

Atmospheric Turbulence in a Jar

By Matthew R. Francis

Atmospheric models of the Earth are essential, informing a wide range of work in climate models, weather prediction, and atmospheric modeling of other planets in our solar system and beyond. However, attempting to model the Earth's atmosphere presents many challenges for researchers, particularly at length scales where tropical cyclones, polar vortices, and other major weather-driving phenomena form. The heart of the problem is the spectrum of energy transfer, driven by solar heating in warmer latitudes and the rotation of the planet, which leads to various types of atmospheric waves and turbulence. This challenge has attracted scientists from many overlapping disciplines offering a wide variety of solutions.

Basic fluid dynamics models predict very different energy spectra than what is measured by aircraft, as well as the results of numerical simulations. To resolve the discrepancy, researchers constructed an atmospheric analog in the lab that involves a rotating fluid in a cylindrical container with a strong heating gradient to simulate the flow of energy from the equator to the pole (see Figure 1). The results and mathematical analysis provide a guide for theoretical and numerical models to come.

"With this kind of setup, we have all the fundamental physics in the atmosphere," Shanshan Ding, an atmospheric researcher at the University of Oxford, said. In a recent study published in *Physical Review Letters* [1], she and her colleagues described the analog experiment and what it reveals about energy dispersion at larger distance scales.

Between theoretical work and the new experiments, Ding and her colleagues reproduced the energy spectrum from airplane measurements. More than that, however, they identified the energy transfer processes driving atmospheric turbulence, known as an enstrophy cascade.

Scaling the Heights

Research on atmospheric turbulence and its connection to weather dates back to the 19th century but the ability to systematically measure winds in the upper troposphere—the lowest layer of the atmosphere where most weather happens—only became possible in the second half of the 20th century. Researchers instrumented long-range commercial airplanes in the 1970s, which provided wind-speed data over distances ranging from a few kilometers to tens of thousands of kilometers. By converting this data to a spectrum, they found the kinetic energy scaled roughly with the wavelength

of the atmospheric disturbances cubed, λ^3 , at large wavelengths and $\lambda^{5/3}$, at smaller wavelengths [3].

Development on the theoretical side, however, lagged due to the extreme nonlinearity of turbulent systems. Additionally, two-dimensional (2D) calculations produced the opposite spectral relationship: cubic depen-

dence on wavelength at small wavelengths, and shallower $5/3$ power scaling at larger wavelengths [2], directly contradicting experimental airplane measurements.

"This poses a puzzle to understand why the atmosphere doesn't obey the rules," said Peter Read, an atmospheric fluid

See *Atmospheric Turbulence* on page 4

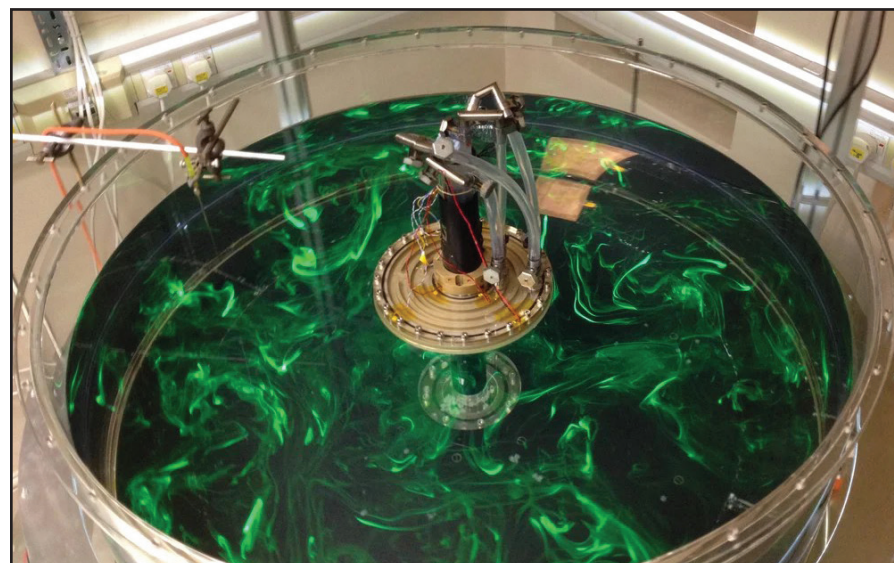


Figure 1. A one-meter-diameter rotating cylindrical tank containing a water-glycerol mixture with tracer particles appearing in green. The water is heated by a ring at the bottom of the tank, and cooled by the disk at the top. The turbulent eddies exhibit the same wavelength scaling as observed high in Earth's atmosphere. Photo courtesy of H. Socolan of Claude Bernard University Lyon 1.

Machine Learning for Dynamical Systems and Vice Versa: From Data Assimilation to Optimal Control

By Wei Kang, Liang Xu,
and Qi Gong

The topics discussed in this article about data assimilation (DA) and optimal control are unified by a common philosophy: leveraging machine learning (ML) for dynamical systems and, conversely, using system-theoretic insights to improve ML. ML offers new ways to overcome long-standing computational barriers, while dynamical systems theory provides analytical and computational tools that enhance the efficiency and reliability of learning. Our recent research sits

precisely at this intersection of systems theory and ML [1].

The primary goal of DA is to combine experimental observations with numerical models to obtain an optimal estimate of the system state, or its parameters, of a dynamical system. Current methods include variational approaches and sequential filters, but they are all impacted by the phenomenon known as the *curse of dimensionality*. For dynamical systems that arise from the discretization of partial differential equations (PDEs), the implementation cost of existing DA methods can easily exceed the memory and computational limits of modern com-

puters. In general, algorithms that provide more detailed uncertainty information require significantly greater computational resources (see Figure 1). Many numerical models—such as those used in atmospheric dynamics and other large-scale engineering systems—have extremely high dimensions that make it infeasible to compute or represent full-error covariance information with conventional DA methods. Although the examples shown in Figure 1 do not represent an exhaustive list of applications, the colored regions illustrate the practical trade-off between model dimension and estimation capability. The upper-right corner of the diagram—where high dimensional systems intersect with the need to estimate full probability distributions—remains largely unexplored. These are precisely the types of problems where emerging artificial intelligence (AI)-based tools may lead to transformative advances.

A fundamental question in AI-based DA is: how much data is sufficient? When the data form a time sequence generated by an underlying dynamical system, this question is closely related to the ideas of observability and detectability, a classical concept in control theory. In [1], we refine this concept. For simplicity, consider the linear system

$$\begin{aligned} \mathbf{u}(k+1) &= \mathbf{A}\mathbf{u}(k), & \mathbf{u} &\in \mathbb{R}^n, & k &\in \mathbb{N}_0 \\ \mathbf{y}(k) &= \mathbf{H}\mathbf{u}(k) + \mathbf{v}(k), & \mathbf{y} &\in \mathbb{R}^m \\ \mathbf{z}(k) &= \mathbf{W}\mathbf{u}(k), & \mathbf{z} &\in \mathbb{R} \end{aligned}$$

where $\mathbf{A} \in \mathbb{R}^{n \times n}$, $\mathbf{W} \in \mathbb{R}^{1 \times n}$, $\mathbf{H} \neq 0$ and $\mathbf{H} \in \mathbb{R}^{m \times n}$, with $\mathbf{z}$ representing the quantity of interest (QoI). In classical control theory, the system is said to be observable if the initial state $\mathbf{u}(0)$ can be uniquely deter-

See *Machine Learning* on page 5

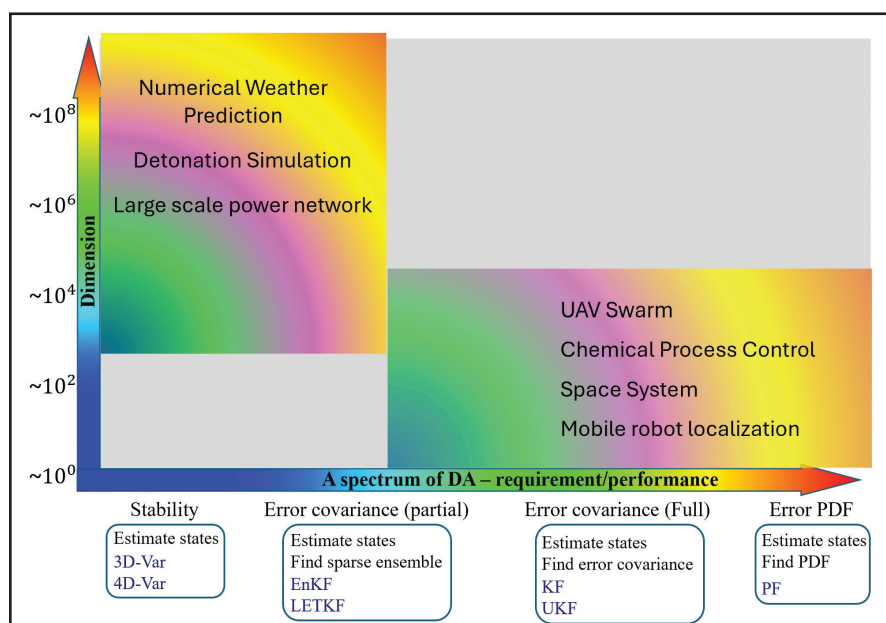


Figure 1. Data assimilation (DA) methods. The vertical axis represents the dimension of the dynamical model while the horizontal axis includes a spectrum of algorithms for DA. On the left are methods such as four-dimensional (4D)-Var and the ensemble Kalman filter (EnKF), which provide state estimates with limited information about the error covariance. Moving toward the right, methods such as the Unscented Kalman Filter and particle filters aim to provide richer information, including full covariance matrices or even the probability distribution of the estimated variables. Figure courtesy of Wei Kang.

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- 6

Game Theory for the Evolution of Collective Intelligence
In the absence of any central planner, how do living beings in the natural world get together to tackle their problems? Lakshmi Chandrasekaran details new mathematical modeling research on this topic that employs a game theoretic approach to demonstrate the evolution of collective intelligence, a type of group behavior in which collaboration within a group of individuals can be the best way to successfully tackle complex problems.
- 8

The Role of Quantization in Quantum Computing
The use of topological data analysis for quantum computing has gained momentum within the applied mathematics community, opening doors for the creation of novel hybrid methods that employ several branches of mathematics. Arni Srinivasa Rao and Steven Krantz propose a quantization method that could aid in processing massive data sets at the atomic level by imagining a flexible topological space of the data, followed by functional mapping onto a space in the atom.
- 9

The Cognitive Science Revolution
Devdatt Dubhashi reviews *The Laws of Thought: The Quest for a Mathematical Theory of the Mind* by Tom Griffiths. The book overviews the evolution of cognitive science, highlights key ideas and important figures, and explores how recent advances in artificial intelligence could impact the field.
- 12

Dealing with the After-math: Finding Your Niche in the Mathematical Sciences
Caoimhe Rooney recounts her personal career journey, including the passion that drove her to study mathematics and her previous work applying mathematical models to understand the atmospheres of distant exoplanets. She also describes her current position as a senior ground segment systems engineer at Astroscale, a company developing sustainable technologies for space exploration.



Obituary: Cleve Moler

A Gentle Giant of Numerical Computing

By Jack Dongarra, John Gilbert, and Rob Schreiber

Cleve Barry Moler, a pioneer in applied mathematics and computational science, passed away on May 20, 2026, at the age of 86. With Cleve’s passing, our field and all who had the privilege of knowing him share a deep loss. Cleve was indeed a gentle giant: brilliant, generous, and kind. He was a towering figure in numerical computing, a trusted colleague, and a dear friend. His contributions to the field of computing and the SIAM community will live on for many years to come.

Through MATLAB,¹ his best-known contribution to computing, Cleve made numerical methods accessible and interactive; his work at MathWorks² empowered mathematicians, engineers, and students to build, test, and understand mathematical models with ease. What had once required difficult programming and deep knowledge of computing systems suddenly became available for quick and intuitive exploration. Cleve gave the computational science community a new language for discovery.

Cleve was born in Salt Lake City, Utah, in 1939. As a student, he already displayed the combination of mathematical curiosity, practical intelligence, and quiet determination that would define his career. He received his bachelor’s degree in mathematics from the California Institute of Technology in 1961, where he encountered an emerging culture of computational mathematics. He went to Stanford University for graduate study, earning his Ph.D. in mathematics in 1965 under the supervision of George Forsythe, a founder of modern numerical analysis and computer science. That intellectual lineage mattered, as Forsythe’s view on the centrality of computing to mathematics and engineering became part of Cleve’s own outlook.

Cleve held professorships at the University of Michigan, Stanford University, and the University of New Mexico (UNM). He taught mathematics and computer science for nearly two decades; throughout those years, he remained committed to teaching students about the practical use of computation. His research and educational work grew from the conviction that numerical methods should be efficient, reliable, and available to all who need them.

That conviction led to multiple groundbreaking software contributions. Cleve was an author of EISPACK³ and LINPACK,⁴ two of the first libraries to make high-quality numerical linear algebra software available free of cost. EISPACK and LINPACK represented an important paradigm shift: rather than requiring individual researchers to write and debug their own

numerical routines, the community could rely on portable, carefully designed, and well-tested software. The LINPACK Users’ Guide, which Cleve coauthored with Jack Dongarra, Jim Bunch, and Pete Stewart, set a standard of quality for documentation and dissemination of numerical software [1].

Cleve’s most widely recognized contribution began as an educational tool. While teaching at UNM, he wanted his students to be able to access the numerical power of LINPACK and EISPACK without first needing to become Fortran programmers. The result was the inaugural version of MATLAB (short for “matrix laboratory”). It provided students and researchers with an interactive environment where matrices, vectors, algorithms, and experiments could be explored easily, without the time and energy required to write and debug Fortran code. What began as a teaching aid became a new way to think about computation.

In 1984, Cleve co-founded MathWorks with Jack Little to continue developing and disseminating MATLAB. Since its inception—when PC-MATLAB was distributed by mail on floppy disks—MathWorks has grown into one of the most important institutions in technical computing. Cleve remained, until his passing, the intellectual and moral center of MATLAB’s evolution and delighted in interacting with and influencing the next generation of MATLAB developers.

MATLAB gave mathematical programmers easy, interactive access to graphics; complex arithmetic; matrices and tensors as objects; and built-in random numbers and constants (e.g., π and i). MATLAB quickly became part of the daily practice of countless people in industry and academia, reshaping how students learn linear algebra, how engineers develop designs, and how scientists test theories. Through MATLAB, Cleve’s approach to computing touches nearly every area where computation and mathematics meet. His judgment, taste, and deep understanding of numerical computation shaped the software at every level — not only its algorithms, but its spirit.

In addition to his work in software, Cleve also contributed greatly to matrix algorithms. With Pete Stewart, he developed the QZ algorithm for the generalized eigenvalue problem, an important and enduring method for computing the generalized Schur decomposition of a matrix pencil. The QZ algorithm is now a fundamental tool in numerical linear algebra, with applications in stability analysis, control theory, and differential equations. It reflects Cleve’s characteristic style: deep mathematical insight combined with algorithmic practicality



Cleve Moler, 1939-2026. Photo courtesy of Institute of Electrical and Electronics Engineers Computer Society.

cality and the ability to foresee how methods would behave computationally.

Beyond his mathematical skills, Cleve had a gift for exposition. His writing was clear, lively, and memorable, imbued with his talent for explaining complex numerical issues in an understandable way. He co-wrote *Computer Methods for Mathematical Computations* with George Forsythe and Michael Malcolm, a book that influenced generations of students and practitioners [2]. With Charlie Van Loan, he wrote the celebrated paper “Nineteen Dubious Ways to Compute the Exponential of a Matrix,” a classic work that combined humor, insight, and technical depth [3]. That paper went beyond reviewing methods for computing matrix exponentials to teach readers how to think critically about numerical algorithms, stability, conditioning, and the gap between mathematical formulas and reliable computation.

Cleve was a pioneer in bringing numerical linear algebra into the era of distributed-memory parallel computing. While in the supercomputing division at Intel, he helped shape early thinking on the expression and implementation of dense matrix computations on massively parallel machines, where data had to be distributed across many processors and communication costs were as important as arithmetic speed. This work came at a formative moment, when scalable parallel computers were emerging but the necessary algorithms, software structures, and performance models to make them useful for scientific computing were still being invented. During this period, Cleve gave scientific computing one of its most memorable phrases: “embarrassingly parallel.” The term captured, with characteristic wit and clarity, a class of problems whose computations could be divided almost trivi-

See Cleve Moler on page 4

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Obituary: Richard A. Tapia

By Rodrigo Bañuelos and
Anthony J. Kearsley

Richard Alfred Tapia, a distinguished mathematician in the fields of mathematical optimization and iterative methods for nonlinear problems, passed away on May 22, 2026, at the age of 88. His work and influence on both SIAM and the applied mathematics community will continue to inspire generations of mathematicians to come.

Richard was born on March 25, 1938, in Santa Monica, California. His parents, Amado and Maria Magdalena Tapia, had five children after immigrating to the U.S. from Mexico. Richard had a twin brother Bobby. In their youth, the Tapia brothers successfully built and raced cars; they made drag-racing history with a 6.54-second quarter-mile — a world record at the time and an achievement recognized by the National Hot Rod Association. Richard attended Los Angeles Harbor College, a public community college in Wilmington, California, where he earned his associate's degree before transferring to University of California, Los Angeles (UCLA) for his undergraduate degree. Despite showing enormous promise as an undergraduate, Richard was not initially offered financial support as a graduate student at UCLA, something that is puzzling in hindsight. He instead worked as a mathematician at the Todd Shipyards in San Pedro, California, and later as a part-time scientific programmer at IBM Los Angeles. Later, he garnered graduate student support from the Office of Naval Research that ultimately enabled him to earn his Ph.D. from UCLA in 1967. Throughout his life Richard was always a proud Angeleno.

Richard defied classification: he was as knowledgeable and enthusiastic about chrome, paint, and horsepower as he was about mathematical optimization and its applications. Throughout his career, Richard demonstrated that the successful examination of deeply technical obstacles—whether a nonlinear system of equations or a high-gloss paint on a Chevelle—depends on deep thinking about modeling, careful iteration of ideas, and knowing when to step back and admire the result. Richard's technical work quickly earned him the reputation of an applied mathematician of distinction; he produced fundamental contributions to optimization theory and computational mathematics. In his Ph.D. thesis, under the direction of Charles Tompkins, Richard generalized Newton's method by extending it to solve a wider class of nonlinear and variational problems, including the Euler-Lagrange equations. His work provided a rigorous framework for the analysis of the convergence of Newton's Method when solving complex optimization problems. Richard often reflected on his time at UCLA as a formative period of growth, shaped by wise guidance from a supportive and distinguished faculty. Among them, David A. Sánchez, a member of his thesis committee, offered insightful criticism and encouragement, serving as an important sounding board throughout the development of this work. Richard's research was also closely connected to the broader research program of Magnus Hestenes, whom Richard frequently credited in later lectures during his tenure at Rice University as having exerted a lasting and profound influence on his career and research in optimization.

Richard continued his advances in mathematical theory for solving complex nonlinear problems efficiently and reliably. His research contributed to a broader understanding of how iterative algorithms converge and their adaptation to a wide variety of sophisticated mathematical models. For example, when one looks at

nearly all contemporary machine learning (ML) and deep learning systems, they are all trained by solving optimization problems. The objective is to minimize a loss function over millions or even billions of parameters. Many of today's large-scale models rely on variants of gradient descent, for which the underlying theory of optimization—including convergence analysis, curvature information, constrained optimization, and Newton-type methods—draws heavily from the mathematical foundations developed by researchers like Richard. Many advanced optimization techniques used in ML can be understood as descendants or adaptations of ideas that emerged from the broader Newton and nonlinear programming traditions to which Richard made important contributions. He made profound additions to the analysis of secant methods, which helped to establish a deeper theoretical understanding of these powerful gradient-based techniques for solving nonlinear equations and optimization problems. His work in nonlinear programming advanced the mathematical foundations of constrained optimization and provided new insights into the behavior and convergence of iterative algorithms. Among his many achievements was the study of so-called indicator functions, where he developed elegant formulations for understanding constraints and optimality. Motivated by Georg Cantor's diagonalization proof, Richard created the Diagonalized Multiplier Method, an innovative approach for solving complex constrained optimization problems. Yet another influential contribution was his work on the equivalence of optimality systems, an analysis that described fundamental relationships among alternative formulations of necessary and sufficient conditions for optimality and strengthened the conceptual framework underlying mathematical programming. Through his skillful mathematical rigor, practical insight, and unique creativity, Richard shaped optimization as a discipline, leaving a legacy that continues to influence research today.

Richard moved from Los Angeles to the University of Wisconsin–Madison, where, in 1968, he was a freshly minted Ph.D. working at the Mathematics Research Center. He described his decision to move to the University of Wisconsin as one of the best decisions of his life. He frequently reported to his colleagues and students that being surrounded by his mathematical heroes—people like Michael Golomb and Isaac Jacob Schoenberg—and the effort that they made to include him, seek his opinion, and to befriend him had an enormous effect. He described their exchange of research and opinions, and only in hindsight did Richard realize he was being mentored. He attributes his time in Madison as being where he learned that professional kindness need not be in conflict with greatness, a lesson he imparted to his students and those who worked with him. From Madison, he went to Rice University where he would spend the majority of his career, save a one-year sabbatical at Stanford University between 1976 and 1977. At Rice, Richard held various positions; he started as an assistant professor of mathematical sciences—now the department of computational applied mathematics and operations research—in 1970, eventually being promoted to full professor in 1976, and served as a department chair from 1978 to 1983. At the time of his death, he still served as a university professor and the Maxfield-Oshman Professor of Engineering. He also served as the director of Rice University's Alliances for Graduate Education and the Professoriate Program¹ and director of

the Tapia Center,² whose mission is to “empower everyone to be able to pursue education and careers in STEM.”

Richard also worked with SIAM many times throughout his career. For his contributions to linear and nonlinear programming and efforts to increase diversity, Richard was inducted into the inaugural class of SIAM Fellows³ in 2009. He was also awarded the SIAM Prize for Distinguished Service to the Profession⁴ in 2004, an award that recognized his work advancing mathematics and strengthening the scientific community. He was a member of the SIAM Board of Trustees⁵ as well as an editor of the *SIAM Journal on Numerical Analysis*⁶ and *SIAM Journal on Optimization*.⁷ He was an officer for SIAM's Activity Group in Optimization,⁸ served on SIAM's Diversity Advisory Committee⁹ for 11 years, and a frequent organizer of SIAM events, including co-chairing the organizing committees for SIAM meetings in Houston and Rio Grande, Puerto Rico. Richard was often invited to give plenary seminars around the world; he delivered a Centennial Lecture, titled “The Emergence of Numerical Optimization,”¹⁰ at the 1988 SIAM Annual Meeting in Minneapolis, and delivered “Some Mathematical Insights Into Car and Bicycle Racing”¹¹ for the I.E. Block Community Lecture at the 1999 SIAM Annual Meeting in Atlanta.

In addition to Richard's contributions to SIAM, he became the first U.S.-born Hispanic individual to be elected to the National Academy of Engineering, and later received the Presidential Award for Science, Mathematics, and Engineering Mentoring from President Bill Clinton. Richard served on the National Science Board from 1996 to 2002 and was later awarded the National Medal of Science in 2011 by President Barack Obama. More of Richard's accolades include the National Science Foundation's Vannevar Bush Award and the American Association for the Advancement of Science's Public Engagement with Science Award; he was the charter recipient of the Computing Research Association's A. Nico Habermann Award in 1994, and he was named a Fellow of the American Mathematical Society in 2021. There is no doubt that Richard played a unique role highlighting underrepresented groups of mathematicians, which helped foster a more inclusive and vibrant mathematical community. The Association for Computing Machinery established the annual Richard Tapia Celebration of



Richard A. Tapia, 1938-2026. Photo courtesy of Luis Melara.

Diversity in Computing Conference¹² where the Richard A. Tapia Achievement Award for Scientific Scholarship, Civic Science and Diversifying Computing¹³ is awarded. There is also the biennial Blackwell–Tapia Conference,¹⁴ where the Blackwell–Tapia prize is awarded in honor of the legacies of both Richard and David Blackwell, whose positive influences on the African American and Hispanic communities in the mathematical sciences are undeniable.

Richard's commitment to excellence, opportunity, and inclusion is further recognized through the MAA-SIAM-AMS Hrabowski-Gates-Tapia-McBay Lecture,¹⁵ which honors individuals who have advanced these ideals within the profession. Through these initiatives and by personal example, Richard earned extraordinary respect, admiration, and affection from generations of mathematicians, particularly those from underrepresented backgrounds. This is a true testament to the breadth, depth, and enduring significance of his influence. Earlier this year, Harris County in Texas honored him by renaming the Beltway 8 Ship Channel Bridge as the Dr. Richard A. Tapia Bridge, a recognition that meant a great deal to Richard [1].

Interestingly, what made Richard most proud was the cadre of students, postdocs, and younger colleagues he brought together. He was a wizard at assembling personalities and stitching together different technical skills to maximize output. The research inclusion he appreciated so much at University of Wisconsin–Madison as a postdoc was very much practiced and imparted to his collaborators, especially students and postdocs. Through the decades he spent as a tireless sounding board for ideas and in a campus office with a door that was always open, Richard's international network of friends is immense. When word spread of his serious illness, the response was almost instantaneous. The network lit up with phone calls, messages, schedules being rearranged, and flights to Houston being booked, from friends near and far. This was a powerful demonstration of friendship and profound debt felt by those who were lucky enough to cross paths with Richard. In addition to the clarity, con-

See **Richard A. Tapia** on page 5

² <https://tapiacenter.rice.edu/>

³ <https://www.siam.org/programs-initiatives/prizes-awards/fellows-program/>

⁴ <https://www.siam.org/programs-initiatives/prizes-awards/major-prizes-lectures/siam-prize-for-distinguished-service-to-the-profession/>

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⁶ <https://epubs.siam.org/journal/sjnam>

⁷ <https://www.siam.org/publications/siam-journals/siam-journal-on-optimization/>

⁸ <https://www.siam.org/get-involved/connect-with-a-community/activity-groups/optimization/>

⁹ <https://www.siam.org/get-involved/connect-with-a-community/committees/diversity-advisory-committee/>

¹⁰ <https://archive.siam.org/meetings/pdf/AN1988.pdf>

¹¹ <https://archive.siam.org/meetings/an99/tapia.htm>

¹ <https://strides.rice.edu/>

¹² <https://tapiaconference.cmd-it.org/>

¹³ <https://tapiaconference.cmd-it.org/about/richard-tapia-award/>

¹⁴ https://icerm.brown.edu/program/special_event/se-24-btc

¹⁵ <https://www.siam.org/programs-initiatives/prizes-awards/joint-prizes/maa-siam-ams-hrabowski-gates-tapia-mcbay-lecture/>

Atmospheric Turbulence

Continued from page 1

dynamics researcher at the University of Oxford and coauthor of the Physical Review Letter [1]. “What don’t we understand about this system?”

Turning in the Widening Gyre

For cyclones and other phenomena of interest at approximately 1000-kilometer scales, Earth’s atmosphere can be modeled in 2D because the horizontal distances are much larger than the vertical thickness of the troposphere. That also means researchers can apply a mathematical trick to simulate the atmosphere in a cylindrical tank where the simulated “north pole” lies at the axis and the “equator” at the outer radius. This type of experiment reproduces many large-scale phenomena such as Rossby waves, which are responsible for jet streams.

Ding, Read, and their collaborators designed a modified version of this experiment, using a meter-diameter cylinder to provide a much larger volume than prior work (which used chambers approximately 10 centimeters in diameter). In addition, they built the base of the meter-diameter cylinder to be a shallow cone, which simulated the varying strength of the Coriolis effect at different latitudes on Earth. They applied heat to the fluid via a heating coil at the bottom outer circumference and increased the temperature differential by fixing a cold plate to the top of the cylinder at the axis (see Figure 2).

The varying depth, temperature gradient, and rotation of the apparatus produced a misalignment between pressure and density inside the fluid, which created swirls known as baroclinic eddies like those seen in the troposphere. Since the boundary conditions for concentric cylinders are vastly different than the real atmosphere, the large size and geometry of the experiment meant the measurements focused on the middle annual region away from both walls — which also correspond most closely to mid latitudes of interest.

“The temperature difference in the vertical direction will cause the convection that will lead to plumes,” Ding said. “The temperature contrast between the equator and the north can lead to Rossby waves and baroclinic eddies.”

Due to the impossibility of reproducing all the physical characteristics of the upper troposphere, the research group used a mixture of water and glycerol. That fluid has two advantages: it lowered the threshold for turbulence to emerge, and it allowed them to suspend small particles to trace that turbulence using a video camera. The onset of turbulence is governed by the (dimensionless) Reynolds number:

Re = \frac{L\bar{v}}{\nu}

where L is a characteristic length, $\bar{v}$ is a characteristic velocity for the fluid, and ν is the viscosity.

“We want the Reynolds number to be large enough, and when we scale down the system, the Reynolds number won’t be very large if we use air,” Ding said. Even so, the Reynolds number in the water-glycerol mixture is much lower than that of the highly turbulent upper troposphere simply because the viscosity of air is so tiny while the characteristic scales of atmospheric phenomena are large.

“The distribution of heating and cooling is at least qualitatively similar to what we find in the atmosphere,” Read said. “The basic instabilities that arise are physically and qualitatively the same as mid-latitude weather systems in the real atmosphere. We are trying to match the order of magnitude of certain dimensionless variables that allow us to do what aerodynamicists do in terms of scaling experimental test systems.”

Cascades and Alien Worlds

To study the range of spectra, the researchers fixed all quantities in the system, such as temperature differentials, while taking measurements at rotational speeds between 0.5 and 10 revolutions per minute to simulate different latitudes. The water-glycerol mixture is viscous enough

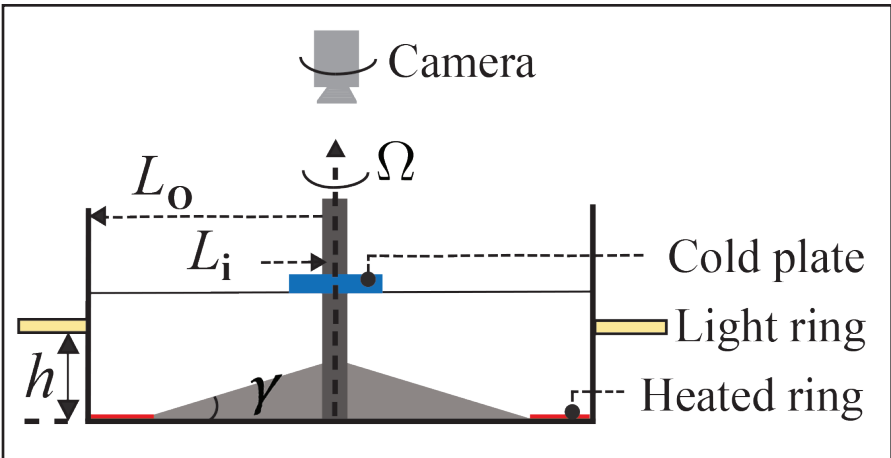


Figure 2. Cutaway diagram of the experiment, showing the inclined base of the cylindrical tank and the positions of the heating and cooling elements. Figure adapted from [1] and available via the Creative Commons Attribution 4.0 International license.

that the experiments could run for several hours without the suspended particles settling out of solution; this enabled the experimenters to measure velocities throughout the cylinder. The 2D energy spectrum

E(k) = \frac{1}{2dk} \sum_{k_{mn} \in k \pm dk/2} \langle \hat{u}_{mn} \cdot \hat{u}_{mn}^* \rangle_t

as a function of the wavenumber $k = 2\pi / \lambda$ involved taking the Fourier-Bessel transform of the measured velocity fields $\mathbf{u} \rightarrow \hat{\mathbf{u}}$ in polar coordinates, which makes them complex numbers (“*” denotes the complex conjugate), then averaging over time as indicated by the brackets.

To quantify the way kinetic energy dissipates in turbulent fluids, dynamics researchers calculate the enstrophy spectrum, which is defined in terms of the gradient of the fluid velocity field. Determining the spectrum of the enstrophy flow from the experiment revealed a cascade that drove the steep λ^3 energy spectrum at large scales. These results indicate horizontal energy transfer from small fluctuations to large ones, while the angular momentum transfer goes the other direction from large vortices to small.

Interestingly, the researchers also noted that at the highest rotational speeds, the kinetic energy tended to follow latitudinal zones, a phenomenon that astronomers

observe on fast-spinning gas giant planets like Jupiter. Read pointed out that in order to extend cylinder experiments to study these worlds, the fluids would need to be spun significantly faster, which produces a concave surface like a whirlpool, further complicating matters. However, the qualitative understanding of energy transfer could apply to atmospheric models for Mars or even exoplanets, potentially helping researchers grasp alien worlds with intense temperature gradients known to orbit other stars.

“It’s one thing to reproduce the [atmospheric] pattern in an engineering sense,” Read said. “It’s another to understand why you get the answer that you actually observe.”

References

[1] Ding, S., Bobas, H., Scolan, H., Young, R.M.B., & Read, P.L. (2026). Stratification-dependent enstrophy-controlled regime in geostrophic turbulence. *Phys. Rev. Lett.*, 136, 114101.
[2] Kraichnan, R.H. (1967). Inertial ranges in two-dimensional turbulence. *Phys. Fluids*, 10(7), 1417-1423.
[3] Nastrom, G.D., Gage, K.S., & Jasperson, W.H. (1984). Kinetic energy spectrum of large- and mesoscale atmospheric processes. *Nature*, 310, 36-38.

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Cleve Moler

Continued from page 2

ally among many processors, with little or no need for communication. Like so many of Cleve’s turns of phrase, it was both technically apt and instantly understandable.

Cleve’s work at Intel served as a bridge between classical numerical analysis and the emerging field of scalable high-performance computing. It reflected the same philosophy that animated much of his career: make powerful numerical methods usable on the machines of the day, in a way that opens the door for others. Whether through MATLAB, LINPACK, EISPACK, or his parallel computing work, Cleve helped turn advanced computation from a specialist’s craft into a practical tool for science and engineering.

Cleve’s accomplishments were recognized by some of the highest honors in computing, engineering, and applied mathematics. He was elected to the National Academy of Engineering⁵ in 1997 for the conception and development of widely used mathematical software. He received the Institute of Electrical and Electronics Engineers (IEEE) Computer Society Computer Pioneer Award in 2012 and the IEEE John von Neumann Medal⁶ in 2014. He was named a Fellow of the Computer History Museum⁷ in 2017, in recognition of the historic significance of MATLAB and his contributions to numerical computing. In 2023, he received the International Council for Industrial and

Applied Mathematics Industry Prize⁸ for his extraordinary impact on mathematical and computational tools for science and engineering; he was then elected to the National Academy of Sciences⁹ in 2026.

Cleve’s relationship with SIAM was especially deep and lasting. He was a committed member of the SIAM community, served on the SIAM Board of Trustees,¹⁰ and served as President of SIAM from 2007 to 2008. He was named a SIAM Fellow¹¹ in 2009 for his outstanding contributions to numerical analysis and software, including the invention of MATLAB. His involvement with SIAM reflected his belief that applied mathematics, numerical analysis, computer science, engineering, and industrial practice belong in close conversation. Cleve understood SIAM not merely as a professional society, but as a community — one built around ideas, service, collegiality, and the shared responsibility to make mathematics useful in the world.

Through his deep commitment to both SIAM and MathWorks, Cleve played a pivotal role in the creation of the MathWorks Math Modeling Challenge¹² (M3 Challenge). Cleve served as a trusted bridge between SIAM and MathWorks; he championed the partnership and believed that MathWorks was the ideal collaborator for a program that aimed to inspire and empower the next generation of mathematical thinkers. For several years he attended the annual M3 Challenge Final

Event in New York City as an honorary judge, where his enthusiasm for mathematics, technology, and education was evident. Students were delighted to meet the co-founder of MathWorks and the creator of MATLAB, which many of them used in their own work.

Cleve’s legacy is not only in software, algorithms, and institutions. It is also in the people he encouraged, challenged, and inspired. He was a colleague, mentor, and friend to many. He could ask a pointed question that went directly to the heart of the matter, but always with curiosity, humor, and goodwill. Those of us who saw him at conferences will remember that unmistakable booming voice, his careful attention to the technical details, and the warm smile that so often followed.

Cleve represented the best of our community: intellectual depth, practical insight, humility, and humanity. He helped build the foundations on which much of modern computational science rests, yet he remained approachable, generous, and deeply engaged with others. He had the rare ability to see both the mathematical essence of a problem and the practical needs of the person trying to solve it. That combination—rigor joined with usefulness, and brilliance joined with kindness—made him unique.

Cleve Moler leaves behind a remarkable legacy that will continue through the tools he created, the ideas he advanced, the company he helped build, and the many people whose work and lives he influenced. Cleve is survived by his wife, Patsy, his three daughters—Kathryn, Teresa, and Antonia—and his two stepdaughters, Carolyn and Byerly. We will miss his wisdom, his questions, his presence, and his insight. Most of all, we will miss our friend.

References

[1] Dongarra, J.J., Moler, C.B., Bunch, J.R., & Stewart, G.W. (1979). *LINPACK users’ guide*. Philadelphia, PA: Society for Industrial and Applied Mathematics.
[2] Forsythe, G.E., Malcolm, M.A., & Moler, C.B. (1977). *Computer methods for mathematical computations*. Upper Saddle River, NJ: Prentice Hall Publishing.
[3] Moler, C.B., & Van Loan, C. (1978). Nineteen dubious ways to compute the exponential of a matrix. *SIAM Rev.*, 20(4), 801-836.

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⁵ <https://www.nae.edu/>
⁶ <https://corporate-awards.ieee.org/award/ieee-john-von-neumann-medal/>
⁷ <https://computerhistory.org/hall-of-fellows/>

⁸ <https://iciam.org/prize/iciam-industry-prize>
⁹ <https://www.nasonline.org/>
¹⁰ <https://www.siam.org/about-us/governance-leadership/>
¹¹ <https://www.siam.org/programs-initiatives/prizes-awards/fellows-program/>
¹² <https://m3challenge.siam.org/>

Machine Learning

Continued from page 1

mined from the sequence of observations $\{\mathbf{y}(k)\}_{k=0}^K$. We broaden this notion by introducing a quantitative measure of estimation uncertainty [1]. Specifically, define

$$\rho^2 = \max_{\{\hat{\mathbf{u}}(k)\}} (W\hat{\mathbf{u}}(K) - z(K))^2,$$

$$\text{subject to } \hat{\mathbf{u}}(k+1) = A\hat{\mathbf{u}}(k),$$

$$k = 0, 1, \dots, K-1,$$

$$\sum_{k=0}^K \|\mathbf{H}\hat{\mathbf{u}}(k) - \mathbf{H}\mathbf{u}(k)\|_2^2 \leq \epsilon^2.$$

The quantity ρ measures the worst-case estimation error over all trajectories whose dynamics and observation satisfy the constraints above. For this reason, we call ρ the ambiguity bound of the estimation problem. When using estimators that exploit additional information beyond the observation data, one may obtain estimates that outperform this worst-case bound. Nevertheless, it provides a fundamental benchmark: the error of a well-designed estimator should be smaller than ρ , as a smaller value of ρ typically indicates stronger observability of the quantity.

This definition generalizes the classical notion of observability. If the QoI $z(K)$ is the initial state $\mathbf{u}(0)$, then the ratio ϵ^2/ρ^2 equals the smallest eigenvalue of the system's observability Gramian. For nonlinear systems, the value of ρ can be approximated using empirical Gramians, which provides a practical way to assess the information content of observations.

In ML-based surrogate DA models, such as our approach developed in [1], a neural network is trained to learn the mapping from observations to the system state. Using a two-dimensional viscous Burgers equation as an example, we illustrate how the ambiguity bound can guide the design of such learning-based DA methods.

The ambiguity bound helps determine whether the observation sequence $\mathbf{y}(k)$ within a given time window contains sufficient information about the QoI. Ideally, the value of ρ should be smaller than the desired estimation error tolerance; even a well-trained neural network is unlikely to produce accurate estimates if ρ is larger. In addition, for a QoI defined over a spatial region, D_{QoI} , the ambiguity bound can be used to identify an effective region, denoted by $D_{\text{effective}}$, that contains D_{QoI} . Outside this region, the value of ρ becomes essentially constant; this indicates that the dynamics beyond $D_{\text{effective}}$ have little or no influence on the estimation of the QoI. This observation enables a substantial reduction in computational cost. When training data is generated for the surrogate DA model, the numerical simulation can therefore be restricted to the effective region with an appropriate artificial boundary condition, thereby forming a reduced-order model that avoids solving the PDE over the entire domain.

We now turn to optimal control, where similar principles guide the integration of ML and dynamical systems. From autonomous vehicles to spacecraft maneuvering, modern engineering systems require controllers that operate reliably in complex and uncertain environments. Optimal control provides a systematic framework to design such control laws and plays a central role in a wide range of engineering applications.

A typical optimal control problem seeks a control input $\mathbf{u}(\cdot)$ that minimizes a performance metric subject to a dynamical system,

$$\begin{cases} \text{minimize } J[\mathbf{u}(\cdot); \mathbf{x}_0] = \mathcal{F}(\mathbf{x}(t_f)) + \\ \quad \int_0^{t_f} \mathcal{L}(t, \mathbf{x}, \mathbf{u}) dt, \\ \text{subject to } \dot{\mathbf{x}}(t) = f(t, \mathbf{x}, \mathbf{u}), \\ \mathbf{x}(0) = \mathbf{x}_0. \end{cases}$$

For a given initial condition $\mathbf{x}_0$, the optimal trajectory and control $\mathbf{x}^*(t)$, $\mathbf{u}^*(t)$ must satisfy the necessary conditions of Pontryagin's minimum principle, whose solutions produce an *open-loop* optimal control.

In many applications, however, it is desirable to design a *feedback* control law that depends on the current state of the system, $\mathbf{u}(t, \mathbf{x}(t))$. This improves robustness to disturbances and modeling uncertainties and enables real-time implementation, hence the use of dynamic programming, whose central object is the *value function* $V(t, \mathbf{x})$, which represents the minimum achievable cost starting from a given state. This function satisfies the Hamilton-Jacobi-Bellman (HJB) equation, a nonlinear PDE whose solution encodes the optimal feedback control through a Hamiltonian minimization condition. Despite its benefits, this approach is also subject to the curse of dimensionality, as it becomes intractable at high dimensions. As a result, classical optimal feedback control methods are limited to special cases such as linear quadratic regulators or low dimensional nonlinear systems.

Recent advances in ML offer new ways to address these challenges. Neural networks are remarkably effective at representing complex high-dimensional functions. It is now possible to use neural networks to approximate value functions or control policies and revisit optimal feedback design for high-dimensional nonlinear systems. These developments have led to growing interest in ML-based approaches to optimal feedback control, but they come with their own set of challenges. Applying neural networks to the HJB equation may admit multiple solutions, among which only the viscosity solution corresponds to the true value function. Consequently, minimizing the residual of the HJB equation at sampled points, as done in physics-informed neural networks, does not guarantee that the learned solution corresponds to the viscosity (and hence optimal) value function. A promising path forward is to incorporate structural insights from optimal control theory into the learning process, rather than rely purely on generic function approximation.

Although computing optimal *feedback* control is difficult for high-dimensional systems, many powerful algorithms exist for computing *open-loop* optimal solutions. These include indirect methods that solve the boundary value problems arising from Pontryagin's minimum principle and direct methods that discretize the optimal control problem into a finite-dimensional optimization problem. Such techniques have been successfully applied to complex nonlinear systems. By repeatedly solving open-loop optimal control problems for randomly selected initial conditions $\mathbf{x}$, one can generate datasets that contain not only the optimal cost $V^{(i)}$ but also the optimal trajectory $\mathbf{x}^{*(i)}$, control $\mathbf{u}^{*(i)}$, and costate variables $\boldsymbol{\lambda}^{(i)}$. These quantities provide rich and highly accurate information about the structure of the optimal solution and can be used to train and validate neural network approximations of the value function and the associated feedback control.

Additionally, physics-informed training must be integrated. Rather than training neural networks solely to fit sampled values of the value function, the learning process can incorporate structural relationships from optimal control theory. For example, the training objective for a data set $\mathcal{D}$ may combine multiple loss terms,

$$\mathcal{L}(\theta; \mathcal{D}) = \text{loss}_V + \mu_\lambda \text{loss}_\lambda(\theta; \mathcal{D}) + \mu_u \text{loss}_u(\theta; \mathcal{D}),$$

where loss_V is the mean squared error of value function and the additional loss terms— loss_λ , based on costate data, and loss_u , based on control data—enforce relationships derived from optimal control theory. In

particular, the costate variable is the gradient of the value function, and the optimal control is determined from the value function through the Hamiltonian minimization condition, thus significantly improving training efficiency and accuracy.

Control-theoretic insights can also guide the design of neural network feedback architectures. Although neural networks can achieve high training accuracy, controllers obtained purely from data may fail to stabilize the system when deployed in closed-loop operation. Recent work addresses this with classical optimal control designs such as the linear quadratic regulator. These hybrid architectures combine the reliability of classical control with the expressive power of deep learning. To further enhance both efficiency and reliability, components can be integrated into an adaptive learning framework (see Figure 2). In this scheme, an initial controller trained on a limited dataset generates approximate closed loop trajectories and costates, which serve as effective initial guesses to warm-start open-loop optimal control solvers. This significantly improves convergence and computational efficiency of the open-loop solvers. Moreover, it enables targeted data acquisition, particularly in regions where the current controller underperforms. Through this iterative, performance-driven process, the controller is progressively refined toward accurate and efficient optimal feedback control.

These ideas have been tested on several high-dimensional control problems, from spacecraft attitude control to systems of PDEs [2] and nonlinear unmanned aerial vehicles [3]. Together, recent developments point toward a new paradigm in which data, models, and control are tightly integrated through learning. By combining theoretical insights from dynamical systems with the expressive power of neural networks, researchers can address problems once deemed intractable. Continued progress along this direction may significantly expand the scope and impact of DA and optimal control in science and engineering.

Wei Kang delivered a plenary talk on this topic, titled “Deep Learning Meets System Theory: A Collaborative Frontier in Control Engineering and Data Science,”¹ at the 2025 SIAM Conference

¹ https://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=85139

Richard A. Tapia

Continued from page 3

fidence, and professional enlightenment he bestowed on so many, he left us with the best memories, fantastic stories, and important lessons to pass on to the next generation.

Acknowledgements: Rodrigo Bañuelos and Anthony J. Kearsley wish to express their sincere gratitude to Luis Melara, professor of mathematics at Shippensburg University and SIAM Vice President for Equity, Diversity, and Inclusion, for suggesting that they write this obituary and for encouraging them throughout its preparation.

References

- [1] Smith, B. (2026, March 20). Harris County names Ship Channel Bridge for Rice mathematician Richard Tapia. *Rice University Office of Public Affairs*. Retrieved from <https://>

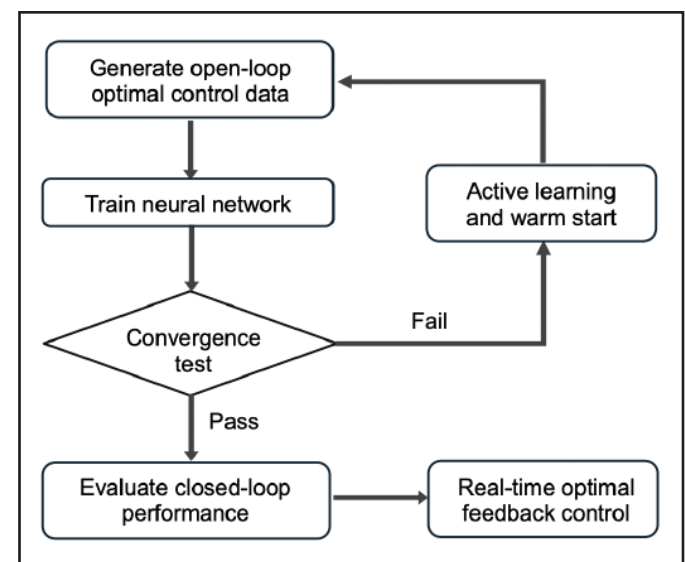


Figure 2. Adaptive learning of optimal feedback control. Figure courtesy of the authors.

on Control and Its Applications² (CT25), which took place last July in Montréal, Québec, Canada. Kang, Qi Gong, and Lars Grüne also delivered a two-session minisymposium³ on this topic at CT25.

References

- [1] Kang, W., Xu, L., & Zhou, H. (2024). Observability and effective region of partial differential equations with application to data assimilation. *SIAM J. Sci. Comput.*, 46(3), 249-271.
- [2] Nakamura-Zimmerer, T., Gong, Q., & Kang, W. (2021). Adaptive deep learning for high dimensional Hamilton-Jacobi-Bellman equations. *SIAM J. Sci. Comput.*, 43(2), A1221-A1247.
- [3] Nakamura-Zimmerer, T., Gong, Q., & Kang, W. (2022). Neural network optimal feedback control with guaranteed local stability. *IEEE Open J. Control Syst.*, 1, 210-222.

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² <https://www.siam.org/conferences-events/past-event-archive/ct25/>

³ https://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=84515

news.rice.edu/news/2026/harris-county-names-ship-channel-bridge-rice-mathematician-richard-tapia

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Game Theory for the Evolution of Collective Intelligence

By Lakshmi Chandrasekaran

A swarm of bacteria exhibits social behavior by following a communication protocol called *quorum sensing* to coordinate gene expression and forage for food [2]. The animal kingdom is social, too, as birds and mammals have fine-tuned their collective behavior over millennia to tackle predation, gain vital resources, and gather information in dynamic environments [1].

In humans, collaboration within a group of individuals (as opposed to efforts made by a single individual) can be better suited to successfully tackle complex problems. This type of group behavior is known as *collective intelligence*. “The best real-world examples of collective intelligence are probably the stock markets and, more recently, prediction betting markets,” Joshua Plotkin of the University of Pennsylvania said.

Engineers have extensively studied the phenomenon of collective intelligence and designed algorithms to allow a group of independent agents (e.g., servers) to combine limited information (e.g., distributed computation nodes) in an intelligent manner for successful collective outcomes. However, critical open questions remain. For instance, in the absence of any central planner, how do living beings in the natural world get together to tackle their problems?

To address this question, Plotkin and his colleagues Guocheng Wang, Qi Su, and Long Wang employ a game theoretic approach to demonstrate the evolution of collective intelligence in their new study [3].

To develop a mathematical model, the researchers consider the environmental outcome Y as a function of m random factors $\{X_1, X_2, \dots, X_m\}$. Here, x_i represents the value of factor X_i at any given time over discrete time steps. Each random factor is sampled independently from a Gaussian distribution with mean zero and variance σ_i^2 (see Figure 1a).

The researchers assume a linear relationship between the factors and the environmental outcome:

$$Y = \alpha_0 + \alpha_1 X_1 + \dots + \alpha_m X_m \quad (1)$$

The constant linear coefficients $\alpha_i (i \in M)$ denote the weight of each factor X_i 's contribution to the outcome. These weights are unknown to the individuals.

“It would be helpful to think about this as, what is the temperature [outcome] tomorrow?” Plotkin said to provide an example. “There are so many random factors that will affect the temperature.”

The authors consider a population of n individuals who are tasked with predicting the environmental outcome based on observations of the factors, which serve as the determinants. Predicting the outcome by linearly combining various factors may, at first blush, seem straightforward. “Here’s what makes it hard,” Plotkin said. “Each individual in the population can only look at one factor.”

Based on their observation of a single factor and personal belief about the correlation between the observed factor and the outcome, each individual makes a

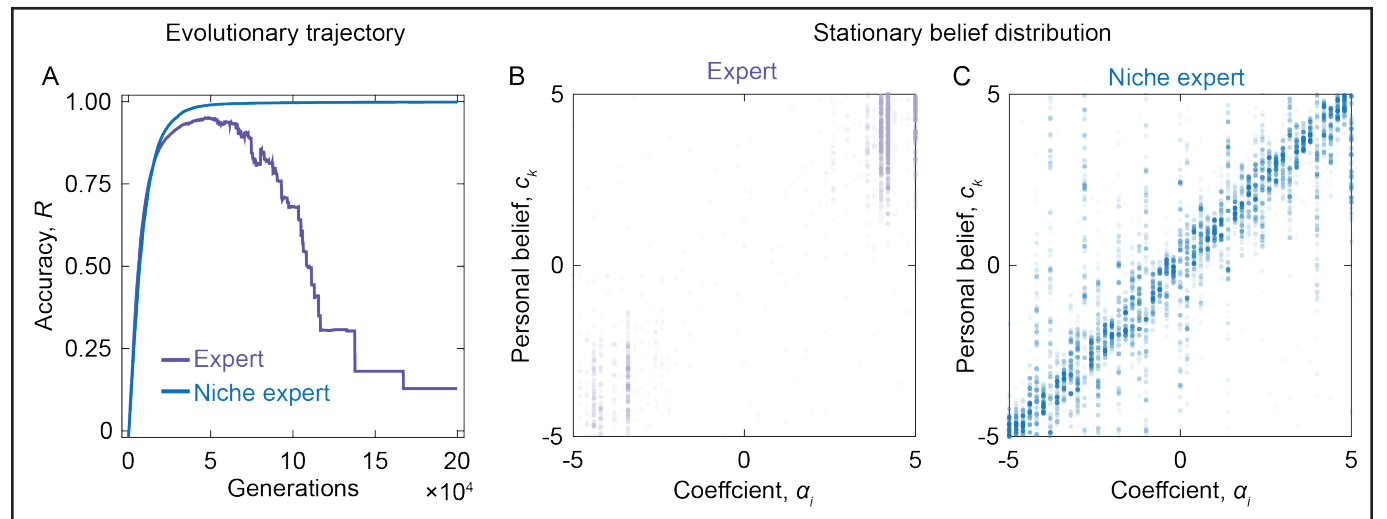


Figure 2. The niche-expert payoff scheme can facilitate collective intelligence. **2a.** The expert payoff scheme promotes collective accuracy in the early stages of the behavioral evolution but collapses over time. The niche-expert incentivizes individuals to observe different factors and that guarantees long-term collective accuracy. **2b–2c.** Each dot represents the individual’s strategy including which factor they choose to observe and their belief. Compared to the niche-expert structure, the stationary strategy distribution for the expert structure is less diverse. Figure courtesy of [3].

prediction, quantified by the following framework:

$$Y_A = c_A x_{eA}. \quad (2)$$

This equation represents the personal prediction made by individual A , who has chosen to observe factor e_A with value x_{eA} . Here, c_A represents the individual’s belief. If c_A is positive, then A believes that the factor is positively correlated with the outcome Y . The absolute value of c_A reflects the strength that the individual believes this factor has for the outcome (see Figure 1b).

Aggregating all of the individual predictions produces a collective prediction for the group. Typically, this collective prediction is expressed as a function of individual predictions: $\hat{Y} = g(Y_1 + Y_2 + \dots + Y_n)$ (see Figure 1c). The authors use two different approaches to combine individual predictions into a group prediction: (i) average across the entire population or (ii) only average across clusters (i.e., average belief among individuals who observe the same factor). When averaging across the entire population, the collective prediction is denoted by

$$\hat{Y} = \frac{Y_1 + Y_2 + \dots + Y_n}{n} \quad (3)$$

For each factor, the aggregate prediction can be expressed in terms of the predicted coefficients:

$$\hat{\alpha}_i = \frac{1}{n} \sum_{A \in N_i} c_A. \quad (4)$$

Here, N_i denotes the group of individuals who observe the factor X_i . The collective prediction $\hat{Y}$ can thus be expressed as $\hat{Y} = \hat{\alpha}_0 + \hat{\alpha}_1 X_1 + \dots + \hat{\alpha}_m X_m$.

In contrast, the clustering aggregation method is based on the framework that each individual only predicts the contribution of the factor that they have observed rather than predicting the final outcome. The predicted coefficient $\hat{\alpha}_i$ is then calculated as the average belief of individuals who observe the same factor, instead of averaging over all the individuals:

$$\hat{\alpha}_i = \frac{1}{n_i} \sum_{A \in N_i} c_A. \quad (5)$$

Here, n_i denotes the size of cluster i of individuals who observe the same factor X_i .

The collective prediction $\hat{Y}$ is then determined by adding the predicted effects of all observed factors. Any unobserved factor (such that $n_i = 0$) is excluded from the sum:

$$\hat{Y} = \sum_{i=0}^m \frac{1}{n_i} \sum_{A \in N_i} Y_A. \quad (6)$$

After the individuals form their own predictions, they are rewarded with payoffs based on their performance. Individuals then imitate each other’s strategies, with a tendency to imitate those with greater payoffs — a process called *social learning*. In their paper, Plotkin and his colleagues explore varying payoff schemes that could foster collective intelligence via social learning.

They first considered an obvious payoff scheme, which rewards the “expert” who has made the most accurate prediction. This payoff structure is represented as

$$\pi_A = -(Y_A - Y)^2, \quad (7)$$

such that individuals whose predictions Y_A are closer to the actual outcome Y will receive higher payoffs. However, such a payoff scheme presents a risk to the evolution of population-level behavior. Other individuals in the group will tend to imitate the expert and adopt the most lucrative strategy; while this approach may enhance collective accuracy in the initial stages of behavioral evolution, it can fall apart over time because little information from other unobserved factors (which may have an influence on the final outcome) are incorporated into the reward structure. “Since everyone will be copying one person’s strategy, that won’t work the next round,” Plotkin said. “It’s as if there’s not enough diversity of opinions in the population.”

Figure 2a displays the collapse in collective accuracy over time in the expert

payoff scheme (purple curve). As shown in Figure 2b, simulations reveal that personal beliefs are focused on a limited number of factors, leading to poor collective accuracy.

To compensate for the shortcomings of the “expert” reward scheme, the researchers introduced a modified version called the “niche expert.” Under this structure, individuals imitate others who are good predictors, particularly when these predictors focus on “very rare” factors that impact the final outcome. “What this does is that it preserves diversity of the population,” Plotkin said. “People copy each other’s strategies, but they never end up all looking at the exact same factor because again, you don’t copy people who are looking at a common factor.”

The “niche expert” payoff structure is formulated as

$$\pi_A = -\rho_{eA} (Y_A - Y)^2, \quad (8)$$

where ρ_{eA} denotes the proportion of the population that observes the same factor as individual A . Under the niche payoff structure with a diversity of observed factors, simulations demonstrated that personal beliefs fall close to the real coefficients (see Figure 2c). Assuming the clustering aggregation rule in (5) and using Monte Carlo simulations, Figure 2a shows that the niche expert scheme (blue curve) can drive the population to produce more accurate long-term predictions than the expert scheme. In fact, the niche expert scheme will indeed eventually ensure collective accuracy.

Both the expert and niche expert reward structures are based on the idea of incentivizing personal accuracy. However, would these reward structures be efficient even under diverse problem settings? For example, consider that rather than predicting the temperature, the problem has now been changed to predict how sunny it will be. While this modified problem may still depend on the same factors that were used to predict the temperature, the researchers mathematically demonstrate that the niche expert payoff structure may not be robust enough to tackle diverse problems.

The introduction of a perturbation or “environmental shock” leads to a drop in prediction accuracy for the niche expert scheme (represented by the blue curve in Figure 3a, on page 7). A jolt to the system leads to a resampling of the coefficients to ensure that the population finds a solution to the new prediction task. However, the niche expert model does not immediately recover its high accuracy level because it does not allow for a diversity of beliefs after equilibrium is reached for the first time, as well as exhibits a reduction of belief variance within clusters (see Figure 3b, on page 7). This is clearly not ideal.

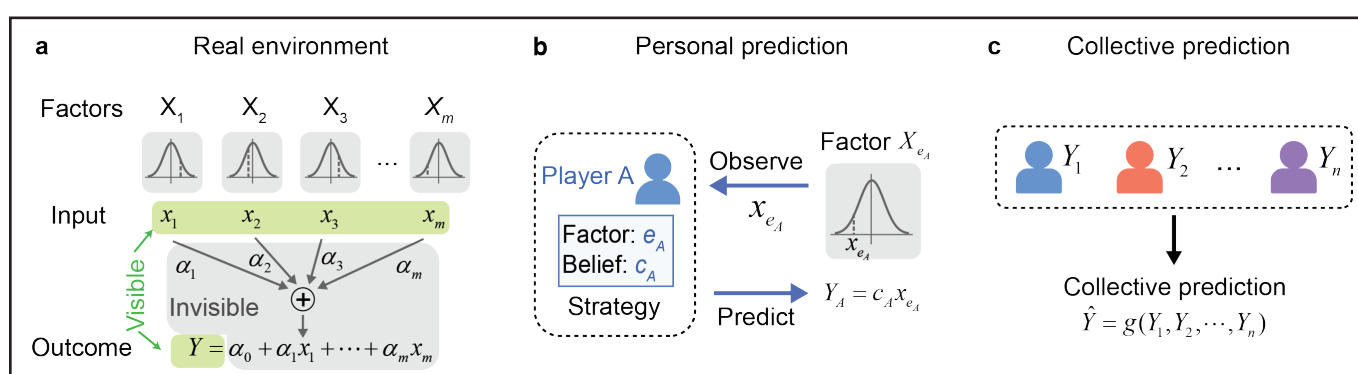


Figure 1. Individual and collective predictions. **1a.** The factors are sampled independently from a Gaussian distribution. Each individual can observe only a single factor to make their predictions for the outcome of the environment. **1b.** An individual A has a strategy composed of which factor they choose to observe and how the individual believes the factor is correlated with the outcome. **1c.** The collective prediction is calculated by combining all the individual predictions expressed by different aggregating functions. Figure courtesy of [3].

Collective Intelligence

Continued from page 6

“In a changing environment where you have to keep adapting to new decisions, maintaining some exploration and variation is crucial,” Simon Levin, a Princeton University professor who was not associated with this study, said.

To strengthen their model’s robustness to various problem settings, the study authors propose another reward structure called the “feedback” payoff scheme. “By feedback, we mean how much did that person’s contribution move the collective prediction to the true value,” Plotkin said.

Under the feedback scheme, individuals whose contributions improve the overall accuracy of the collective prediction $\hat{Y}$ are rewarded even if their individual predictions are not necessarily accurate. In this scheme, “You have to know three things — what was their individual prediction, what was the collective prediction, and what was the truth?” Plotkin said.

The orange curve in Figure 3a demonstrates that when the model encounters a shock, the feedback scheme is able to rapidly adapt back to an almost perfect collective accuracy. “[This structure] rewards people who effectively get the wrong answer, but in the right direction,” Levin said. “That is people who, by choosing answers that are different than the average of the group, pull it back towards the middle and maintain the diversity in the system. I think it’s very clever.”

Under the feedback payoff scheme, the equilibrium state supports a diversity of beliefs that allows the model to bounce back quickly and converge to collective accuracy. The error of collective prediction is now modeled as

$$\pi_A = Y_A(Y - \hat{Y}), \tag{9}$$

for the feedback payoff structure. If the collective prediction $\hat{Y}$ is lower than the real outcome Y , then individuals with predictions larger than $\hat{Y}$ will receive a higher payoff. This scheme rewards “reformers” in the group, whose predictions help bring the population average closer to the true outcome. The researchers used a Lyapunov function to prove that the feedback reward scheme is guaranteed to produce collective accuracy. These results demonstrate that identifying and rewarding good team players, rather than experts, enhances collective intelligence. Other advantages to the feedback payoff scheme include better performance when the factors are correlated (see Figure 3c).

The current model framework can provide insights into the emergence of collective intelligence in real-world settings. For instance, it may be natural to consider a feedback payoff scheme in the stock market, where a trader’s prediction may move the market price towards the true outcome even if their individual predictions may not be accurate. In the future, the model could be extended to study collective intelligence when the relationship between the observable factors and the outcome is nonlinear, or when individuals intentionally report false value of factors to gain higher payoffs.

References

[1] Couzin, I.D. (2025). Collective intelligence in animals and robots. *Nat. Commun.*, 16, 9574.
[2] Miguel, C.M.T.S., Santos, C.A., Lima, E.M.F., Rahme, L.G., & Pinto, U.M. (2026). Quorum sensing in bacteria: From mechanism to applications in foods. *Curr. Opin. Food Sci.*, 69, 101394.
[3] Wang, G., Su, Q., Wang, L., & Plotkin, J.B. (2025). Individual incentives that promote collective intelligence. *Proc. Natl. Acad. Sci.*, 122(51), e2516535122.

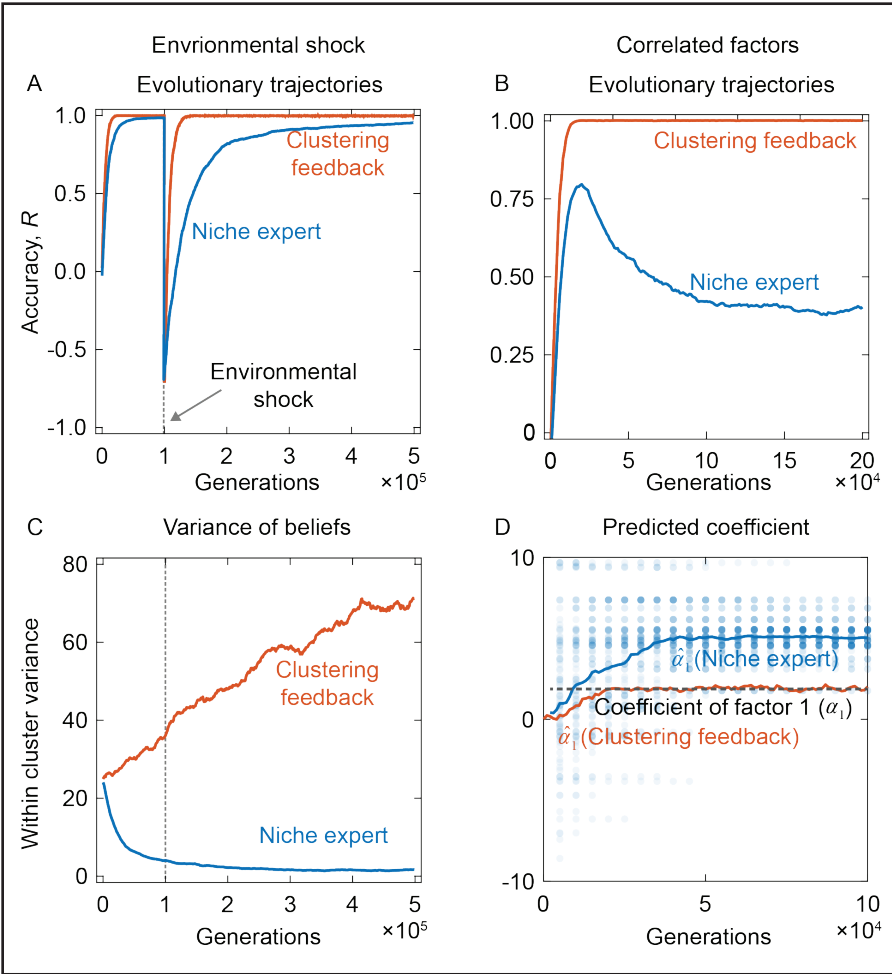


Figure 3. When there is variation in the nature of tasks, the feedback payoff structure is more robust than niche-expert. **3a.** Under feedback payoff, the population is able to rapidly recover to near-perfect collective accuracy upon the introduction of a shock to the system. **3b.** The within cluster belief variance reduces over time in the niche-expert scheme compared to feedback scheme, whose equilibrium state allows for a diversity of beliefs. **3c.** Compared to the niche-expert, the feedback scheme still produces collective accuracy when the factors are correlated. **3d.** For the niche-expert scheme, the predicted coefficient never equals the real coefficient, in contrast to the feedback scheme. Figure courtesy of [3].

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The Role of Quantization in Quantum Computing

By Arni S.R. Srinivasa Rao
and Steven G. Krantz

The use of topological data analysis for quantum computing has gained momentum within the applied mathematics community [2, 4]. This has opened doors for the creation of novel hybrid methods that employ several branches of mathematics for processing and digesting the very large, ever-increasing data sets that many mathematicians work with daily.

Quantum computing allows mathematicians to process this high-dimensional, complex data through either qubits or quantum bits. These qubits consist of two states: two polarizations of a photon or two spin positions of an electron [6]. Practically, the state of a qubit is still at a model stage due to (i) the several uncertainties in defining atomic structure, and (ii) the fact that the measurement of a qubit follows a probability approach.

Recent demonstration of the supremacy of quantum computing in laboratory-level experiments shows promise [1]; however, these experiments were not based on direct comparison with classical computing algorithms, and instead compared partial patterns using random pattern circuits for benchmarking. Suppose $F(x_i)$ is the cross-entropy benchmarking fidelity [5] of a measurable bitstring, say, x_i ; then

$$F(x_i) = 2^n P(x_i) - 1, \tag{1}$$

where $P(x_i)$ is the probability of detecting a bitstring x_i . When $P(x_i)$ follows a uniform sampling distribution, $F(x_i) \rightarrow 0$ (since $P(x_i) \rightarrow 1/2^n$).

These experiments of benchmarking might see real-world, implementable quantum computing hardware and software in the future, but the technicalities of quantization [3] could instead be used to process massive, real-world data sets.

Quantization Hypothesis

We propose a quantization method that could aid in processing massive data sets at the atomic level by imagining a *flexible topological space* of the data, followed by functional mapping onto a space in the atom. These techniques potentially offer better analysis of complex data sets, but were left unexplored in previous experiments that used one- and two-qubit proof of concepts.

Flexible Topological Space

The massive data sets created from individual and societal metrics are continuous and unavoidable, thus the ability to optimally process such data sets is of high value. This is where the quantum computing framework could assist. Quantization can break down data at large scales, allowing it to be conceptualized as a topological space containing hidden or visible multilevel information. This breakdown of smaller spaces need not be unique; they depend on the requirements of a given study design and the goals of an experiment.

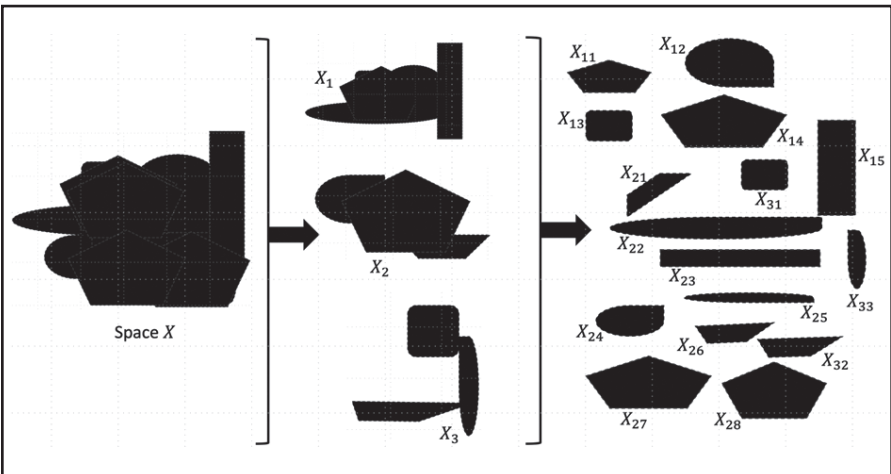


Figure 1. Decomposition of massive topological data X into flexible spaces for processing and modeling. The space X is the union of smaller disjoint subspaces X_1 , X_2 , and X_3 . The subspaces X_{ij} , X_{ik} , and X_{il} for $i = 1, 2, 3$ are themselves a union of smaller disjoint subspaces: X_{1j} for $j = 1, \dots, 5$; X_{2k} for $k = 1, \dots, 8$; and X_{3l} for $l = 1, 2, 3$. Figure courtesy of the authors.

Given a topological space X that represents a massive data set, one can partition X into smaller subspaces that can be processed more efficiently by a quantum machine (see Figure 1).

Functional Mapping of Quantized Spaces

Once data is decomposed into smaller subspaces, it can be used to model the phenomenon of interest. Different quantized subspaces can lead to different forms of individual model equations, allowing for effective analysis of the original massive dataset. By reducing the data space through quantization, computations are significantly quicker.

Given current computational capabilities, and the existing cloud computing facilities for any time-sensitive research objectives and goals, collecting and analyzing complex data sets from various sources is a challenging endeavor. Every organism on the planet continuously leaves massive data footprints, ranging from health-related and scientific information to climate and food-related global information. This complex multilevel data varies by individual, region, and several other factors. The topology of quantum data can be thought of as wired deformations of multilevel data, where each wire can be imagined representing a space of the quantized data at the atomic level. Such deformations are precisely needed for understanding if *atomic uncertainty* exists.

Atomic Uncertainty

Let us denote by X the space of massive complex multilevel data. Here X could be dynamic over time, and we express it here as

$$X = \bigcup_{i \in A} \int X_i di, \tag{2}$$

where $X_i \subset X$ and A is the set of different components of data in X that needs quantization while processing them through an atom. We expressed X in (2) as an integral to give the impression that the X_i 's are continuous pieces of the total space X . For example, when only three components of the data, say i, j , and k , are present in X , then it can be expressed as

$$X = \int X_i di \cup \int X_j dj \cup \int X_k dk. \tag{3}$$

Carlsson's methods of data transformation [1] could help us to write X in smaller quantities, then the data can be processed into normal or traditional computing machines (non-quantum type). The continuous data flux can be broken into smaller continuous fluxes of bits of data. Suppose Q_X is a space of the quantized data at the atomic level, then it is rather difficult to establish a one-to-one correspondence: $Q_X \sim X$, because the atomic structure is yet to be clear to scientists.

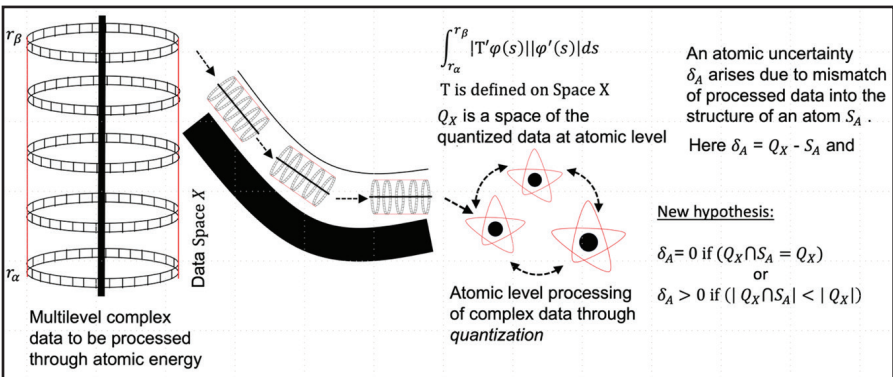


Figure 2. Processing complex data into an atomic structure, quantization, and the possibility of the creation of uncertainty in the atomic space. Figure courtesy of the authors.

If we let the atomic structure be denoted by S_A . We propose a new hypothesis:

$$\delta_A = 0 \text{ if } (Q_X \cap S_A = Q_X) \tag{4}$$

Or

$$\delta_A > 0 \text{ if } (|Q_X \cap S_A| < |Q_X|),$$

where $\delta_A = Q_X - S_A$, a quantity representing atomic uncertainty. The norm $|Q_X \cap S_A|$ indicates the level of matching quantized data with the atomic structure.

Even though quantization techniques can provide bits of continuous data flux, the uncertainty in the atomic structure makes quantum computing prone to error. This is especially true when working with massive data inputs, as the errors of quantum computing (until we have clarity on atomic structure) could generate large-scale errors. Even so, the random pattern bitstring benchmarking that typically assists in small-scale proof of concepts can be used to measure uncertainty. The quantization techniques proposed, combined with circuits, can help break down complex multilevel data into single-layer data in a faster way (see Figure 3, on page 10).

A Basic Example

Let us try to understand the computation of atomic uncertainty in a disease modeling framework within a population. Let the multilevel data X be formed out of four layers of distinct information, say X_1 , X_2 , X_3 , and X_4 , where:

- X_1 is basic demographic information of the population (e.g., population density by age or gender)
- X_2 is socioeconomic status of individuals (e.g., proportions of low-income versus high-income, etc.)
- X_3 is the infection status of each individual in the population (e.g., susceptible, infected, or recovered)
- X_4 is living conditions of work and house (e.g., living in a condo opposed to an individual house, etc.).

For the sake of illustration, let us assume:

$$\begin{aligned} X_1 &= [0.35, 0.45, 0.15], \\ X_2 &= [0.15, 0.25, 0.11, 0.28, 0.16, 0.21], \\ X_3 &= [0.23, 0.67, 0.10], \\ X_4 &= [0.13, 0.42, 0.15, 0.16, 0.25]. \end{aligned}$$

See Quantum Computing on page 10

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The Cognitive Science Revolution

By Devdatt Dubhashi

The Laws of Thought: The Quest for a Mathematical Theory of the Mind. By Tom Griffiths. Henry Holt and Co., New York, NY, February 2026. 400 pages, \$31.99.

While the world is transfixed with the artificial intelligence (AI) revolution—most recently with the development of Claude¹ and ChatGPT,² the large language model (LLM) products of the companies Anthropic³ and OpenAI⁴ respectively—the field of cognitive science is simultaneously having its own revolution. *The Laws of Thought: The Quest for a Mathematical Theory of the Mind* gives a panoramic overview of this revolution, from the viewpoint of an active participant at the epicenter of this new wave: cognitive scientist Tom Griffiths of Princeton University. The book describes, in a lucid and accessible way, key ideas in the discipline, from ancient history to the present state of the art, and paints short biographical sketches of the most important figures in the revolution.

Griffiths highlights the three main pillars of the cognitive science revolution: logic, neural networks (and other computational models), and probability—listed roughly in historical order of their contribution.

Logic dates back to Aristotle and the syllogisms he designed to explain the formation of arguments and inferences. The great polymath genius Gottfried Wilhelm Leibniz later attempted to reduce all human thought to mathematical calculation:

“Let us calculate, without further ado, to see who is right,” he declared. In 1879, workmen fixing a leaky roof discovered a mysterious machine discarded in the corner in an attic at the University of Göttingen; with its cylinders of polished brass and oaken handles, the artifact was identified as one of several early mechanical calculating devices that Leibniz invented in the late 17th century. Leibniz in turn was influenced by Ramon Llull—a Majorcan philosopher, logician, and mystical thinker—and his work *Ars magna* (or “ultimate general art”) from 1308. The modern foundations of logic, though, were laid by the English autodidact George Boole who invented the system known today as Boolean algebra in his great work *An Investigation into the Laws of Thought*, which inspired the title of Griffiths’s book. The logic tradition led eventually to the birth of modern computer science via the foundational insights of Alan Turing in his solution to David Hilbert’s famous *Entscheidungsproblem* from 1928.

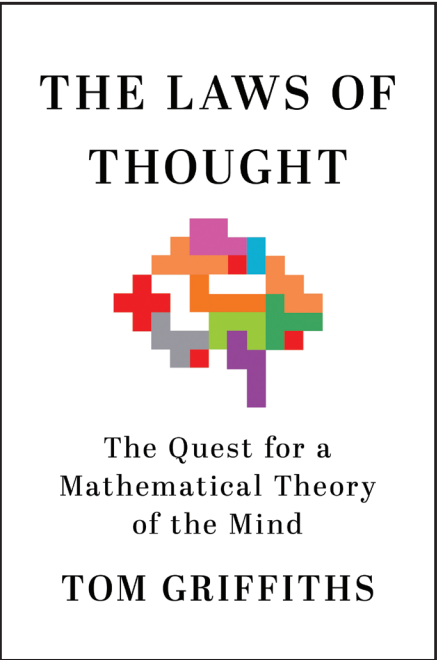
The second pillar is connectionism, or the tradition of using neural networks to model cognition by drawing inspiration from the brain. Griffiths cites the early origins of connectionism in Frank Rosenblatt’s Perceptron and the first model of neural networks by Warren McCulloch and Walter Pitts. This area of cognitive science suffered a temporary setback but was revived again by scientists like David Rummelhart and Jay McClelland with their influential book *Parallel Distributed Processing*, published in 1987. Connectionism has reached its apogee today following the deep learning revolution that lies at the heart of modern AI, for which Geoff Hinton received both the 2024 Nobel Prize in physics⁵ and the 2018 Turing Award.⁶ Modern neural networks lie behind the major successes of modern deep learning both in perception systems based on vision models and language modelling based on LLMs. These

systems demonstrate amazing capabilities for logic and reasoning, most recently achieving a gold-medal-worthy performance in the International Mathematics Olympiad and also solving or assisting in high-level mathematics.

The third pillar, *probability*, supplies the fundamental concepts and toolkit to deal with uncertainty within how cognitive systems understand the world around us. Griffiths effectively shows how probabilistic reasoning extends logical reasoning to handle uncertainties. Perhaps the single most important concept here is the simple principle called the Bayes Rule, a concept introduced by Reverend Thomas Bayes in 1763 that lays the foundations for a systematic approach to updating beliefs when new evidence is presented. Today, Bayesian statistics is an active field of research that provides sophisticated, data-driven computational techniques to implement such belief-updating schemes. Bayesian thinking also provides a fruitful approach to addressing the vexed problem of causation as opposed to mere correlations between events: discovering cause and effect relationships between events is central to cognition. Judea Pearl, engineer and computer scientist, received the Turing Award in 2011 for creating a calculus of probabilistic rules to deduce cause and effect relationships from observational data. Griffiths explains how these principles can be used by many cognitive scientists—such as deep thinkers like Roger Shepard and Amos Tversky—to uncover the principles of cognition. Readers with computer science backgrounds such as myself may have some familiarity with the topics and figures discussed in this

BOOK REVIEW

By Devdatt Dubhashi

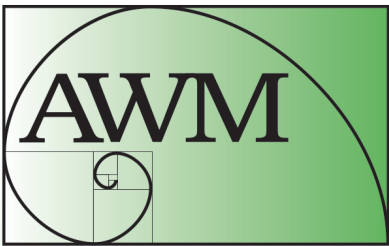


The Laws of Thought: The Quest for a Mathematical Theory of the Mind. By Tom Griffiths. Courtesy of Holt.

¹ <https://claude.ai/login>
² <https://chatgpt.com/>
³ <https://www.anthropic.com/>
⁴ <https://openai.com/>

⁵ <https://www.nobelprize.org/prizes/physics/2024/summary/>
⁶ <https://awards.acm.org/about/2018-turing>

See *Cognitive Science* on page 11



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Quantum Computing

Continued from page 8

The space of X is then given by $X = \cup_{i=1}^4 X_i = [0.10, 0.11, 0.13, 0.15, 0.16, 0.21, 0.23, 0.25, 0.28, 0.35, 0.42, 0.45, 0.67]$.

Let us obtain Q_X values by quantization to one decimal place of the X values above (i.e. rounding to nearest 0.1). This gives us

$$\begin{aligned} Q_{X_1} &= [0.4, 0.5, 0.2], \\ Q_{X_2} &= [0.2, 0.3, 0.1, 0.3, 0.2, 0.2], \\ Q_{X_3} &= [0.2, 0.7, 0.1], \\ Q_{X_4} &= [0.1, 0.4, 0.2, 0.2, 0.3]. \end{aligned}$$

We thus obtain $Q_X = \cup_1^4 Q_{X_i} = [0.4, 0.5, 0.2, 0.2, 0.3, 0.1, 0.3, 0.2, 0.2, 0.2, 0.7, 0.1, 0.1, 0.4, 0.2, 0.2, 0.3]$.

Hence $Q_X = [0.1, 0.2, 0.3, 0.4, 0.5, 0.7]$. Let us assume the atomic structure $S_A = [0.05, 0.1, 0.13, 0.35, 0.4, 0.45, 0.6, 0.7]$.

The atomic uncertainty δ_A is computed by $Q_X - S_A = [0.2, 0.3, 0.5] \neq 0$. Since the atomic uncertainty is present, the computation errors are present in understanding the disease spread.

Conclusion

Quantization techniques could offer great assistance in processing complex data that would not otherwise be processable through a classical computing approach in a standard bit; however, the potential for atomic uncertainty due to the random structures and patterns involved remains. The quantum computing world is still in a proof-of-concept stage and raises speculation of various kinds of data manipulation at the atomic level, but the possible applications of quantum computing are considerable if the hardware

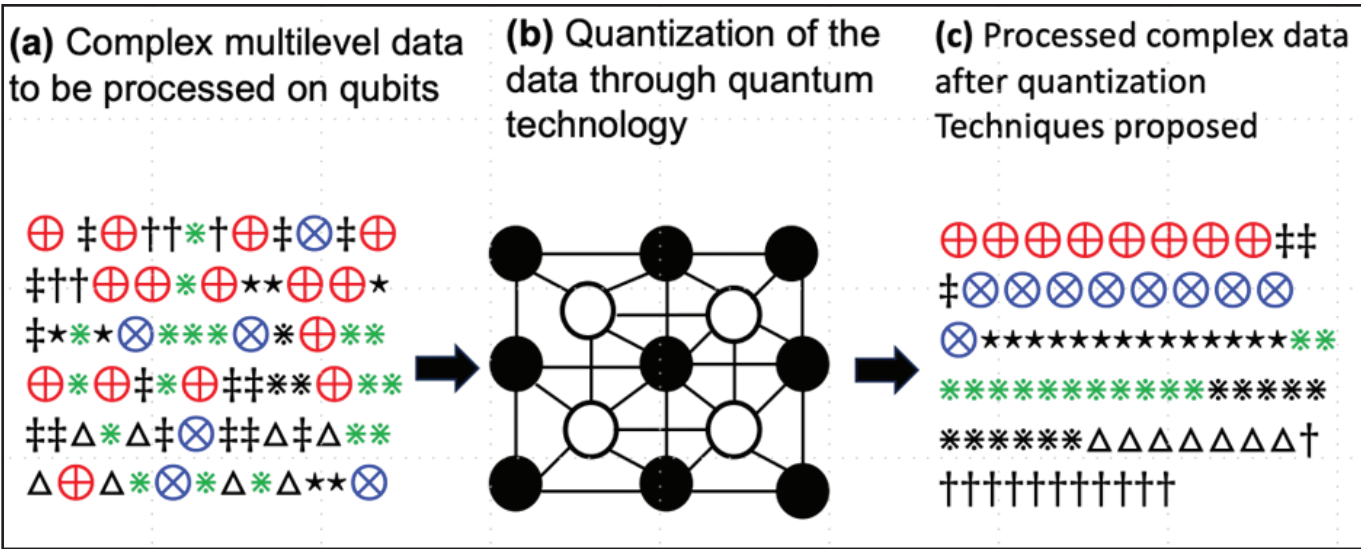


Figure 3. Simplification of the processing of complex data through quantization and qubit circuits. Figure courtesy of the authors.

eventually becomes available. However complex, the patterns within a data set and their complexities need to be precisely understood in order to map data into the structure of a multi-partite quantum system. The role of quantization would reduce the gap in decoding the patterns in the data and preparing it to be processed into a qubit.

For more information, a complete list of references, and an expanded introduction, please visit our website.¹

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References

[1] Arute, F., Arya, K., Babbush, R., Bacon, D., Bardin, J.C., Barends, R., ... Martinis, J.M. (2019). Quantum supremacy using a programmable superconducting processor. *Nature*, 574, 505-510.
[2] Carlsson, G. (2009). Topology and data. *Bull. Amer. Math. Soc.*, 46(2), 255-308.
[3] Graf, S., & Luschgy, H. (2000). *Foundations of quantization for probability distributions*. New York, NY: Springer Science & Business Media.
[4] He, Y.H., Heyes, E., & Hirst, E. (2023). Machine learning in physics and geometry. In S.G. Krantz, Arni S.R. Srinivasa Rao, & C.R. Rao (Eds.), *Artificial intelligence: Handbook of statistics* (Vol. 49) (pp. 47-81). Amsterdam, Netherlands: Elsevier.

[5] Parthasarathy, K.R. (2005). *Mathematical foundation of quantum mechanics* (Vol. 35). New York, NY: Springer.

[6] Schumacher, B. (1995). Quantum coding. *Phys. Rev. A*, 51(4), 2738-2747.

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¹ <https://www.math.wustl.edu/~sk/Krantz-Rao>

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A Strange Collision

Like many dogs, our dog Lina likes to fetch sticks. On one of our walks, I threw a stick for her to fetch and accidentally hit a tree. Instead of bouncing off, the stick did something unexpected. It seemed as if the stick wrapped around the tree and changed direction (see Figure 1a). It was almost as though the tree pulled the stick and deflected its path in the direction opposite to the expected.

Was my perception distorted? Could the stick actually change direction in this unexpected way, at least in principle?

Figures 1b and 2a illustrate a possible scenario in which a strange deflection can indeed happen.¹ According to Figure 1b, it is possible to have no previous collisions

¹ Gravity is ignored throughout since we are interested in the top view. All our sticks have uniform mass distribution.

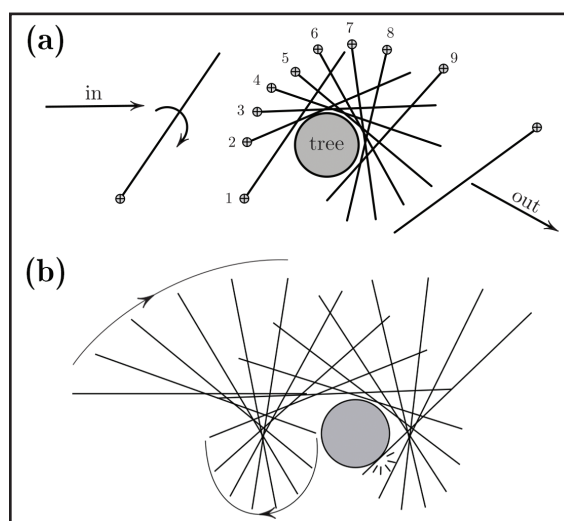


Figure 1a. The tree seemed to have pulled the stick, diverting its path as shown. This picture is a bit misleading: the stick cannot remain in prolonged contact with the tree, as explained later (compare with 1b). **1b.** No collisions happened before the one shown. These are a schematic figures and not results of a computation.

before the fateful impact, something that may not be as clear in Figure 2a.

Some Geometry

Material points of a freely moving stick describe trochoids; two of them are shown in Figure 2a. A trochoid may or may not contain loops, depending on whether the speed of the midpoint is smaller or greater than the speed of the point in question relative to the midpoint. If these speeds are equal, the trochoid becomes a cycloid (see Figure 2b). One can produce a cycloid by spinning a stick on its end and releasing it; both ends will then describe cycloids, moving exactly like the ends of the diameter in Figure 2b.

Hidden Cycloids

Returning to Figure 1a, we ask: can the flying segment “wrap” around the tree, remaining tangent to it for longer than an instant? Leaving this question aside for now,² here is a partial answer: if the disk’s boundary in Figure 1a is not circular but contains an arc of a cycloid, then the answer is “yes.” This is because of the beautiful fact that *the envelope generated by a segment that rotates and translates simultaneously is a cycloid* [1] (see Figure 2b).

In particular, all lines in Figure 1b are tangent to the same cycloid (not shown).

² Wishing to avoid discussing impacts, friction, etc.

The Envelope is a Cycloid: A Proof

Here is the short proof of the claim that a rolling wheel or an equivalently freely flying stick is tangent to the cycloid of half the size of the cycloid that is generated by a diameter’s endpoint. Figure 3 shows AB to be a material diameter painted on a disk that rolls without sliding. Consider also the disk of half the diameter (inner) tangent to the large disk at the contact point C . Let P be the foot of the perpendicular onto AB from C .

Since C is the instantaneous center of rotation of AB (no sliding), velocity $v_P \cdot CP = 0$. However, this means that P lies on the envelope, which is characterized by the property that the enveloping curves (AB s in this case) have zero velocity, normal to the envelope at the point of contact. So, P traces out the envelope of AB s as the disk rolls.

On the other hand, P always lies on the small circle, since $\angle OPC = \pi/2$ and CO is a diameter. Furthermore, from the already established $v_P \parallel AB$ we have $v_P \parallel PO$. To summarize, point P on the small circle has its velocity pointed at the top point O at all times. But this is the characteristic property of a cycloid! Thus, P traces out a cycloid generated by the small circle, in addition to tracing the envelope, proving that the envelope is a cycloid. Q.E.D.

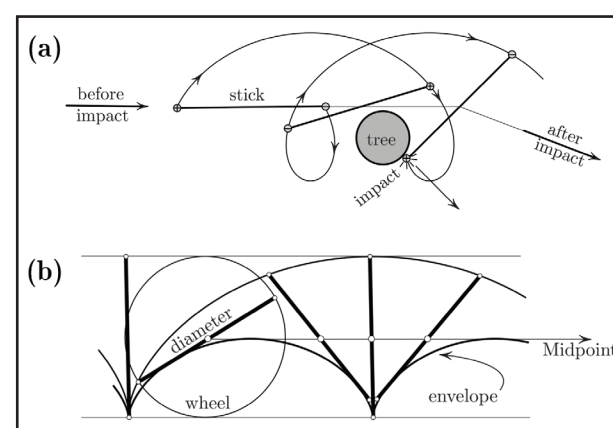


Figure 2a. The (first and only) collision happens from the unexpected right side of the tree. **2b.** The diameter of a wheel rolling without sliding (or equivalently a freely flying stick), is tangent to the cycloid of half the size of the cycloid that is generated by a diameter’s endpoint.

References

[1] Lockwood, E.H. (1961). *A Book of Curves*. Cambridge, U.K.: Cambridge University Press.

The figures in this article were provided by the author.

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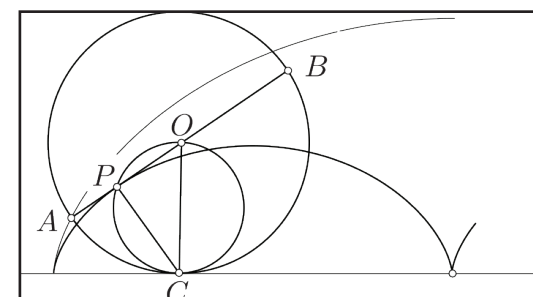


Figure 3. The envelope of the family of diameters is a cycloid of half the size of the cycloids generated by the diameter’s endpoints.

Cognitive Science

Continued from page 9

book, but this aspect of cognitive science research offered many new insights.

Griffiths also explores the value of language; language occupies a very special and central position in cognitive science and is one of the defining properties of human cognition that separates us from other animal species. At the start, language was studied in the symbolic tradition, with Noam Chomsky’s seminal Syntactic Structures as the fountainhead for the study of language via grammar. Later, pioneering explorations of the Soviet mathematicians Andrey Andreyevich Markov and Claude Shannon introduced probabilistic approaches to studying language. This avenue was pursued at IBM Research,⁷ where manager and director Fred Jelenik quipped: “Every time I fire a linguist, the performance of the system goes up.” IBM was eventually displaced by Google, and the statistical approach reached its peak with Google Translate⁸ around 2006. This approach itself was overtaken by the neural approach using modern deep neural networks, which lies behind the current version of the application. Finally, the ongoing development of LLMs and their innovative properties is based heavily on large neural networks, such as Transformer architecture, which allows AI tools to use self-attention mechanisms to ingest heaps of data from the internet.

The debate over whether LLMs can truly “understand language” or are merely “stochastic parrots” that reproduce statistical patterns in their training data is still ongoing. Even further, the question of if LLMs can pave the way to fully-fledged artificial general intelligence (AGI) with the potential to overcome human capabili-

ties remains. Griffiths doesn’t pronounce a definitive verdict on this, but gives clear hints that he does not believe LLMs are the final word. In a recent paper [1], his group countered the claims of a Microsoft group that LLMs are showing “sparks of AGI” by arguing that they actually *betray* the “embers of prediction problems” they are trained on. Griffiths argues instead for a new approach that brings the probabilistic approach to bear on neural networks using modern Bayesian techniques, writing that “hundreds of years after the death of the Reverend Thomas Bayes, thinking in terms of probabilities is still useful for understanding intelligent systems.” Indeed, the future of both AI and cognitive science lies in “neuro-symbolic” systems that create “world models” that can bring together all three pillars of the cognitive science revolution, or the Laws of Thought that govern all intelligent systems.

References

[1] McCoy, R.T., Yao, S., Friedman, D., Hardy, M.D., & Griffiths, T.L. (2024). Embers of autoregression show how large language models are shaped by the problem they are trained to solve. *Proc. Natl. Acad. Sci.*, 121(41):e2322420121.

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Dealing with the After-math: Finding Your Niche in the Mathematical Sciences

By Caoimhe Rooney

“Maths? What on Earth are you going to do with that?” Throughout my education in mathematics, I heard this question more times than I can recall, yet the question itself reveals less about the subject and more about the widespread misunderstanding of its significance.

Mathematics is the unseen architecture of the modern world, too often dismissed as abstract or impractical. Such attitudes reflect not the limitations of mathematics, but rather a failure to recognize its profound influence on systems, ideas, and innovations.

The real question, then, is not what one can do with mathematics, but what could be done without it.

That said, even though I was captivated by the subject itself, I was never entirely sure what path I would pursue after my undergraduate graduation. Mathematics felt logical and structured, with each idea building naturally on the last, forming a coherent whole when viewed in full. Yet it was also profoundly creative. Solving a problem was never merely a matter of applying rules; it required pattern recognition, the ability to make connections, and the drive to discover new pathways and shortcuts to a solution. In that sense, mathematics resembled a vast and intricate puzzle that could be approached from countless directions, each offering a new insight into the underlying structure.

These ideals were omnipresent in academia. Every course, project, and tutorial problem seemed designed to reinforce the delicate balance between structure and imagination — the discipline of following logic while also being free to explore new ideas. But beyond that world, I wasn’t sure where those qualities took root. I hadn’t yet seen how the kind of thinking I loved—the careful reasoning, the creative problem-solving, the thrill of uncovering elegant solutions—translated into the work people did in industry.

My desire to deepen my understanding of mathematics and its practical applications ultimately motivated me to pursue a Ph.D. in industrially-focused mathematical modeling¹ at the University of Oxford. My classes and projects illustrated how I could turn fundamental mathematical ideas into innovative solutions to real-world problems. I began to see how mathematics could drive tangible progress, not just in theory. What struck me most was that the very qualities that first drew me to mathematics—the logical reasoning, puzzle-like problem solving, and the satisfaction of uncovering solutions—remained at the heart of these approaches, now expressed in ways that were both practical and impactful.

I began to wonder where else math could be applied. I came to realize that I could combine my love of mathematics with my

long-standing fascination for space, where analytical thinking, modeling, and creative problem-solving are essential. I started to see how the mathematical principles I had studied in school could be used to model astrophysical objects, understand orbits, optimise trajectories, analyse satellite data, and tackle the complex challenges of space exploration. It was exciting to think that the skills I had developed could contribute to something as vast and inspiring as the space industry.

Following my Ph.D., I received a postdoctoral fellowship at the National Aeronautics and Space Administration (NASA) Ames Research Center,² where I applied mathematical models to understand the atmospheres of distant exoplanets. Since we cannot physically travel to these bodies, we rely on the light that reaches us to infer the chemical composition of the atmospheres of these planets. By collecting this light through powerful telescopes and breaking it down into its electromagnetic spectrum, we can identify subtle patterns and features, commonly referred to as a spectral signature. Each element or molecule leaves a unique fingerprint in the spectrum, which allows us to infer the chemical composition of the planet’s atmosphere. The most exciting part of this job was seeing how the mathematical skills I had developed throughout my studies proved fundamental in developing these models that can understand and decipher real exoplanetary atmospheres.

Studying exoplanets was an incredible experience; it deepened my understanding of how mathematics can reveal the unseen and expanded my sense of curiosity about the universe. It also inspired me to broaden my skills and apply them to something closer to home, where the impact could be more immediate and tangible. I wanted to use the same mathematical and analytical approaches to contribute to real missions that could make a direct difference to life on Earth while advancing our ability to explore space. I was drawn to the idea of being part of a mission from start to finish; I wanted to help design, develop, and operate systems that could fly, turning equations and theory into something real.

That dream brought me to Astroscale³—a company focused on the development of innovative solutions for space sustainability, including actively removing debris and servicing satellites in orbit—where I joined as a flight dynamics engineer. The flight dynamics team offered me the bridge between mathematics and engineering that I’d been looking for: a position where I could apply my problem-solving skills, analytical thinking, and coding experience to develop algorithms for cutting-edge space missions. Beyond the algorithm development, I had the rare opportunity to operate an active



Caoimhe Rooney representing the National Aeronautics and Space Administration (NASA) at the International Astronautical Congress in Paris. Photo courtesy of Caoimhe Rooney.

space mission, demonstrating some of the world’s first achievements in mission called ADRAS-J.⁴ This experience allowed me to rapidly grow from a space mathematician into a fully-fledged space engineer, bridging theory and practice on real missions.

One of the things that struck me most at Astroscale was how deeply collaborative space missions are. Every mission is the product of multidisciplinary effort. Beyond the critical work of the flight dynamics engineers, the flight software team designs the algorithms on board the spacecraft; the Guidance, Navigation, and Control team design how a spacecraft will navigate and orient itself in orbit; the mechanical team designs and builds

the spacecraft itself; and the systems team designs the mission plan while making sure every part worked seamlessly together (just to name a few). Everyone works on a critical component needed to achieve

a single, shared goal. The entire company moves together to make one mission succeed, fostering a sense of collective purpose that I feel is incredibly rewarding. It was a refreshing change from academia, where work could often feel more independent or isolated. The more I learned about systems engineering and how all the pieces come together to create a functioning, reliable spacecraft, the more I wanted to challenge myself to apply my mathematical and analytical skills in this broader, integrated context.

This prompted my move to the ground segment team, where I now work as a senior ground segment systems engineer. In this role, I work on flight dynamics in addition to the broader systems needed for mission control and planning — from the ground station network to the software and processes that enable the safe and effective operation of the spacecraft. Although my day-to-day work is less mathematical than before, I still rely heavily on the same problem-solving mindset and logical thinking that mathematics trained me to develop. Much of my role

involves understanding how all the different elements fit together to meet the mission requirements and ensuring the systems are robust, reliable, and ready for operations. My background as a mathematician has given me the versatility, adaptability, and analytical thinking to approach complex engineering problems from multiple angles — skills that have proven invaluable well beyond equations. And thankfully, I still have the opportunity to code up a solution to a software problem every now and again!

Looking back, my love of mathematics led me to one of the most exciting and versatile careers. It has taken me from exploring distant exoplanets to operating real spacecraft in orbit, and throughout it all, I’ve realized that mathematical skills allow me to do so much more than simply solve equations. They present a different way of thinking, a logical, analytical, and creative approach to understanding and tackling complex problems. Mathematics has taught me how to break challenges down, find patterns, and build solutions — skills that extend far beyond the classroom and can be applied across countless industries, roles, and disciplines. It’s a universal language for problem-solving, and it continues to shape the way I approach every new challenge.

Looking back again on all the times I was asked, “Maths? What on Earth are you going to do with that?” I have to laugh, because it turns out the answer was “something not on Earth at all.”

Caoimhe Rooney is a senior ground segment systems engineer at Astroscale U.K., where she works on the design and development of mission-critical space systems to tackle the ever-increasing space-debris problem. She previously conducted research on exoplanet atmospheres at the National Aeronautics and Space Administration (NASA) Ames Research Center, focusing on the characterization of distant worlds. Caoimhe holds a Ph.D. in applied mathematics from the University of Oxford, where her work combined advanced mathematical modeling with industrial, real-world problems.

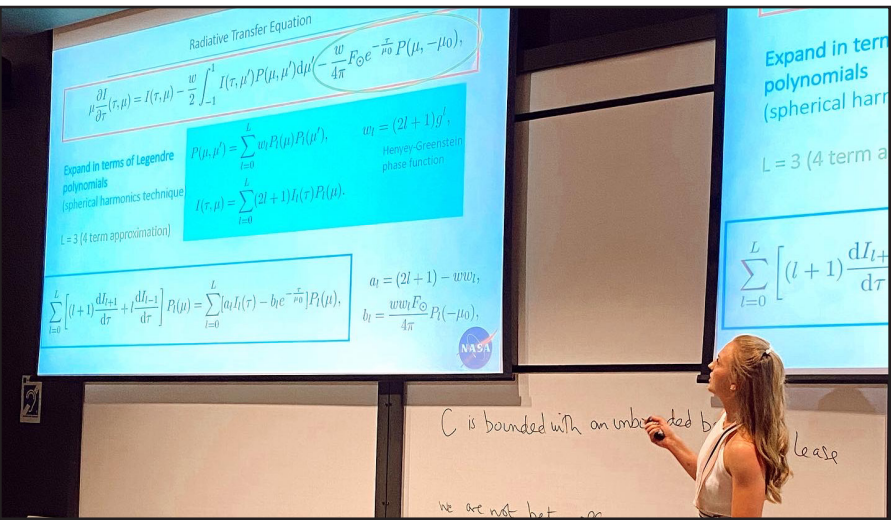
CAREERS IN MATHEMATICAL SCIENCES

¹ <https://www.maths.ox.ac.uk/study-here/postgraduate-study/industrially-focused-mathematical-modelling-epsr-cdt>

² <https://www.nasa.gov/ames/>

³ <https://www.astroscale.com/en>

⁴ <https://www.astroscale.com/en/missions/adras-j>



Caoimhe Rooney presents her research into mathematical models for the electromagnetic spectra of exoplanet atmospheres at a conference at the University of Oxford. Photo courtesy of Caoimhe Rooney.



Caoimhe Rooney graduated with her Ph.D. in applied mathematics from the University of Oxford. Photo courtesy of Caoimhe Rooney.



A Place to Network and Exchange Ideas

Upcoming Deadlines

The following conferences will be held jointly:



SIAM Conference on Mathematics of Data Science (MDS26)

November 16–20, 2026 | Salt Lake City, Utah, U.S.
siam.org/mds26 | #SIAMMDS26

ORGANIZING COMMITTEE CO-CHAIRS

Andreas Mang, *University of Houston, U.S.*
Rebecca Morrison, *University of Colorado Boulder, U.S.*
Rebecca Willett, *University of Chicago, U.S.*

SIAM Symposium on Imaging Science (IS26)

November 16–19, 2026 | Salt Lake City, Utah, U.S.
siam.org/is26 | #SIAMIS26

ORGANIZING COMMITTEE CO-CHAIRS

Weihong Guo, *Case Western Reserve University, U.S.*
Yifei Lou, *University of North Carolina at Chapel Hill, U.S.*

SIAM Symposium on Data Mining (SDM26)

November 19–20, 2026 | Salt Lake City, Utah, U.S.
siam.org/sdm26 | #SIAMSDM26

GENERAL CO-CHAIRS

Arindam Banerjee, *University of Illinois Urbana-Champaign, U.S.*
Michał Pilipczuk, *University of Warsaw, Poland*

TRAVEL SUPPORT DEADLINE FOR MDS26, IS26 and SDM26: August 17, 2026

EARLY REGISTRATION RATE and HOTEL RESERVATION DEADLINE FOR MDS26, IS26 and SDM26: October 19, 2026

The following conferences will be held jointly:



SIAM Conference on Computational Science and Engineering (CSE27)

February 22–26, 2027 | Pittsburgh, Pennsylvania
siam.org/cse27 | #SIAMCSE27

ORGANIZING COMMITTEE CO-CHAIRS

Edmond Chow, *Georgia Institute of Technology, U.S.*
Andy Nonaka, *Lawrence Berkeley National Laboratory, U.S.*
Anna-Karin Tornberg, *KTH Royal Institute of Technology, Sweden*

SUBMISSION AND TRAVEL SUPPORT DEADLINES

July 27, 2026: Minisymposium Proposal Submission Deadline
August 25, 2026: Contributed Lecture, Poster, Minisymposium, and Minisymposium Presentation Abstract Submission Deadline
November 30, 2026: Travel Support Application Deadline

SIAM Conference on Applied and Computational Discrete Algorithms (ACDA27)

February 22–24, 2027 | Pittsburgh, Pennsylvania
siam.org/acda27 | #SIAMACDA27

SUBMISSION AND TRAVEL SUPPORT DEADLINES

September 8, 2026: Short Abstract and Submission Registration Deadline
September 15, 2026: Archival Proceedings Papers and Non-Archival Extended Abstracts Submission Deadline
November 9, 2026: Posters Submission Deadline

Information is current as of July 16, 2026.

Visit www.siam.org/conferences for the most up-to-date information.

Upcoming SIAM Events

SIAM Conference on Mathematics of Planet Earth

July 6–8, 2026
Cleveland, Ohio, U.S.
Sponsored by the SIAM Activity Group on Mathematics of Planet Earth

SIAM Conference on the Life Sciences

July 6–9, 2026
Cleveland, Ohio, U.S.
Sponsored by the SIAM Activity Group on Life Sciences

2026 SIAM Annual Meeting

July 6–10, 2026
Cleveland, Ohio, U.S.

SIAM Conference on Applied Mathematics Education

July 9–10, 2026
Cleveland, Ohio, U.S.
Sponsored by the SIAM Activity Group on Applied Mathematics Education

SIAM Conference on Mathematics of Data Science

November 16–20, 2026
Salt Lake City, Utah, U.S.
Sponsored by the SIAM Activity Group on Data Science

SIAM Conference on Imaging Science

November 16–19, 2026
Salt Lake City, Utah, U.S.
Sponsored by the SIAM Activity Group on Imaging Science

SIAM International Conference on Data Mining

November 19–20, 2026
Salt Lake City, Utah, U.S.
Sponsored by the SIAM Activity Group on Data Science

ACM-SIAM Symposium on Discrete Algorithms

January 24–27, 2027
Philadelphia, Pennsylvania, U.S.
Sponsored by the SIAM Activity Group on Discrete Mathematics

SIAM Symposium on Algorithm Engineering and Experiments

January 24–25, 2027
Philadelphia, Pennsylvania, U.S.

SIAM Symposium on Simplicity in Algorithms

January 25–26, 2027
Philadelphia, Pennsylvania, U.S.

SIAM Conference on Computational Science and Engineering

February 22–26, 2027
Pittsburgh, Pennsylvania, U.S.
Sponsored by the SIAM Activity Group on Computational Science and Engineering

SIAM Conference on Applied and Computational Discrete Algorithms

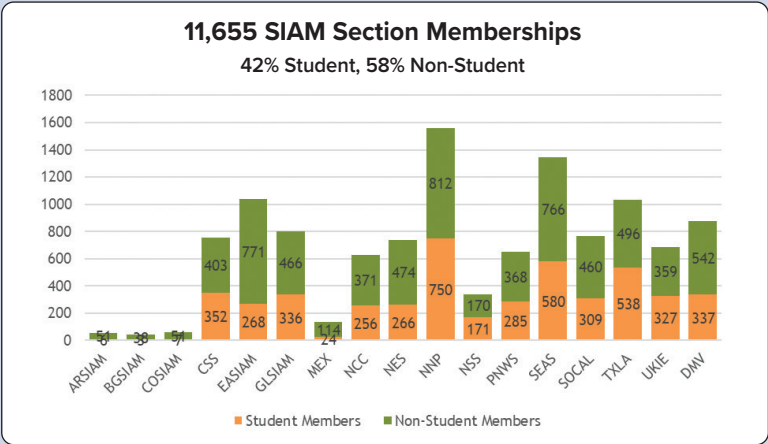
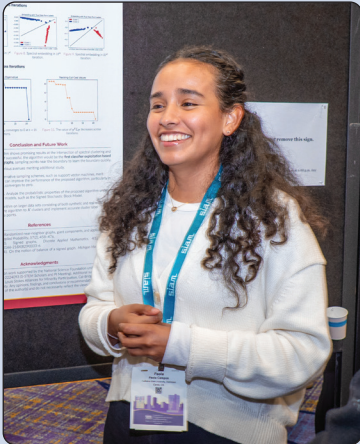
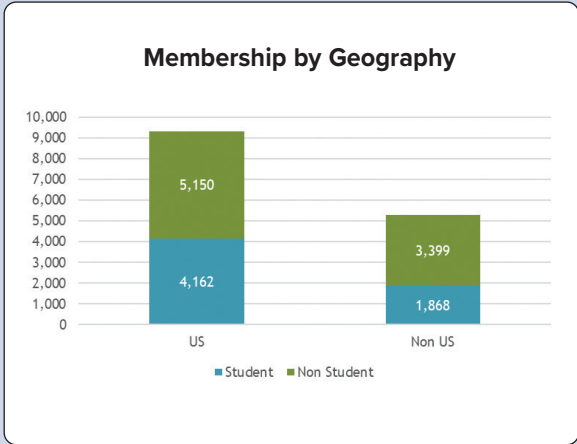
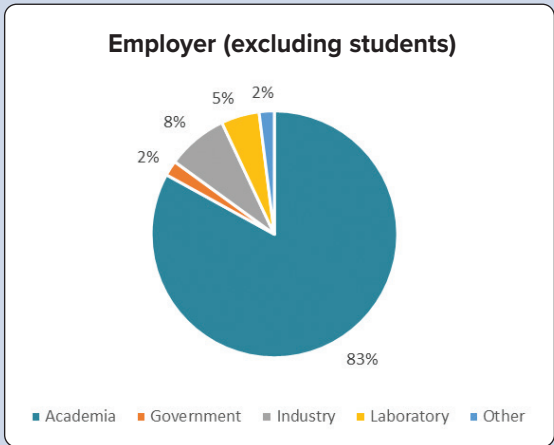
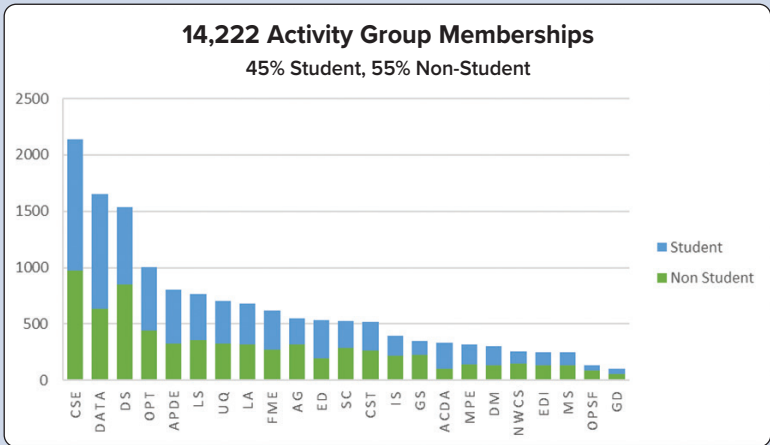
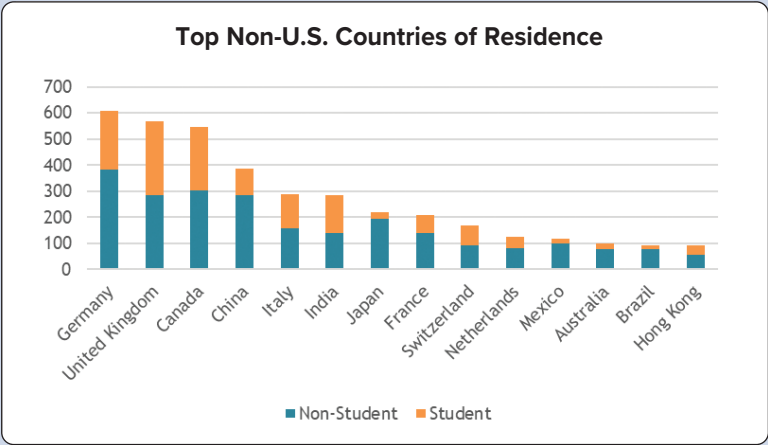
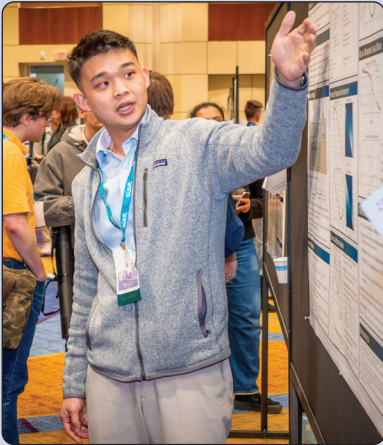
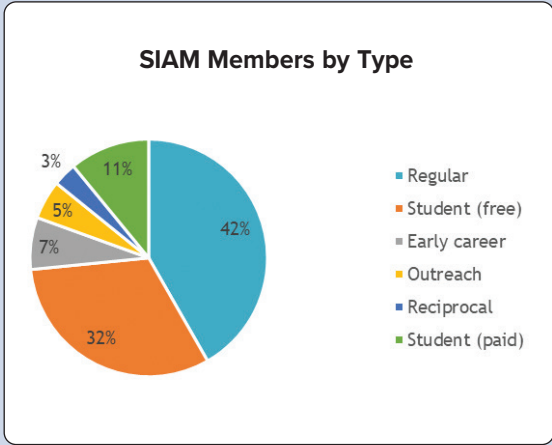
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Pittsburgh, Pennsylvania, U.S.
Sponsored by the SIAM Activity Group on Applied and Computational Discrete Algorithms

SIAM International Meshing Roundtable Workshop

February 22–26, 2027
Pittsburgh, Pennsylvania, U.S.
In partnership with Sandia National Laboratories

Annual SIAM Membership Report

As of December 31, 2025, SIAM had 14,577 members from 102 countries and 238 active student chapters representing 30 countries. Thirty-six percent (36%) of SIAM members reside outside the United States. The charts on this page show information about SIAM members.



When You Renew Your SIAM Membership This Fall...

Please consider checking the “auto-renew” box to have your membership automatically renew at the end of 2026. You can update or cancel your auto-renewal information and preferences any time at my.siam.org.

SIAM-Simons Undergraduate Summer Research Program Seeks Mentors

The program seeks mentors for the summer 2027 cycle. Funding covers food, lodging, travel, and stipend for participants and stipend for mentors. Applications to be a mentor for two students for eight weeks open in July and close at the end of August. Visit siam.org/simons for more information.

Nominate two of your students for free membership!

SIAM members (excluding student members) can nominate up to two students per year for free membership. *student* to make your nominations.

Nominations are open for the SIAM Fellows Class of 2027!

Support your profession by nominating up to two colleagues for excellence in research, industrial work, educational activities, or other activities related to the goals of SIAM. Class of 2027 Fellows nominations are being accepted at nominatefellows.siam.org until October 15, 2026.



Quality and Value in Mathematical Science Literature

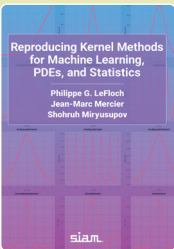
New from SIAM

Reproducing Kernel Methods for Machine Learning, PDEs, and Statistics

Philippe G. LeFloch, Jean-Marc Mercier, and Shohruh Miryusupov

This monograph develops a unified, application-driven framework for kernel methods grounded in reproducing kernel Hilbert spaces and optimal transport. The primary goal is to tackle industrial cases from computational physics and mathematical finance and discuss applications across various areas, such as statistics, or artificial intelligence (physics-informed systems, reinforcement learning, machine learning, generative methods, etc.). This book is divided into two parts, theoretical principles and the techniques employed in their applications; contains numerous applications in engineering, finance, and machine learning; and provides a framework for designing numerically efficient, large-scale dataset strategies.

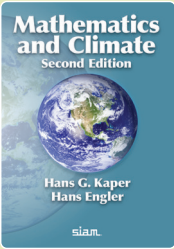
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List \$69.00 / SIAM Member \$48.3 / OT212



Mathematics and Climate, Second Edition
Hans G. Kaper and Hans Engler

Mathematics and Climate, Second Edition reflects advances since the award-winning 2013 first edition. The book emphasizes conceptual models capturing key components of Earth's climate system, explaining how and why phenomena occur. These models introduce methods from applied mathematics, focusing on dynamical systems, bifurcation and stability theory, Fourier analysis, and statistics, including regression and extreme value theory. It also includes a layered atmosphere greenhouse model, a new chapter on glacial cycles, updated paleoclimate reconstruction sections, and end-of-chapter exercises to stimulate critical thinking and communication.

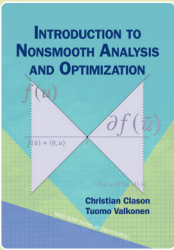
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Introduction to Nonsmooth Analysis and Optimization
Christian Clason and Tuomo Valkonen

This book provides a unified, rigorous introduction to infinite-dimensional analysis and the algorithmic solution of nonsmooth optimization problems arising in imaging, inverse problems, machine learning, and optimal control of differential equations. It covers essential tools of nonsmooth analysis alongside modern algorithms, including their convergence and stability. The text examines first- and second-order methods in infinite dimensions and offers a self-contained treatment of set-valued and variational analysis, including new calculus results. Julia code is included to reproduce numerical results.

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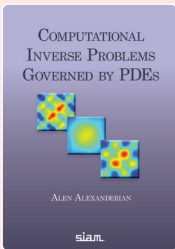


Computational Inverse Problems Governed by PDEs

Alen Alexanderian

This textbook focuses on computational methods for inverse problems that are governed by partial differential equations (PDEs). The author considers deterministic and Bayesian formulations and highlights how traditional tools from deterministic inversion can be integrated into solution methods for Bayesian inverse problems. Advanced topics such as post-optimality sensitivity analysis, optimal design of experiments, and Bayesian inversion under model uncertainty are also included. The book offers readers a balance of theoretical and computational insight, an example-driven approach that provides an accessible presentation, and over 150 theoretical and computational exercises

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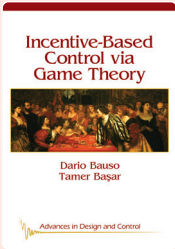


Incentive-Based Control via Game Theory

Dario Bauso and Tamer Başar

Engineering systems in the new economy are increasingly shaped by sharing—shared logistics, joint investments, virtual power plants, and more—a shift that calls for models that capture decentralized decision making, resource pooling, and cooperative infrastructure. Across production, transportation, logistics, and energy, a central challenge is modeling and optimizing collective actions. This book aims to bridge game theory and systems and control by developing tools for incentive-based control in dynamic, uncertain, and networked multiagent systems; it extends both cooperative and noncooperative game models into a control-oriented framework, emphasizing feedback, learning, and decision making under uncertainty.

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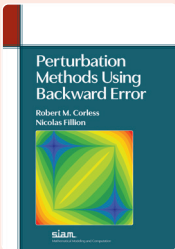


Perturbation Methods Using Backward Error

Robert M. Corless and Nicolas Fillion

Engineering systems in the new economy are increasingly shaped by sharing—shared logistics, joint investments, virtual power plants, and more—a shift that calls for models that capture decentralized decision making, resource pooling, and cooperative infrastructure. Across production, transportation, logistics, and energy, a central challenge is modeling and optimizing collective actions. This book aims to bridge game theory and systems and control by developing tools for incentive-based control in dynamic, uncertain, and networked multiagent systems; it extends both cooperative and noncooperative game models into a control-oriented framework, emphasizing feedback, learning, and decision making under uncertainty.

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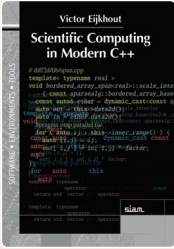


Scientific Computing in Modern C++

Victor Eijkhout

This book targets aspiring computer programmers with a basic knowledge of C++ who want to deepen their understanding for scientific computing. It approaches the subject from a software perspective, covering key topics such as advanced C++ features, domain-specific idioms, parallelism, and hardware and performance considerations. It also introduces practical “carpentry” tools like CMake to improve productivity. Rather than providing a complete treatment of C++, it emphasizes computing-focused concepts and applications, deliberately selecting relevant topics while excluding others. The book also explores broader scientific computing topics that extend beyond the language itself.

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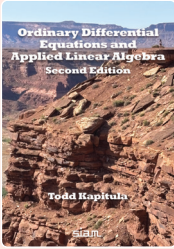


Ordinary Differential Equations and Applied Linear Algebra, Second Edition

Todd Kapitula

This text helps students master linear algebra and ordinary differential equations (ODEs) in a one-semester course. It develops the linear algebra needed for applications first and continually references it when solving ODEs. The ODE discussion emphasizes linear problems, most relevant to undergraduate science courses, while also covering separable equations and nonlinear systems. The second edition expands learning with end-of-chapter case studies on SIR models, lead poisoning, and strongly damped forced oscillators, plus projects exploring modeling, solutions, and physical implications. It also includes access to 300+ online homework problems in myOpenMath.

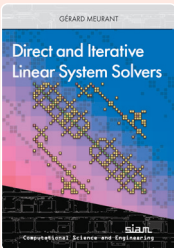
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Direct and Iterative Linear System Solvers
G rard Meurant

Solving linear systems of equations is ubiquitous in scientific computing; therefore, numerical algorithms for solving them are paramount. This book describes the state of the art in direct and iterative methods for solving nonsingular linear systems of equations. The author considers several variants of elimination methods, classical iterative methods, variants of the conjugate gradient method, and Krylov methods for nonsymmetric systems and describes many preconditioners. Finite precision arithmetic and numerical experiments are emphasized.

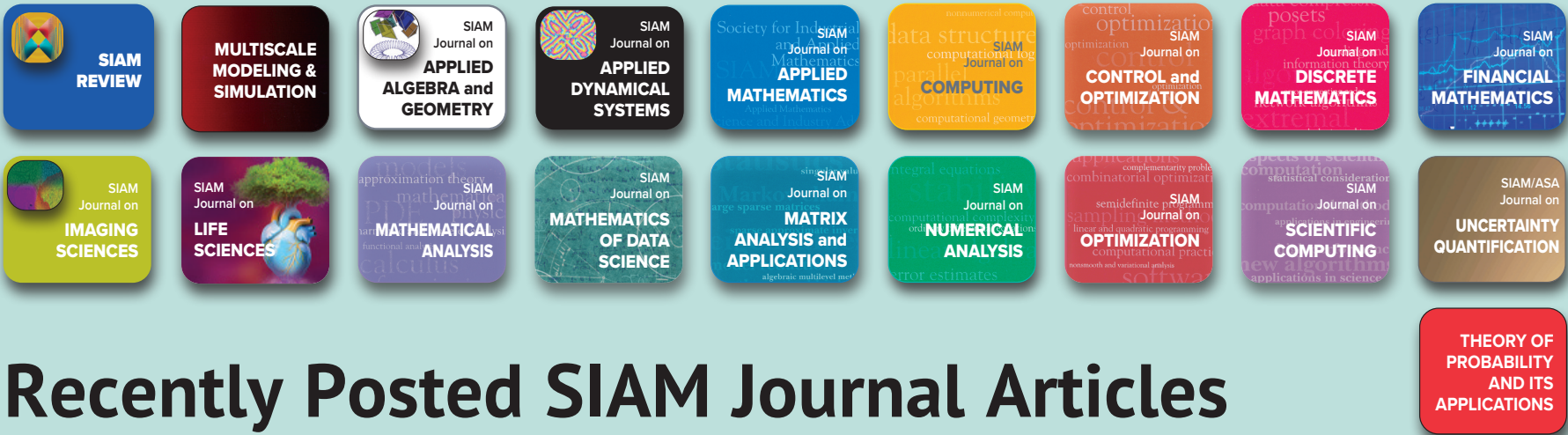
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