of slow manifold techniques to cross-diffusion and Fokker-Planck PDEs and highlight novel developments in the blow-up method for various classes of reaction-diffusion systems. In the context of the blow-up method an interesting observation is that hidden transport terms can be made visible via blow-up.

Christian Kuehn
Technical University of Munich
ckuehn@ma.tum.de

MS43

Resilience Optimization of Tipping-Prone Systems Through Spatial Heterogeneities

Spatially extended multistable systems often exhibit tipping and front propagation, where small local changes can drive large-scale transitions. In many real settings, such as dryland vegetation or simplified climate subsystems, in-

terventions can be implemented. Limited resources, however, mean they cannot be applied everywhere. As a result, they must be spatially targeted. This raises a mathematical question: where should spatial heterogeneity be introduced to most effectively enhance resilience? We develop a constrained optimisation framework that selects an optimal spatial perturbation under PDE dynamics subject to size constraints. For illustration, we consider the one-dimensional Allen–Cahn equation with spatial heterogeneity,

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2} + y(1 - y^2) + \mu(t) + \mu_{\text{het}}(x) + \mu_{\text{pert}}(x),$$

where $\mu_{\text{pert}}(x)$ is the spatially dependent perturbation whose shape we optimise. Its behaviour is governed by two key quantities of multistable gradient systems: the homogeneous saddle-node bifurcation threshold, which marks the tipping point, and the Maxwell point, which controls front propagation and pinning. Our results show how the optimal perturbation interacts with these quantities: small, localised interventions can prevent collapse, arrest or reverse invading fronts, or spatially confine tipping. This framework offers a rigorous foundation for designing efficient, spatially targeted interventions in PDE models where limiting critical transitions is essential.

Aurora Ragani
Utrecht University
a.faureragani@uu.nl

Robbin Bastiaansen
Mathematical Institute
Utrecht University
r.bastiaansen@uu.nl

MS44

A Floquet Spectral Localizer

The periodic driving of lattice systems can induce localized edge states that propagate unidirectional around lattice defects. These so-called Floquet insulators are associated with topological invariants, such as the Chern number. A new approach for diagnosing topology in Floquet insulators will be presented. The approach leverages a spectral localizer framework to probe the topology of genuinely finite and even disordered systems; that is, systems lacking spatial periodicity.

Stephan Wong, Alexander Cerjan
Sandia National Laboratory
stewong@sandia.gov, awcerja@sandia.gov

Justin Cole
University of Colorado Colorado Springs
jcole13@uccs.edu

MS44

Radiative Decay of Edge States in a Time-Forced SSH Model

We study the effect of time-periodic forcing on the edge state of the semi-infinite SuSchriefferHeeger (SSH) model, a 1D tight-binding model. Numerical simulations and an asymptotic expansion demonstrate that if the frequency of forcing is in resonance with the continuous spectrum of the unforced Hamiltonian, then on a time scale proportional to the inverse square of the forcing amplitude, the edge state decays in amplitude due to the radiation of its energy

into the bulk. A proof is work in progress, and makes use of a new dispersive decay estimate for the time-evolution induced by the Hamiltonian.

Remy Kassem
Columbia University
rhk2130@columbia.edu

Michael I. Weinstein
Columbia University
Dept Appl Phys & Appl Math
miw2103@columbia.edu

Amir Sagiv
NJIT
amir.sagiv@njit.edu

MS44

Geometric analysis of high frequency wave behavior in smoothly graded flat periodic plates

We investigate the propagation of high frequency waves on smoothly graded flat periodic plates, and we propose a method for studying high frequency waves by transforming the physical problem into a geometric one. When the wavelength is comparable to the characteristic length of the periodic medium waves propagate through, the periodic structure acts as a series of scatterers. However, at shorter wavelengths, waves can be described through the geodesics of the medium. It is known that material grading, meaning variations in stiffness, density, or thickness of the plate, changes the local wave speed and leads to refraction. Therefore, there are distinct behaviors for the wave based on the separation of scales between the characteristic length of the periodic medium and the wavelength of the propagating wave and a threshold exists at which the behavior of the wave fundamentally changes. By transforming the physical problem into a geometric one, the thin plate can be thought of as a two-dimensional manifold embedded in a three-dimensional space. Variations of the wave speed are accounted for through the curvature of the manifold. The new method is validated numerically and experimentally and can be used to efficiently predict the behavior of waves in media and design complicated patterns towards novel waveguides.

Panagiotis Koutsogiannakis, Massimo Ruzzene
University of Colorado, Boulder
Panagiotis.Koutsogiannakis@colorado.edu, massimo.ruzzene@colorado.edu

MS44

Multimodal Solitary Waves on Periodic Domains

We consider the existence and spectral stability of multimodal solitary wave solutions in two Hamiltonian systems – a fifth-order Korteweg-De Vries equation (KdV5) and a fourth-order nonlinear Schrödinger equation (NLS4) – when these are posed on a periodic domain. For both equations, there exists a discrete family of multi-modal solutions which is characterized by the distances between successive copies of the primary solitary wave. Spectral stability of these solutions is determined by small eigenvalues near the origin, which can be computed by a finite-dimensional reduction. Of particular interest in the periodic setting is instability bubbles which form when eigenvalues of opposite Krein signature collide on the imaginary axis. Both

analytic and numerical results are presented.

Ross Parker
Center for Communications Research - Princeton
rosshparker@gmail.com

MS45

On the Lazutkin-Treschev Invariant of Multipulse Solitary Wave

In this talk, we are interested in the weakly spectral (in)stability of a family of multi-pulse solitary waves arising from a dispersive PDE. A solitary wave is a steady state for the PDE written in a moving frame, which tends to the same finite limit at infinity. For some PDEs, the solitary waves can be considered as solutions of an Hamiltonian ODE with two degrees of freedom and are homoclinic orbits of the Hamiltonian ODE. Based on the Hamiltonian structure, we introduce the geometric invariant of Lazutkin-Treschev (LT) that is linked to the spectral (in)stability of the solitary wave. [Kapitula & Stefanov 2014; Chardard & Bridges 2015]. When such a solitary wave has oscillating decaying tails, there is a countably infinite number of homoclinic orbits [Devaney 1976]. Some of them correspond to 2-pulse symmetric solitary waves, which look like two copies of 1-pulse, denoted by γ , glued together by some oscillations. We denote them by $2(k)$, where $2k \in \mathbb{N}^*$ is the number of oscillations. Our main result is the following relationship:

$$LT(2(k)) = c_k(-1)^{k+1}LT(\gamma)^2, \quad c_k > 0.$$

As a consequence we may deduce the instability of some of these 2-pulse. Some numerical simulations in the case of Kawahara equation will be presented to illustrate our theoretical result.

Frederic Chardard
Universite Jean Monnet/Institut Camille Jordan
France
frederic.chardard@univ-st-etienne.fr

Laetitia Paoli
Université de Saint-Etienne
Laetitia.Paoli@univ-st-etienne.fr

Nhan-Trung Tran
Institut Camille Jordan
Université de Saint-Étienne
nhan.trung.tran@univ-st-etienne.fr

MS46

Global Asymptotic Stability in a 2-Dimensional Front Model

We consider the problem of the asymptotic stability of a front type solution to a weakly two-dimensional model for the propagation of undular bores. This model includes solutions of the KdV-Burgers equation, so this can be considered a study of the stability of those solutions within a more general two dimensional model. We prove that for domains that are sufficiently narrow in the transverse direction and for a range of the dispersion parameter, the family of fronts is a global asymptotic attractor.

Olivia C. Clifton
University of Illinois at Urbana-Champaign, U.S.

ocannon2@illinois.edu

MS46

Bifurcations of Bands of Modulation Instability for Stokes Water Waves

We study the modulational stability problem for the traveling periodic water waves (called Stokes waves) in an infinitely deep fluid by using pseudo-differential operators in conformal variables. We derive the criteria and the normal forms for four bifurcations which are repeated cyclically when the steepness of the Stokes wave is increased towards the limiting wave with the peaked profile. The four bifurcations are observed in the following order: (a) the birth of new figure-8 bands at each extremal point of speed, (b) the vertical slope of the double-degenerate figure-8 bands, (c) the birth of new closed bands surrounding the origin at each subharmonic (double-period) bifurcation, and (d) the reconnection of bands at the figure- ∞ at each extremal point of energy. For the first and second bifurcation cycles, we compute coefficients of the normal forms numerically and show the excellent agreement between the normal form theory and the numerical approximations of the modulational stability bands. This is a joint work with S. Dyachenko (SUNY Buffalo) and R. Marangell (Sydney).

Dmitry Pelinovsky
McMaster University
Department of Mathematics
pelinod@mcmaster.ca

MS46

Orbital Stability of Cnoidal Waves in the Korteweg-De Vries Equation Against Localized Perturbations

The Korteweg-de Vries (KdV) equation arises as a universal amplitude equation in the study of shallow-water waves. It admits a three-parameter family of periodic traveling waves. A longstanding open problem concerns the stability of these so-called cnoidal waves with respect to localized perturbations. Standard stability arguments in Hamiltonian systems, building on the foundational work of Grillakis, Shatah, and Strauss, which characterize solutions as constrained minimizers of the Hamiltonian, break down in the setting of periodic waves subject to localized perturbations. In this talk, we propose an alternative framework that integrates a modulational ansatz – originating from the stability theory of periodic waves in reaction-diffusion systems – with variational arguments. Our approach yields the first orbital stability results for periodic waves in the KdV equation with respect to L^2 -localized perturbations.

Björn de Rijk
Karlsruhe Institute of Technology
bjoern.rijk@kit.edu

Emile Bukieda
Karlsruhe Institute of Technology (KIT)
emile.bukieda@kit.edu

MS47

On the Interaction of Surface Waves and Vortices

The interaction between free-surface waves and localized vorticity structures is a fundamental problem in fluid dynamics, with relevance to geophysical flows. We study nonlinear free-surface flows in the presence of a point vor-

text using the general framework for two-dimensional water waves with arbitrary vorticity developed by Ionescu-Kruse and Ivanov (J. Differential Equations 368, 2023). In the small-amplitude long-wave Boussinesq and KdV regimes, we derive coupled evolution equations for the free surface and the vortex dynamics. Our analysis shows that the interaction with the vortex does not destroy the surface solitary waves and, for a significant range of the vortex strength, the solitary waves remain practically unaffected. This observation leads to a further simplification of the model, in which the vortex motion beneath propagating solitons is described by a decoupled system of ordinary differential equations, capturing the qualitative features of the interaction. Analytical results and supporting numerical simulations are presented (see D. Ionescu-Kruse, R. Ivanov and M. Todorov, J. Nonlinear Sci 36, 2026).

Delia Ionescu-Kruse

Institute of Mathematics of the Romanian Academy
(IMAR)
dionescu@imar.ro

MS47

Integrable Models for Interacting Intermediate Internal Waves and Currents

We consider a geophysical model of an interfacial internal wave formed at the pycnocline (or thermocline) in the ocean. Internal waves arise when there is a change in density stratification in a fluid, which may occur due to variations in temperature, salinity, or other fluctuations. In addition, a current profile with depth-dependent currents in each domain is considered. An example of the physical situation described above is clearly illustrated by the equatorial internal waves in the presence of the Equatorial Undercurrent (EUC). We consider wave propagation in the so-called intermediate long wave approximation, when one of the two fluid layers is of much smaller depth than the other layer. We derive the nonlinear integrable Intermediate Long Wave Equation (ILWE) as a propagation model. As a limiting case we obtain the integrable Benjamin-Ono (BO) equation as an internal wave model, incorporating underlying sheared current in both fluid layers. We show that the BO soliton characteristics are strongly affected by the shear current parameters.

Rossen I. Ivanov

Technological University Dublin, Ireland
rossen.ivanov@tudublin.ie

MS47

Azimuthal Flows: Exact Solutions, Stability/instability and Wave Breaking

We address a question of importance in the analysis of nonlinear partial differential equations: when does a solution to a nonlinear partial differential equation develop singularities and what is the nature of those singularities? The particular type of singularity that we consider is wave breaking which is defined as the situation when the wave remains bounded up to the maximal existence time at which its slope becomes infinite. More specifically, the wave breaking result concerns the geophysical nonlinear water wave problem for an inviscid, incompressible, homogeneous fluid, written in cylindrical coordinates together with the free surface and bottom boundary conditions.

Calin I. Martin

University of Vienna, Austria

calin.martin@univie.ac.at

MS48

Stability of Standing Waves in Next-to-Nearest Neighbour DNLS Using Borel-Pad Methods Aniceto

In this talk I will present a study of the stability of solutions of the discrete nonlinear Schrödinger models combining nearest (NN) and next-nearest (NNN) neighbor interactions, motivated by experiments in waveguide arrays. In particular, I will analyse the key eigenvalues for the stability of the pulse-like stationary states in regimes where the interactions are either cooperating or competing. To obtain these eigenvalues - their exponential dependence on the coupling parameter and prefactors, including the so called Stokes multipliers - we find that one needs to recover beyond all orders, subleading exponential contributions which cannot be obtained via the standard matched asymptotic expansions. I will present how to this can be done systematically through the methods of Borel-Padé exponential asymptotics, which provide a general asymptotic template for studying parametric problems that require the calculation of subdominant Stokes multipliers. This talk is based on the following work: Lustri, CJ, Aniceto, I, Kevrekidis, PG, "Borel-Padé exponential asymptotics for the discrete nonlinear Schrödinger model with next-to-nearest neighbour interactions", arXiv:2506.21120[nlin.PS]

Ines Aniceto

University of Southampton
i.aniceto@soton.ac.uk

MS48

Dispersive Shock Waves in D2 Lattices

We examine traveling waves and dispersive shock waves (DSWs) in a two-dimensional Fermi-Pasta-Ulam-Tsingou lattice. Using a variational approach, we demonstrate the existence of both periodic and solitary traveling waves and develop a numerical scheme that computes these solutions efficiently. The numerical results show good agreement with predictions from a KdV-type reduction. We then explore line DSWs that arise from jump initial data and propagate while remaining constant in one spatial direction. Although their shape depends on direction, key properties such as speed and amplitude remain largely unchanged. Comparisons between simulations and KdV-based approximations show close agreement in the small-jump regime.

Christopher Chong

Bowdoin College
cchong@bowdoin.edu

MS48

Phase-Shifted Nanopterons in Lattices and Related Models

A spring dimer Fermi-Pasta-Ulam-Tsingou (FPUT) lattice is an infinite, one-dimensional chain of coupled oscillators in which the coupling potentials alternate in strength. If one of the two types of couplings is taken to be infinitely strong, or "stiff," then the particles that it connects fuse together, and the dimer lattice reduces to the well-understood monatomic FPUT lattice. In particular, the monatomic FPUT lattice has solitary traveling waves. We

present recent and ongoing investigations into how these monatomic solitary waves perturb when one of the couplings is “very” stiff, but not infinitely so, and provide comparisons with other “material” limits in which a dimer FPUT lattice naturally reduces to a monatomic lattice at an extreme value for some material data.

Timothy E. Faver
Kennesaw State University
tfaver1@kennesaw.edu

MS48

Fronts in Dissipative Fermi-Pasta-Ulam-Tsingou Chains

In a dissipative Fermi-Pasta-Ulam-Tsingou chain particles interact with their nearest neighbors through nonlinear potentials (e.g. Hertzian) and linear dissipative forces. We show the existence of front solutions connecting two different uniformly compressed (or stretched) states at $\pm\infty$ using an implicit function argument starting at a suitable continuum limit in the case of very large damping. The main technical difficulties arise when showing continuity of the relevant derivatives for a setting that includes Hertzian potentials requiring tools for exponentially weighted, fractional Sobolev spaces including the Kato-Ponce inequality. A more detailed analysis allows us to determine sharp exponential decay rates which imply monotonicity of the waves.

Karsten Matthies
University of Bath
k.matthies@bath.ac.uk

Guillaume James
Grenoble INP - Ensimag
guillaume.james@univ-grenoble-alpes.fr

MS49

From explicit formulas to Wave Kinetic theory for the Benjamin-Ono equation

In this talk, we begin by introducing an explicit solution formula for the Benjamin-Ono (BO) equation recently obtained by Patrick Grard. We show how building on this representation yields new tools for understanding the long-time dynamics of BO, both theoretically and numerically. In particular, we focus on the setting of randomized initial data, where by rescaling and iterating this explicit formula, combined with non-commutative Khintchine inequality, we obtain insight on the Wave Kinetic theory for BO, up to the physically relevant kinetic time scale. This is a joint work with Gigliola Staffilani (MIT) and Felipe Hernandez (Penn State).

Yvonne Alama-Bronsard
Massachusetts Institute of Technology
yvonneab@mit.edu

MS50

Rogue-Wave Breaking in Nonlinear Dispersive Equations

The one-dimensional focusing nonlinear Schrödinger equation is a standard model for self-focusing effects in weakly nonlinear dispersive systems. The generic focusing behavior in the small-dispersion regime is a region of rapid but bounded oscillations expanding from a critical breaking

point. In 1965, Talanov discovered a solution of the dispersionless focusing NLS system that blows up in finite time and appears to model a different, non-generic type of focusing characterized by a localized peak of large amplitude. In 2017, Suleimanov identified a solution of the focusing NLS equation (with dispersion) that formally regularizes Talanov blowup. Our recent work proves Suleimanov’s conjecture for a family of initial conditions posed by Talanov. We further show that this non-generic Suleimanov-Talanov focusing is universal in the sense that it occurs for other initial conditions in the NLS equation as well as pure and mixed flows in the NLS hierarchy, such as the Hirota and LPD equations. This work is joint with Robert Jenkins and Peter Miller.

Robert J. Buckingham
Dept. of Mathematical Sciences
The University of Cincinnati
buckinrt@uc.edu

MS51

Direction Matters: Spreading of Planar Patterns Into Unstable States

In pattern-forming systems with two unbounded spatial directions, steady patterns often arise from an unstable homogeneous ground state in the wake of an invading planar front interface. Even though many systems are invariant under rotation, it turns out that the direction of the planar front in relation to the orientation of the pattern matters. I will demonstrate this by constructing slow planar invasion fronts, which leave a hexagonal pattern in their wake, in a 2d Swift-Hohenberg-type system using spatial dynamics and centre manifold theory. The primary focus of the analysis is on how the propagation direction of the front affects the resulting dynamics. Specifically, the analysis of the linear operator in the spatial dynamics systems is used to identify three different scenarios, which result in different reduced systems on the centre manifold.

Bastian Hilder
Technical University of Munich
bastian.hilder@tum.de

MS51

Metastability for Stochastic Planar Waves

We discuss the behaviour of planar travelling front solutions to reaction-diffusion equations under the influence of noise, posed both on cylindrical domains and the full space. We analyze the stochastic corrections to the speed and shape of the waves together with the dimension-dependent timescales over which stability can be maintained.

Hermen Jan Hupkes
Leiden University
hhupkes@math.leidenuniv.nl

Mark Van Den Bosch
Leiden University, Mathematical Institute
m.van.den.bosch@math.leidenuniv.nl

MS51

Patterns in the Wake of a Parameter Ramp

We study the formation of patterns in the light-sensitive CDIMA system in the presence of a slowly-varying opaque mask which induces a Turing instability as it moves in

one spatial dimension. Previous work (Ref. 3) used a sharp mask to control the formation of patterns. Here the parameter ramp induces an asymptotically constant front from which Turing patterns form in the wake of the ramp. Through numerical simulation and continuation we consider the impact of the slow ramp on wavenumber selection curves. We then study complex Ginzburg-Landau equation as a prototypical pattern-forming equation to observe these phenomena. For the cGL equation, we find that a slow passage through an absolute instability governs the leading order front position and asymptotic wavenumber. We then seek to rigorously establish existence of pattern-forming fronts through a GSPT framework and observe a novel slow-passage phenomenon through a Hopf-like singular point.

Benjamin G. Krewson
Boston University
krewson@bu.edu

MS51

Weakly Nonlinear Analysis of Localized Patterns for Reaction-Diffusion Systems

Spatially localized patterns occur for a wide variety of two component reaction-diffusion systems in the singular limit of a large diffusivity ratio. Such localized, far-from-equilibrium, patterns are known to exhibit a wide range of different instabilities such as spot annihilation and spot self-replication behavior. In a 2-D setting prior numerical simulations of the Schnakenberg and Brusselator systems have suggested that a localized peanut-shaped linear instability of a localized spot is the mechanism initiating a fully nonlinear spot self-replication event. By implementing a weakly nonlinear theory for shape deformations of a localized spot, it is shown through a normal form amplitude equation that a peanut-shaped linear instability of a steady-state spot solution is always subcritical for both the Schnakenberg and Brusselator reaction-diffusion systems. Similar weakly nonlinear theories predicting subcritical bifurcations characterizing the onset of the annihilation behavior of various spike and spot patterns is discussed.

Michael J. Ward
Department of Mathematics
University of British Columbia
ward@math.ubc.ca

Tony Wong
University of California, Los Angeles
kawahtony@gmail.com

Theodore Kolokolnikov
Dalhousie University
tkolokol@mathstat.dal.ca

MS52

Using Matrix Homotopy to Classify Material Topology, and the Ongoing Need for Sparse Pfaffian Algorithms

We present a unified computational and mathematical framework for diagnosing topological phases in materials by leveraging matrix-homotopy methods and efficient numerical techniques for skew-symmetric matrices. In particular, we consider quasicrystalline low-energy models, as well as fully-wave models, and show how the sign of a particular matrix Pfaffian serves as a topological invariant for such materials. However, this means that the efficient and

numerically robust computation of the Pfaffian for sparse matrices is a key bottleneck in scaling the approach to realistic material Hamiltonians and disordered or aperiodic systems. Although dense and banded Pfaffian routines exist, there remains a pressing need for sparse-matrix factorization algorithms tailored to exploit locality, sparsity and structure (symmetry, block-diagonality, low-rank updates) in topological material contexts. We illustrate the synergy of these ideas in a model tight-binding system and a photonic crystal analog, demonstrating (i) how matrix-homotopy invariants detect fragile topology even in finite, disordered, and gapless systems, and (ii) how a sparse factorisation approach accelerates the sign-Pfaffian computation by orders of magnitude, making the diagnostic practical for large scale simulations. This work thus bridges advanced topological classification of materials with computational algorithmic demands, paving the way for robust large-scale application of matrix-homotopy diagnostics in material science.

Alexander Cerjan
Sandia National Laboratory
awcerja@sandia.gov

MS52

Discrete Breathers in a Honeycomb Lattice Near a Semi-Dirac Point

Discrete breathers are spatially localized and time-periodic solutions that exist on nonlinear lattices. We develop an effective continuum PDE description of breathers in a 2D dimerized honeycomb lattice near a point in the dispersion landscape known as a semi-Dirac point varying linearly in one direction and quadratically in the other. We then analyze the existence of spectrally stable breathers near the so-called anti-continuum limit and, through numerical continuation seeded from exact solutions in this limit, obtain spatially extended breathers that approach gap solitons of the derived PDE approximation.

Andrew Hofstrand
New York Institute of Technology
ahofstra@nyit.edu

MS52

Chiral Solitary Waves in a Nonlinear Topological Insulator Model

An outstanding challenge in the field of topological insulators is the realization of nonlinear systems that support coherent traveling waves. Highly nonlinear lattices can suffer from significant radiation losses due to Peierls-Nabarro effects. In this talk a nonlinear tight-binding model that supports robust traveling edge states is proposed and examined. This system possess a nontrivial local Chern topology and soliton-like states. This talk will also detail the inelastic interaction when a traveling solitary wave collides with a stationary mode.

Troy Johnson
University of Colorado, Colorado Springs
tjohnso3@uccs.edu

Justin Cole
University of Colorado Colorado Springs

jco13@uccs.edu

MS52

Data-Driven Approximations of Topological Insulator Systems

When measuring topological insulators in a lab, one can reasonably expect to have noisy or disordered data. We have implemented a nonlinear least squares algorithm to derive and compute interaction coefficients for an SSH-type tight binding model for the linear one-dimensional Schrodinger equation for topological insulator systems. We inspect the performance of this algorithm when presented with disordered data, its ability to reproduce the topological properties of the underlying data, and convergence under many realizations.

Michael Nameika

University of Colorado, Colorado Springs
mnameika@uccs.edu

MS53

On Some Integrable Nonlinear Equations with Algebraic Constraints

We consider matrix nonlinear partial differential equations in two independent variables that are integrable with respect to inverse scattering method. Their Lax pairs depend in a rational manner on spectral parameter and are subject to some additional algebraic constraints. The simplest example of such a nonlinear evolution equation is the well-known Heisenberg ferromagnet equation for any simple Lie algebra. Similar but somewhat more complicated examples are given by the Golubchik-Sokolov equation, and the Gerdjikov-Mikhailov-Valchev equation. This talk is dedicated to nonlinear evolution equations that contain those examples as special cases (reductions).

Tihomir I. Valchev

Institute of Math. and Informatics, BAS, Sofia, Bulgaria
tiv@math.bas.bg

MS54

Resonant Vector Bundles, Conjugate Points, and the Stability of Pulse Solutions to the Swift-Hohenberg Equation Using Validated Numerics

Motivated by the goal of providing a computer assisted proof of the (in)stability of pulse solutions to the Swift-Hohenberg equation via the Maslov index, we develop a new technique based on the parameterization method to compute resonant unstable (resp. stable) vector bundles over stable (resp. unstable) manifolds. Such resonant vector bundles are generic to Hamiltonian systems, and obstruct the differentiability of invariant (un)stable vector bundles. This parameterization of the bundles gives us a highly effective coordinate system with which to compute, via validated numerics, the so-called conjugate points associated with the bundles, and hence rigorously count the number of unstable eigenvalues associated with a given pulse solution.

Jonathan Jaquette

New Jersey Institute of Technology
jonathan.jaquette@njit.edu

MS54

Validated Numerics for Differential Equations on

Level Set Manifolds

I'll discuss validated numerical methods for computing (i) chart maps for manifolds defined by level sets, and (ii) invariant objects for vector fields defined on the manifold by either geodesic flow or projection of an external field. This is joint work with J.P. Lessard and Konstantin Mischaikow.

Jason D. Mireles James

Florida Atlantic University
jmirelesjames@fau.edu

MS54

Construction of the Inverse Laplace Operator under Neumann Boundary Conditions Using the Zernike Polynomials

In this talk, we introduce a method for constructing the inverse of the Laplacian in coefficient space based on a Zernike-polynomial expansion for nonlinear elliptic partial differential equations (PDEs) with Neumann boundary conditions on the unit disk. Zernike polynomials form an orthogonal basis on the unit disk, and their polynomial structure together with rotational symmetry enables a matrix representation of the Laplacian on subspaces with fixed azimuthal index. We analyze the Laplacian in terms of Zernike-polynomial coefficients and represent its action as a finite-dimensional transformation matrix. The resulting matrix has a banded structure, making it feasible to construct the inverse Laplacian via direct matrix inversion. This approach provides a framework for computer-assisted proofs based on the Newton-Kantorovich theorem in coefficient spaces for Neumann boundary value problems of nonlinear elliptic PDEs. This is joint work with Tomoya Sakata (University of Tsukuba).

Akitoshi Takayasu

University of Tsukuba
takitoshi@risk.tsukuba.ac.jp

MS54

Computer-Assisted Proofs for Localized Traveling Waves in the Two-Dimensional Suspension Bridge Equation

Traveling waves are a central feature in the dynamics of the suspension bridge PDE. While most studies have focused on the one-dimensional case, we develop computer-assisted proof (CAP) techniques to rigorously establish the existence of periodic and localized traveling waves in two dimensions. Our approach relies on a Newton-Kantorovich framework combined with careful Fourier analysis, where the main challenge is the combination of the exponential nonlinearity and the large amplitudes of the wave. By deriving computable bounds that control aliasing errors, we can control both the residue and the contraction constant of the fixed point operator sufficiently to prove the existence of traveling wave solutions. Moreover, the methodology provides explicit bounds on numerical deficiencies and allows us to enclose the spectrum of the linearized operator rigorously, yielding conclusions about orbital stability. This talk is based on work with and by Lindsey van der Aalst, Matthieu Cadiot, Jean-Philippe Lessard.

Jan Bouwe Van Den Berg

Vrije Universiteit Amsterdam, The Netherlands

janbouwe.vanden.berg@vu.nl

MS55

Vegetation stripes in a nonlocal Klausmeier model

In this talk we consider a nonlocal version of the Klausmeier model describing the interaction between plant biomass and water availability in arid ecosystems. In contrast to the original set of reaction-diffusion equations our model assumes that the spread of plant biomass occurs through long-range dispersal events, which we describe using a convolution operator. In order to understand how this nonlocal form of diffusion affects the formation of periodic patterns we derive an amplitude equation using a multiple-scale analysis and rigorously justify the existence of these structures using Lyapunov-Schmidt reduction. We show that nonlocal diffusion restricts the region in parameter space where periodic patterns can exist and gives rise to striped patterns with narrower bands. In addition, we find that depending on how one defines the convolution operator, the equations may allow for smaller diffusivity ratios when compared to the original Klausmeier model.

Gabriela Jaramillo

Department of Mathematics
University of Houston
gabriela@math.uh.edu

MS55

Pattern Selection Via Three-Wave Interactions in Faraday Waves

Three-wave interactions (3WIs), where the sum of two wavevectors equals a third one, can be used to explain pattern-forming behaviour in the Faraday wave experiment close to onset. The experiment involves periodically forcing a container of fluid up and down and observing the patterns formed on the surface. When the forcing exceeds some threshold, the flat state becomes unstable, which can lead to a variety of patterns. In the case of single frequency forcing, simple patterns with one wavenumber are observed, such as stripes, squares and hexagons. However, if multiple-frequency forcing is used, 3WIs can form between waves with two critical wavenumbers, leading to more complex structures such as superlattices, quasipatterns and spatiotemporal chaos. We consider problems with two critical wavenumbers, where 3WIs form between two waves of a larger wavenumber and a third wave of a smaller wavenumber. The inclusion of a second wavenumber allows for patterns to form on both a rhombic and hexagonal lattice. The dynamics exhibited by these 3WIs can be represented by a system of ODEs. This system is generalised, such that any pattern-forming system exhibiting 3WIs (with two wavelengths) can be reduced to these equations. We present an analysis of the patterns formed by a model PDE, an adaptation of the Lifshitz-Petrich equation with additional nonlinearities. We use weakly nonlinear theory to draw comparisons between the patterns predicted by the ODEs and those observed in the PDE.

Laura Pinkney

University of Leeds
mm18lp@leeds.ac.uk

Alastair M. Rucklidge
School of Mathematics
University of Leeds
A.M.Rucklidge@leeds.ac.uk

Cedric Beaume
University of Leeds
School of Mathematics
c.m.l.beaume@leeds.ac.uk

MS55

(Down)shifting Paradigms in Water Waves: What Processes Drive the Spectral Shifts?

Despite centuries of study, there continue to be many open problems in dynamics of water waves. One such problem is that of frequency downshifting, a phenomenon in water waves where the spectral peak and/or mean of the wave spectra permanently moves downwards over the wave's evolution. Typically this phenomenon is argued to be primarily driven by dissipative processes such as wavebreaking, with energy loss being the only pathway to permanent spectral movement. However, recent work by the authors have demonstrated that this phenomenon can happen conservatively (i.e. without energy loss) and can be mediated by wave-mean flow interactions. Since both processes are shown to contribute to downshifting, and both are present within water wave problems, this provokes the natural question - how do they interact and contribute to the overall process of downshifting in water waves? This talk will start with an overview of downshifting from a wave-mean flow perspective in order to demonstrate how such a process mediates downshifting, by using a Benney-Roskes system to illustrate the fundamentals of the process. We will then introduce (simple) dissipative effects and explore how this impacts movements in the spectral peak as a function of nondimensional depth. Ultimately we find that deep into the modulationally unstable regime dissipation leads to more monotonic and narrow-banded downshifting than that driven by purely mean flow effects, suggesting that dissipation plays a long-term role in arresting the downshifting phenomena.

Daniel J. Ratliff

Northumbria University
daniel.ratliff@northumbria.ac.uk

MS55

Novel Preconditioning Methods for Solving Three-Dimensional Nonlinear Flexural Gravity Wave Problems

We present fully nonlinear three-dimensional flexural-gravity wave solutions (waves under ice) in a variety of physically motivated settings. The problem is reformulated using a boundary integral approach and then discretized to obtain a large nonlinear system. We demonstrate how our novel preconditioner construction, applied within a Newton-Krylov solver, improves convergence and allows us to obtain well-resolved solutions on discretized domains with large numbers of points in a shorter amount of time compared to traditional methods.

Claudia C. Tugulan

The University of Western Ontario
ctugulan@uwo.ca

MS56

Analytical and Computational Methods for Bifurcation Analysis of Collapsing Solutions to Nonlinear Dispersive PDEs

In this talk, the finite-time blow-up in nonlinear dispersive

PDEs will be discussed as a bifurcation problem. First, we will present a universal framework for the identification of self-similar waveforms as stationary solutions in a frame that co-explodes with the solution. This will allow us to perform a spectral analysis of these solutions in the co-exploding frame in order to infer their stability. As prototypical examples, we will consider the 1D Nonlinear Schroedinger (NLS) equation with power-law nonlinearity and generalized Korteweg-de Vries (gKdV) model. Self-similar collapsing branches emanate from the solitary branch at critical nonlinearity exponent therein. However, their stability analysis will reveal the emergence of unstable modes which in turn are intimately connected with symmetries of the systems in the original frame. We will show that these modes do not correspond to true instabilities but rather to neutral eigen-directions. Numerical results both at the existence and stability will be compared with normal forms and WKB results where an excellent agreement will be observed between the two. Then, time-permitting, we will depart from 1D settings and focus our considerations on the 2D NLS model with power-law nonlinearity. The accurate computation of states and their stability requires to use alternative techniques. Indeed, we will present a computational framework that have been developed in the open source software FreeFEM that combines domain-decomposition methods and mesh adaptivity tools. Novel results will be discussed together with the performance of the numerical codes.

Efstathios G. Charalampidis
San Diego State University
San Diego
echaralampidis@sdsu.edu

MS56

Large-amplitude vortex-carry solitary gravity waves

We consider two-dimensional steady gravity waves in finite-depth water. We prove that, for any supercritical Froude number, there exists a continuous one-parameter family of solitary waves in equilibrium with a submerged point vortex. This family bifurcates from an irrotational uniform flow, and, at least for large Froude numbers, extends up to the development of a surface singularity. These are the first rigorously constructed gravity wave-borne point vortices without surface tension. Notably our formulation allows the free surface to be overhanging, and we provide numerical evidence that strongly suggests that some of these waves indeed overturn. This is a joint work with Kristoffer Varholm, Samuel Walsh, and Miles Wheeler.

Ming Chen
University of Pittsburgh
mingchen@pitt.edu

Kristoffer Varholm
Norwegian University of Science and Technology
kristoffer.varholm@pitt.edu

Samuel Walsh
University of Missouri
walhsa@missouri.edu

Miles H. Wheeler
University of Bath

mw2319@bath.ac.uk

MS56

Asymptotic Stability of Smooth Solutions to Peakon Equations

The Camassa-Holm (CH) equation, originally derived as an asymptotic model in shallow water wave theory, is notable for admitting weak soliton solutions known as peakon-solitary waves with a peaked profile and a discontinuous first derivative. Since its discovery, a variety of related peakon equations have been studied, both integrable and non-integrable. Among these, the integrable Degasperis-Procesi (DP) equation stands out. It was identified by applying asymptotic integrability conditions up to third order to a family of nonlinear evolution equations, revealing that only the KdV, CH, and DP equations satisfy these constraints. In this talk, we focus on the asymptotic stability of smooth one-soliton solutions to the DP equation. The analysis presents several challenges: the associated linearized operator includes a non-local term, and the soliton solutions evolve on a nonzero background. Moreover, the Lax pair for the DP equation involves a third-order non-self-adjoint operator, in contrast to the CH equations second-order self-adjoint formulation. These differences significantly impact the techniques required to obtain the stability results.

Stephane Lafortune
College of Charleston
lafortunes@cofc.edu

MS56

A multi-soliton traveling wave emerged from internal solitary wave interactions with infinite depth

We present the discovery and analysis of a novel coherent structure in the (2+1)-dimensional Benjamin-Ono (2DBOII) equation. By modulating and modeling the multi-soliton solutions as shock solutions to an associated system of conservation laws, we derive jump conditions governing this multi-wave structure. This framework is applied to the Mach reflection problem, where an incident soliton encounters an oblique corner. Unlike in other known wave models, this interaction in the 2DBO equation produces four distinct waves: the original, a reflected wave, a dominant "Mach stem," and a unique, small-amplitude fourth wave. Our analytical results, obtained by solving the modulation jump conditions, show great agreement with numerical simulations with a pseudo-spectral scheme, confirming that the multi-soliton structure is a traveling wave solution.

Christina Wuyan Wang
University of Colorado at Boulder
wuyan.wg@gmail.com

Mark A. Hoefler
University of Colorado, Boulder
U.S.
hoefler@colorado.edu

MS57

Spectral stability of large-amplitude shock profiles in scalar hyperbolic-hyperbolic equations

In this talk a brief account of the current stability theory of shock profiles in hyperbolic-hyperbolic equations is

given. Such equations have received increasing attention in recent proposals for theories of viscous relativistic fluids (e.g. Freisthler and Temple (2014), Bemfica, Disconzi, and Noronha (2018)) in which various short-comings of previously existing theories, like lack of causality, have been successfully overcome. Focusing on spectral stability, a small-amplitude result is recalled (B. 2024), and we report on the spectral stability of large-amplitude shock profiles for general scalar semi-linear hyperbolic-hyperbolic equations of the form

$$g(u)_t + f(u)_x = -(\partial_t + c_1 \partial_x)(\partial_t + c_2 \partial_x)u$$

under the sub-characteristic condition

$$c_1 < \frac{f'}{g'} < c_2.$$

The result is part of future work with Sroczinski and Zumbrun on one-dimensional nonlinear stability of shock profiles complementing their multi-dimensional result from 2025.

Johannes Baerlin

Indiana University Bloomington
jbarlin@iu.edu

MS57

Computer Assisted Proof of Stability of Traveling Waves

We discuss recent work toward showing stability of traveling wave solutions of the Kortewegde Vries Burgers equation using computer assisted methods of proof. In particular, our goal is to use rigorous computation to obtain a rigorous error bound on the Evans function to prove stability of traveling waves.

Blake Barker

Brigham Young University
blake@math.byu.edu

Jared Bronski

University of Illinois at Urbana-Champaign, U.S.
bronski@illinois.edu

Vera Hur

University of Illinois Urbana-Champaign
verahur@illinois.edu

Zhao Yang

Academy of Mathematics and Systems Science, CAS
yangzhao@amss.ac.cn

MS57

Stability of discrete shock profiles for systems of conservation laws

This talk deals with the stability analysis of discrete shock profiles for systems of conservation laws. These profiles correspond to approximations of shocks of systems of conservation laws by conservative finite difference schemes. Such discontinuous solutions appear naturally in the study of systems of conservation laws, which can model many physical situations, such as gas dynamics. Existence and stability of discrete shock profiles for each stable shock of the approximated system of conservation laws is seen as an improved consistency condition and implies that the finite difference scheme should approach discontinuities fairly precisely. The aim of the talk will be to review some stability results regarding discrete shock profiles and to present

a recent effort to extend them. More precisely, most results known up until recently are focused on the stability of discrete shock profiles associated with shocks of small amplitude. The talk will focus on a nonlinear stability result for discrete shock profiles in quite a general setting, where the smallness assumption on the shocks amplitude is replaced by a spectral stability assumption on the linear operator obtained by linearizing the numerical scheme about the discrete shock profile. This nonlinear stability result relies on a precise description of the Greens function of the linearization about discrete profiles.

Lucas Coeuret

Université de Lorraine
France

lucas.coeuret@univ-lorraine.fr

MS57

Behavior of Small-Amplitude Waves in Thin Films with Surfactants

We study a coupled system of nonlinear PDE derived from a lubrication model of a thin film with an insoluble surfactant flowing down an inclined plane. Motivated by understanding the stability of viscous shocks in this model, we first use Fourier analytic techniques to understand the stability of uniform states. We find that the gravity-driven nature of the problem yields fast- and slow-wave modes. This physically motivated system is unusual because in some parameter regimes it has a lack of dissipation. For the fast mode, we find the long-time behavior approaches similarity solutions of a certain Burgers' equation. For the slow mode, we observe that the form of the solution is sensitive to the form of regularization present in the system and can yield diffusive or dispersive waves in different regimes. Finally, our linear analysis yields predictions of the diffusion and dispersion coefficients for reduced models for the long-time behavior. This work is joint with Thomas Witelski.

Aric Wheeler

Duke University
USA
aric.wheeler@duke.edu

Thomas P. Witelski

Duke University
Department of Mathematics
witelski@math.duke.edu

MS59

Computations of Traveling Waves and Spiral Waves in Cardiac Arrhythmia

Spiral waves patterns are commonly modeled by reaction-diffusion equations and their linear stability can be probed by computing the spectra of the operator linearized about the pattern. However, this process can be numerically challenging. It is known that when an operator is posed on a spatially extended domain, the norm of the resolvent can grow exponentially with the size of the domain, leading to numerical instabilities and large pseudospectra bounds. This fact has been previously studied in the convection-diffusion operator, but the operators from spiral waves are no different. Thus, when applied to spiral wave problems, standard sparse eigenvalue algorithms result in inaccurate and spurious results. In this work, we showcase that the resolvent norm of spiral wave operators can be bounded by considering the operator in an exponentially weighted space, with the exponential weight derived from the spa-

tial eigenvalues of the asymptotic linearized operator. We demonstrate numerically how the exponential weight stabilizes eigenvalue computations and allows the spectra of relevant spiral wave operators to be efficiently computed using sparse matrix methods. Both the convection-diffusion operator and spiral waves in the Barkley model are used to showcase this phenomenon.

Stephanie Dodson
Colby College
sdodson@colby.edu

MS59

Think Global, Act Local: Inducing Fully Localised Planar Patterns

The existence of localised two-dimensional patterns has been observed and studied in numerous experiments and simulations: ranging from optical solitons, to patches of desert vegetation, to fluid convection. And yet, our mathematical understanding of these emerging structures remains extremely limited beyond one-dimensional examples. In this talk I will discuss how adding a compact region of spatial heterogeneity to a PDE model can not only induce the emergence of fully localised 2D patterns, but also allows us to rigorously prove and characterise their bifurcation. The idea is inspired by experimental and numerical studies of magnetic fluids and tornados, where our compact heterogeneity corresponds to a local spike in the magnetic field and temperature gradient, respectively. In particular, we obtain local bifurcation results for fully localised patterns both with and without radial or dihedral symmetry, and rigorously continue these solutions to large amplitude. Notably, the initial bifurcating solution (which can be stable at bifurcation) varies between a radially-symmetric spot and a dipole solution as the width of the spatial heterogeneity increases. This work is in collaboration with David J.B. Lloyd and Matthew R. Turner (both University of Surrey).

Dan J. Hill
University of Oxford
dan.hill@maths.ox.ac.uk

David Lloyd, Matthew R. Turner
University of Surrey
d.j.lloyd@surrey.ac.uk, m.turner@surrey.ac.uk

MS59

Existence of Stationary Non-Radial Localized Patterns in the Planar SwiftHohenberg Equation

In this talk, we present a constructive approach to proving the existence of smooth, stationary, non-radial localized patterns in the planar SwiftHohenberg equation. Starting from a numerical approximation, we build an approximate inverse of the linearized operator and implement a Newton-Kantorovich framework to obtain a sufficient condition for the existence and local uniqueness of a nearby localized solution. This condition is verified using a combination of analytic estimates and rigorous numerical computations. We also derive and validate a condition ensuring that this localized pattern arises as the limit of a family of spatially periodic solutions as the period tends to infinity. As an application, we provide computer-assisted proofs for the existence of three distinct unbounded branches of periodic solutions converging toward a constructively validated localized planar pattern. This is joint work with Matthieu Cadiot (cole Polytechnique, Paris) and Jean-Christophe

Nave (McGill).

Jean-Philippe Lessard
McGill University
jp.lessard@mcgill.ca

MS59

Anchored Spiral Waves in the Theta Model

We investigate a population of simple oscillators, coupled diffusively, $\partial_t \theta = \Delta \theta + 1 + \mu \cos(\theta)$ on an annulus $\Omega = \{R_i < |x| < R_o\} \subset \mathbb{R}^2$ with $\theta \in S_{2\pi}^1 \cong \mathbb{R}/(2\pi\mathbb{Z})$. For $0 < \mu < 1$, the kinetics are oscillatory; for $\mu > 1$ they are excitable. We show that the system supports rotating waves for all $\mu > 0$, with nonzero degrees as maps from $\Omega \rightarrow S^1$, using a global homotopy to $\mu = 0$. For $R_o \rightarrow \infty$, these rotating waves converge to almost Archimedean spirals, which we investigate in the limit $\mu \sim 0$. We also show some numerical results and comment on stability.

Nan Li
University of Minnesota Twin Cities
li002843@umn.edu

Arnd Scheel
University of Minnesota, Minneapolis
School of Mathematics
scheel@math.umn.edu

MS60

Spiral Waves in Optical Lattices

Nonlinear dispersive systems support a rich variety of fascinating solutions, such as solitons, dispersive shock waves, rarefaction waves, instantons, to name a few. A far less studied class of solutions exhibit spiral rotations and their dynamics. A class of spiral solutions in energy preserving dispersive wave systems is studied; focusing on a set of nonlinear Dirac and Lieb systems that arise as continuum limits from honeycomb and Lieb optical lattices. In the linear limit, envelope dynamics for both lattices are governed by the 2+1 dimensional Klein-Gordon equation. Using stationary phase approximations we find spiral solutions that are algebraically decaying spatially and derive algebraic equations for the structure of the spiral.

Sean Nixon
University of Colorado
sean.nixon@colorado.edu

Justin Cole
University of Colorado Colorado Springs
jcole13@uccs.edu

Mark Ablowitz
Dept. of Applied Mathematics
University of Colorado
mark.ablowitz@colorado.edu

MS60

Stability Theory of Flat Band Solitons in Discrete Nonlinear Schrodinger Equation

In this talk, we study the discrete nonlinear dynamics of waves in a class of lattices, whose low amplitude limit has ‘flat band’ spectrum. In particular, we study the stability of compactly supported, homogeneous-density and symmetric minimal compact solitons, or MCS states, of the

time-dependent discrete nonlinear Schrodinger equation in such lattices. Our approach combines Sherman-Morrison formula with functional analytic tools, and allows us to derive a sharp stability criterion in terms of the explicit form of the nonlinearity and the projection of distinguished vectors onto the flat band eigenspace. We apply this general theory to MCS states in the DNLS for the 1D diamond, 2D Kagome and checkerboard lattices. In lattices where MCS states are unstable, we demonstrate how to engineer the nonlinearity to stabilize small amplitude MCS states. Finally, via systematic numerical computations, we put our analytical results in the context of global bifurcation diagrams.

Cheng Shi
Columbia University, U.S.
cs4223@columbia.edu

Ross Parker
Center for Communications Research - Princeton
rosshparker@gmail.com

Panayotis Kevrekidis
University of Massachusetts, Amherst
kevrekid@umass.edu

Michael I. Weinstein
Columbia University
Dept Appl Phys & Appl Math
miw2103@columbia.edu

MS60

Stability of Ginzburg-Landau Pulses Via Fredholm Determinants of Birman-Schwinger Operators

We introduce a novel computational method to determine the spectral stability of stationary pulse solutions of nonlinear wave equations such as the cubic-quintic complex Ginzburg-Landau equation in one spatial dimension. We show that the point spectrum of the linearization of the equation about a pulse is given by the zero set of the regular Fredholm determinant of a trace-class Birman-Schwinger operator. This operator is defined in terms of a Green's kernel for the linearized equation. We adapt a method of Bornemann to numerically approximate the Fredholm determinant by a matrix determinant, and we quantify the error in this approximation. Numerical results show excellent agreement with existing methods. The new method avoids the computational challenge of solving a stiff system for the Jost solutions that is inherent in the standard approach based on computation of the Evans function. The motivation for developing the method was for future extensions to determine the spectral stability of time-periodic pulses, for which an Evans function most likely cannot be defined.

John Zweck
New York Institute of Technology
jzweck@nyit.edu

Erika Gallo
University of Texas at Arlington
erika.gallo@uta.edu

Yuri Latushkin
Department of Mathematics
University of Missouri-Columbia

latushkiny@missouri.edu

MS61

Anomalous Invasion in a Fisher-Kpp Model with Diffusive Source

It has long been established that speed selection of the Fisher KPP family of fronts is determined by the leading edge of initial conditions and that for compactly supported initial conditions, a unique speed corresponding to a marginally stable front is selected. For coupled FKPP equations the phenomenon of anomalous spreading is observed, that is, a propagation speed is selected that is significantly larger than what is observed in the uncoupled case. Results, relying on comparison principles, exist establishing the stability and selection of these greater, so called anomalous, speeds by compactly support initial conditions. Here we develop a semigroup approach to probe the stability and selection of these anomalous fronts.

Anthony Cortez
University of Minnesota
acortez@umn.edu

MS61

Traveling Waves in a Holling-Tanner System with Prey Cannibalism

We consider a Holling-Tanner type predator-prey model. Earlier work has shown that incorporating prey cannibalism alters the stability of the coexistence equilibrium. In this talk, we discuss how this change in stability affects the existence of traveling fronts in the corresponding reaction-diffusion system.

Anna Ghazaryan
Department of Mathematics
Miami University
ghazarar@miamioh.edu

MS61

Sharp Long-Time Front Asymptotics for Cascading Fisher-Kpp Systems

In this talk, I will present results on the long-time behavior of reaction-diffusion systems of Fisher-KPP type with interacting components which can be associated with multitype Branching Brownian Motion. For a coupled two-component Fisher-KPP system, the interactions modify the classical Bramson logarithmic delay to a $12\log^2 t$ correction. This PDE-based proof confirms previously known probabilistic results for two-type Branching Brownian Motion and extends them to general Fisher-KPP nonlinearities. I will then discuss more general cascading systems of coupled equations associated with Branching Brownian Motion with k -many types and establish sharp front asymptotics and convergence to the minimal-speed Fisher-KPP traveling wave. This provides a PDE proof of the conjectured asymptotics for the median position of the rightmost particle in cascading Branching Brownian Motion.

Alexandra Stavriani
Universitaet Muenster

alex.st@uni-muenster.de

MT1**Computer-Assisted Analysis for Pdes**

In recent years, computer-assisted proofs have become a central tool in the study of PDEs, particularly for establishing constructive existence results for nontrivial solutions. In this mini-tutorial, I present a methodology for constructively proving the existence of solutions to PDEs on R^m that vanish at infinity, such as localized patterns or solitary traveling waves. The tutorial introduces the analytical framework underlying the existence proof, based on a fixed-point argument, together with rigorous numerical techniques used to compute explicit quantitative estimates. These estimates are essential for validating the existence of a genuine solution in a neighborhood of a numerically computed approximate solution. Lecture notes and a code skeleton will be provided to guide participants through both the theoretical and computational aspects. References: [1] M. Cadiot, J.-P. Lessard, J.-C. Nave, SIADS, (2024). [2] M. Cadiot, Nonlinearity, (2025). [3] M. Cadiot, L. van der Aalst, arXiv:2509.16693 (2025). [4] M. Cadiot, arXiv:2502.20644 (2025). Organizer: Matthieu Cadiot

Matthieu Cadiot

McGill University

Department of Mathematics

matthieu.cadiot@polytechnique.edu

MT2**Semiclassical Analysis of Water Waves**

In this mini-tutorial we will revisit the classical topic of water waves propagating over variable currents and depth. We will formulate the problem in terms of the wave amplitude and surface velocity potential using the Dirichlet-to-Neumann operator. Next, we study asymptotic reformulations of this wave system using pseudo-differential operators; in particular, we will see why the semiclassical Weyl quantization of the variable-depth symbol is the correct choice for asymptotic analysis. We then conclude by deriving several classical (and some non-standard) results from the water waves literature from the asymptotic Weyl-calculus and WKB analysis. Throughout the lecture we will use Jupyter Notebooks to explore examples and aid our understanding.

Adrian Kirkeby

Simula Research Laboratory

Norway

adrian@simula.no

PP1**Analysis of a Non-Local Swift Hohenberg Equation**

We study striped pattern formation in a nonlocal Swift-Hohenberg equation in which the nonlocality is introduced by modifying its Fourier symbol and reverse-engineering the corresponding linear operator. Using a multiple-scale expansion, we derive a Ginzburg-Landau amplitude equation near the critical threshold. We then focus on one-dimensional striped solutions and prove their existence rigorously via Lyapunov-Schmidt reduction in the parameter regime where the amplitude description remains valid.

Johnathan A. Andres

The University of Houston

Mathematics Department

jaandres@cougarnet.uh.edu

Gabriela Jaramillo

Department of Mathematics

University of Houston

gabriela@math.uh.edu

PP1**Well-Posedness of Generalized Camassa-Holm Equations**

The classical Camassa-Holm (CH) equation is used to describe dynamics of shallow water waves, and features interesting behavior such as solitons or wave breaking. The study of CH has been extensively investigated in the literature. In this talk, we consider a generalized version of CH where the momentum can be of arbitrarily high order and the nonlinearity can be of any polynomial order. More precisely, the equation reads as

$$m_t + m_x u^p + b m u^{p-1} u_x = -(g(u))_x + (b+1)u^p u_x, \quad \text{where } m = (1 - \partial_x^2)^k u,$$

where $p \geq 1$, $k \geq 1$, b is a real parameter, and $g(u)$ is a smooth function. We prove local well-posedness using Kato's semigroup, where nonlinearity is treated directly using commutator estimates and the fractional Leibniz rule without having to use tricky manipulation. Furthermore, in the case where the momentum is conserved, we show that the solution is in fact global.

Nesibe Ayhan, Bao Quoc Tang

University of Graz

nesibe.ayhan@uni-graz.at, quoc.tang@uni-graz.at

Nilay Duruk Mutlubas

Sabanci University

nilaydm@sabanciuniv.edu

PP1**Orbital Stability of Periodic Waves in Hamiltonian Systems Against Localized Perturbations**

We investigate the stability and long-term behavior of spatially periodic solutions in Hamiltonian systems against localized disturbances. Such periodic waves often correspond to robust structures that arise in a multitude of physical settings, with notable examples including water waves, periodic light pulses in nonlinear optical fibers, or wave trains in Bose-Einstein condensates. To date, nonlinear stability results for periodic waves in Hamiltonian systems have primarily addressed co-periodic or subharmonic perturbations. Their stability with respect to localized perturbations - a natural setting in many physical applications - remains a longstanding open problem, as such perturbations render the wave neither localized nor periodic, placing its stability analysis outside the scope of the classical orbital stability framework for Hamiltonian systems developed by Grillakis, Shatah, and Strauss. We present an alternative approach that combines variational techniques, leveraging conserved quantities tailored to the perturbation equation, with Duhamel-based methods and a modulational ansatz - tools known from the stability analysis of periodic waves in dissipative systems and reaction diffusion models. Using this approach, we establish first orbital stability results in key Hamiltonian models, such as the Klein-Gordon, Korteweg-de Vries and nonlinear Schrödinger equation with respect to L^2 -localized perturbations. This is joint work with Bjrn de Rijk (KIT).

Emile Bukieda

Karlsruhe Institute of Technology (KIT)
emile.bukieda@kit.edu

Björn de Rijk
Karlsruhe Institute of Technology
bjoern.rijk@kit.edu

PP1

Stability of Asymptotically Oscillatory Multipulses: Conjugate Points Lie on a Compact Interval

In this poster we consider the stability of stationary multimodal pulse (multipulse) solutions with asymptotically oscillatory tails for a class of fourth-order scalar PDEs posed on an unbounded domain. Recent efforts have sought to equate the number of unstable temporal eigenvalues to conjugate points using the Maslov Index. We first show that these conjugate points can only exist in a compact interval and then use this fact to conclude that the tail oscillations do not impact the stability of the multipulse. This is consistent with existing conjectures about counting the number of conjugate points and thus unstable eigenvalues directly from a spatial profile of a multipulse.

Cameron Edgar
Boston University
cse1@bu.edu

Margaret Beck
Boston University, U.S.
mabeck@bu.edu

PP1

Coherent Dynamics in Networks of Soft-Threshold Integrate-and-Fire Neurons on the Ring

We study bifurcations in networks of integrate-and-fire neurons with stochastic spike emission, focusing on the effects of the spatial and temporal structure of the synaptic interactions. Using a deterministic mean-field approximation of the population dynamics, we characterize spatial, temporal, and spatiotemporal patterns of macroscopic activity. In the mean-field theory, synaptic delays give rise to uniform oscillations across the population through a subcritical Hopf bifurcation of the stationary uniform equilibrium. With local excitation and long-range inhibition the network undergoes a Turing bifurcation, resulting in a localized area of sustained activity, or stationary bump. When the coupling has both delays, local inhibition, and long range excitation, the network undergoes a Turing-Hopf bifurcation leading to spatiotemporal dynamics, such as standing and traveling waves. When multiple instabilities are excited, we observe other complex spatiotemporal dynamics. We confirm all these predictions of the mean-field theory in simulations of the underlying stochastic model.

Lauren Forbes
Boston University
U.S.
lforbes@bu.edu

Jared Grossman
Boston University
jaredg@bu.edu

Montie Avery
Emory University

montie.stuart.avery@emory.edu

Ryan Goh
Boston University
Dept. of Mathematics and Statistics
rgoh@bu.edu

Gabriel Ocker
Department of Mathematics and Statistics
Boston University
gkocker@bu.edu

PP1

A Study of Benjamin Bona Mahony Type Equations

Pamela Guerrero
University of Tennessee
pguerrer@utm.edu

PP1

A Dominant Balance Series Solution to the Rayleigh Collapse

For the classic Rayleigh Collapse problem for a single spherical vacuum in an infinite fluid, the exact analytic solution to the nonlinear ordinary differential equation that governs the bubble radius during the collapse is first obtained via a very slowly converging direct power series, and this includes a derivation of the formula of the series coefficients in general. Two different asymptotically motivated factorizations are both considered including one previously discussed in the literature, and in both cases, this leads to an acceleration of the convergence of the series. To achieve machine precision, it is shown that the prior methods require an impractical number of terms varying from just over 85 million to just over 1.1×10^{37} series terms. Next, a dominant balance series solution, which is an exact analytic series solution as a function of a gauge variable that removes the singular behavior, was found as well, and a comparison of the estimated numerical errors of the different solution methods was considered. The error analysis shows that the dominant balance series achieves optimum computational efficiency by reaching machine precision with only 35 series terms.

Daniel P. Hobbs, Anthony Harkin, Nathaniel S. Barlow, Steven Weinstein
Rochester Institute of Technology
dph2386@rit.edu, harkin@rit.edu, nsbsma@rit.edu, sjweme@rit.edu

PP1

Tensor-Based Computation of the Koopman Generator via Operator Logarithm

Identifying the governing equations of nonlinear dynamical systems from data is a fundamental challenge. Indirect methods based on the Koopman operator logarithm can avoid the numerical differentiation of data, offering robustness to noise and larger sampling intervals. However, constructing the Koopman operator matrix in high-dimensional settings leads to prohibitive memory and computational costs because of the curse of dimensionality. To address this, we propose a data-driven method to compute the Koopman generator in high-dimensional settings. The key point is to use the Tensor-Train (TT) format, which

enables low-rank approximation. In the proposed method, we compute the Koopman eigenvalues and eigenfunctions in the TT format using the AMUSEt algorithm. Then, we derive the logarithm of an operator represented in the TT format via the logarithm of its eigenvalues. This procedure allows us to compute the Koopman generator from the Koopman operator in the TT format. We demonstrate the effectiveness of our method on 4-dimensional Lotka-Volterra systems and 10-dimensional Lorenz 96 systems. The results show that the proposed method accurately identifies the vector field coefficients. Furthermore, for the 10-dimensional systems, our method significantly reduces memory consumption, demonstrating its scalability for high-dimensional systems.

Tatsuya Kishimoto
Saitama University, Sakura, Saitama, 3388570 Japan
t.kishimoto.622@ms.saitama-u.ac.jp

Jun Ohkubo
Saitama University
johkubo@mail.saitama-u.ac.jp

PP1

Pattern-Formation in Slowly Varying Environments – Weyl’s Law, Resonances, and Branching

We study pattern formation in slowly environments. Specifically, we consider the Allen-Cahn Equation $\Delta u + V(\varepsilon x)u - u^3 = 0$ and the Swift-Hohenberg Equation $-(\Delta + 1)^2 u + V(\varepsilon x)u - u^3 = 0$ with a slowly varying linear part. We focus on highly oscillatory eigenfunctions of the linearization, characterized by multiple nodal lines. In the Allen-Cahn case, their density is given by Weyl’s Law. We are interested in a refined linear analysis and bifurcation theory that detects resonances and induced branching of nodal lines, in simple symmetric geometries, with an eye towards branching patterns in the Swift-Hohenberg equation.

Yeuk Yin Lam, Arnd Scheel
University of Minnesota, Twin Cities
lam00185@umn.edu, scheel@umn.edu

PP1

Physics-Informed Deep Learning for Nonlinear Friction Model of Bow-String Interaction

This study investigates the use of an unsupervised, physics-informed deep learning framework to model a one-degree-of-freedom mass-spring system subjected to a nonlinear friction bow force and governed by a set of ordinary differential equations. Specifically, it examines the application of Physics-Informed Neural Networks (PINNs) and Physics-Informed Deep Operator Networks (PI-DeepONets). Our findings demonstrate that PINNs successfully address the problem across different bow force scenarios, while PI-DeepONets perform well under low bow forces but encounter difficulties at higher forces. Additionally, we analyze the Hessian eigenvalue density and visualize the loss landscape. Overall, the presence of large Hessian eigenvalues and sharp minima indicates highly ill-conditioned optimization. These results underscore the promise of physics-informed deep learning for nonlinear modelling in musical acoustics, while also revealing the limitations of relying solely on physics-based approaches to capture complex nonlinearities. We demonstrate that PI-DeepONets, with their ability to generalize across varying parameters, are well-suited for sound synthesis. Furthermore, we demon-

strate that the limitations of PI-DeepONets under higher forces can be mitigated by integrating observation data within a hybrid supervised-unsupervised framework. This suggests that a hybrid supervised-unsupervised DeepONets framework could be a promising direction for future practical applications.

Xinmeng Luan
McGill University
CIRMMT
xinmeng.luan@mail.mcgill.ca

PP1

Data-Efficient Koopman Operator Learning for Interconnected Nonlinear Systems Utilizing Subsystem Equation Information

There are many studies analyzing interconnected systems. Since such systems often exhibit nonlinear dynamics, the Koopman operator offers a powerful framework for their analysis. Because the Koopman operator is a linear operator that acts on observable functions rather than on the state space, it enables nonlinear dynamical systems to be analyzed within a linear framework. However, these systems are usually high-dimensional. Hence, a sufficient amount of data is needed for learning. We propose an efficient framework for modeling interconnected nonlinear systems using extended dynamic mode decomposition (EDMD), an approximation of the Koopman operator. The proposed method incorporates equation information into EDMD by exploiting the duality between the Fokker-Planck and backward Kolmogorov equations. When subsystem equations are known, the corresponding Koopman matrices are derived from this duality and used as prior knowledge. A sparse global matrix combining these subsystem matrices is then constructed, and the online EDMD algorithm is applied to learn the Koopman matrix using a small dataset. In this poster presentation, we demonstrate that the proposed method learns the Koopman matrix with a smaller dataset than standard EDMD.

Tatsuya Naoi
Saitama University
Graduate school of Science and Engineering
t.naoi.982@ms.saitama-u.ac.jp

Jun Ohkubo
Saitama University
johkubo@mail.saitama-u.ac.jp

PP1

Traveling Fronts in a Three-Component, Multi-Species Klausmeier-Type Model for Dryland Ecosystems

Motivated by the coexistence of multi-species, such as grass and trees, in drylands, we study a three-component Klausmeier-type reaction-diffusion model describing two vegetation species competing for a limiting resource, water. The two species differ in parameter values including growth, harvesting, and diffusion rates, yet exhibit the same qualitative ecological dynamics. It creates a natural hierarchy between them with one acting as a fast-spreading colonizer and the other as a locally superior competitor. Each component is assumed to diffuse at a distinct rate, modeled through two asymptotically small parameters ε and δ , which introduces a separation into three spatial scales. We construct a traveling wave solution connecting states in which one or more species are present and

examine the resulting interspecific competition for the limiting resource using the geometric singular perturbation theory (GSPT) with multiple spatial scales along with the exchange lemma.

Clara Park

University of California, Irvine
yunkyup@uci.edu

Paul Carter

UC Irvine
pacarter@uci.edu

PP1

Periodic Traveling Vegetation Stripes in Arid Ecosystems

The phenomenon of vegetation pattern formation has been observed in a variety of ecological contexts, particularly in semi-arid ecosystems. The formation of such patterns can be seen as an adaptation by the ecosystem to resource scarcity. On desert hillsides, vegetation stripes have been observed to move uphill at a constant rate, over large timescales (mm/yr), while maintaining their profile. We seek to show that these phenomena are well described by periodic traveling wave solutions to the Klausmeier reaction diffusion advection PDE, which describes interaction of water and vegetation. Our work is focused on the analysis of this model in the case of a sloped planar domain, where the advection of water dominates diffusion. We construct a family of far-from-onset traveling wave train solutions using geometric singular perturbation theory. Our aim is to show that such solutions are linearly stable to 2D perturbations using exponential trichotomies and Lin's method.

Daniel J. Shvartsman

University of California Irvine
djshvart@uci.edu

Paul Carter

UC Irvine
pacarter@uci.edu