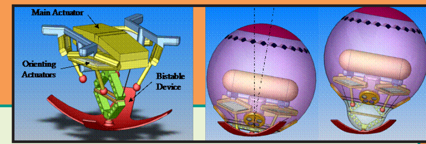
Math Matters

Apply It.



Courtesy MIT Field and Space Laboratory

The math behind... Microbot Locomotion

Technical terms used:

Navier–Stokes equation, Stokes equation, fluid dynamics, viscous effect

Uses and applications:

Microrobots for minimally invasive medicine have the potential to revolutionize modern medical sciences. However, the way they *ought to* move is vastly different from our daily cognitions because of their smallness. Understanding the math behind the locomotion at the small scale is essential in designing effective microrobots.

How it works:

Imagine you are a unicellular organism with hundreds of movable hairlike appendages, called cilia, on your surface. How would you coordinate your cilia to swim? You may want to follow the strategies of Olympic rowing champions: row your cilia (oars) as fast as you can for a powerful thrust and gradually return to the start position for the next cycle. Most important, every cilium must synchronize during the cycle to achieve a good speed. Unfortunately, you are destined to stay at the same spot if you follow what is described above.

This counterintuitive, sad truth is induced by viscous effect of the fluid and can be beautifully explained by math. The equation describing the underlying fluid dynamics is called the Navier–Stokes equation, which is essentially Newton's second law with respect to fluids. When the size of the swimmer is small (say, hundreds of micrometers) or the fluid is very viscous (imagine swimming in a pool of honey), the inertia effect is negligible, and the fluid motion is governed by the viscous effect. In this limit, Navier–Stokes equation reduces to Stokes equation, which is a time-independent linear partial differential equation. Hence the problem is essentially a geometric problem and time irrelevant. That is to say, the resultant flow patterns from strokes are the same no matter how fast or slow the strokes are. Fluid dynamics in Stokes regimes are therefore time reversible, meaning that strokes going back and forth between two configurations (like rowing all your oars in synchrony) would not result in any net movement—and the speed does not matter.

The real microbes, such as the well-known *Paramecia*, know math. Instead of synchronizing their cilia, they coordinate their cilia with a constant phase lag. In this way, the cilia form a traveling wave on the cell surface. Breaking the time reversibility enables the microbe to swim.

Interesting fact:

The fluid dynamics governed by viscous effect is fun to watch. Stirring dye in a glass of corn syrup with a stick will mix the fluid, but moving your stick exactly backward will make the dye return to the original state! (Check out this video on YouTube: https://www.youtube.com/watch?v=_dbnH-BBSNo.)

Reference:

E. M. Purcell, "Life at low Reynolds number." *Am. J. Phys* 45.1 (1977): 3-11.

B.J. Nelson, I. K. Kaliakatsos, and J. J. Abbott. "Microrobots for minimally invasive medicine." *Annu. Rev. Biomed. Eng.* 12 (2010): 55-85.

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