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# Standing Up for Science

On Earth Day, April 22, 150,000 supporters attended the March for Science on the National Mall in Washington, D.C. Attendees braved the rain to advocate for climate science and evidence-based facts that uphold the common good. Satellite marches, rallies, and events took place in 600 cities around the world. Read a report of the march by Hans Kaper and Hans Engler on page 8, and view additional photos on page 9.


*After a morning of speeches, teach-ins, and rallies, March for Science participants marched from the Washington Monument to Capitol Hill. Photo credit: Nicholas Higham.*

# Deep Learning Models in Finance

*By Justin Sirignano*

The finance literature has historically focused on stochastic models and their mathematical analysis. However, unlike in physics or other sciences, there are no fundamental laws in finance (such as Newton's laws) from which to derive models; therefore, some assumptions must be made. A purely data-driven approach, such as machine learning, is potentially superior for some applications. Opportunities exist for both the development of new machine learning models for financial applications and the mathematical analysis of these statistical learning algorithms.

Deep learning, a subfield of machine learning that uses "deep neural networks," has achieved state-of-the-art results in fields such as image and text recognition. A deep neural network is a neural network with many hidden layers, which allow it to model complex nonlinear functions more effectively than single-layer neural networks. Deep learning focuses on the development of specific model architectures and training methods to enhance the performance of multilayer neural networks. Deep neural networks, which have a large number of parameters, are typically trained on large amounts of data to avoid overfitting. Training is very computationally expensive due to the complexity of the deep neural network model and the large amount of data. Models are often trained for multiple days on clusters of graphics processing units.

Deep learning research has made continual advances over the last decade. Researchers have designed new optimization methods (Adam, RMSprop, and others) to better train the highly nonconvex neural networks. Regularization methods such as dropout help reduce overfitting [14], and ever deeper neural networks are trained. For example, [7] trains a neural network with 1,000 layers. Deep reinforcement learning has successfully combined deep neural networks with reinforcement learning algorithms to learn complex tasks. For example, researchers have trained deep neural networks to play a range of Atari video games using only the raw pixels from the screen (similar to how a human watches the game) [10]. An overview of deep learning models and methods can be found in [3].

*See* **Deep Learning** *on page 3*

# Electricity Demand Response and Optimal Contract Theory

*By René Aïd, Dylan Possamaï, and Nizar Touzi*

Part of the equation to achieve the 2015 United Nations Climate Change Conference (COP21) objective of limiting climate change effects to a 2-degree Celsius increase relies on the design of carbon-free electric systems. According to the International Energy Agency's 2015 report on carbon emission from fuel combustion, more than a third of the world's carbon emission for energy systems comes from power generation. The massive development of renewable energy sources worldwide, particularly solar and wind power, is helping us reach the COP21 objective. However, these sources are simultaneously reshaping the management of power systems.

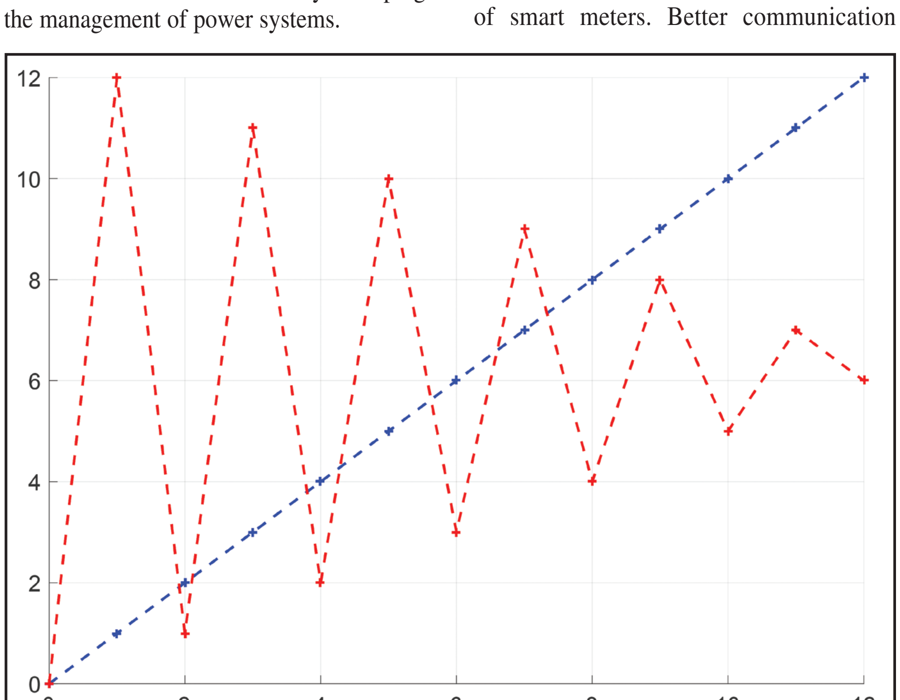


**Figure 1.** *Consumption in red exhibits a quadratic variation* $\langle X\rangle := \sum_t \left(X_{t+1}-X_t\right)^2 = 650$. *Consumption in blue has a quadratic variation* $\langle X\rangle := \sum_t \left(X_{t+1}-X_t\right)^2 = 12$. *Image credit: René Aïd, Dylan Possamaï, and Nizar Touzi.*

## Decarbonation of Power Systems

Renewable energy sources are non-dispatchable and highly intermittent. The root mean square of the error forecast for the production of a wind farm in six hours can reach 20% of its installed capacity. These increases in uncertainty of power generation have put *flexibility* at the heart of system design for large-scale renewable energy sources.

One can increase the flexibility of power systems in two possible ways: acting on the generation side by adding batteries, or acting on the demand side by developing new demand response programs. We are interested here in the second tool. Many Organisation for Economic Co-operation and Development countries have made significant investments in the development of smart meters. Better communication with consumers is necessary to implement efficient demand-response programs. 45 million smart meters have already been deployed in Italy, Sweden, and Finland; there is an ongoing investment of 45 billion euros to reach the level of 200 million appliances in the EU-27, based on the European Commission's Energy Efficiency Directive.[1] Nevertheless, proposed demand-response schemes are generally used to shave peak-load demands. The need for flexibility in new power systems calls for a continuous assessment of large variations of net consumption over time addressed to the grid.

Recent progress in the theory of incentives and optimal contract allows researchers to design mechanisms that adapt demand to the flexibility capacity of power systems by incentivizing consumers to reduce the volatility of their consumption.

## Contract Theory and Electricity Demand Response

Contract theory is a field of microeconomics that analyses the interaction of economic agents linked by contract. This framework covers situations as different as relationships between stockholders and managers, managers and employees, and land owners and farmers. In each case, one side of the contract relation—the principal—is looking for an incentive mechanism that will lead the other side—the agent—to act within the principal's best interests. The problem is complicated by the fact that the principal can only observe and contract on the results, and not on the agent's efforts. Contract theory is thus about finding the optimal incentive mechanism to maximize the principal's utility, while knowing that the agent will take advantage of the contract design only in her

*See* **Electricity Demand** *on page 4*

[1] https://ec.europa.eu/energy/en/topics/energy-efficiency/energy-efficiency-directive

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# Behind the Scenes of SIAM's Prize Program

## *And Why You Should Nominate for Prizes*

FROM THE SIAM PRESIDENT
*By Nicholas Higham*

The call for nominations for SIAM prizes opened on May 1, as announced by the A3 poster included with last month's *SIAM News*. The open prizes range from student and early-career prizes to awards recognizing lifetime achievements. Why are prizes important, and why should you make a nomination?

I see three main reasons. First, prizes recognize and reward outstanding accomplishments, bringing honor to the recipients and their departments and institutions. Second, they demonstrate the importance of their field (and of applied mathematics in general) to everyone from administrators to funders to potential graduate students. Third, they help advance careers, especially for early career researchers, and can be stepping stones to further success.

In order to function successfully, prizes need a reasonable number of nominations. SIAM's prize policy[1] requires a prize to receive a minimum of three new nominations in response to an open call in order for the prize to be awarded, and it directs that a prize be terminated if it fails to achieve this minimum for two consecutive cycles. The message it clear: if a prize is valued, the relevant community should ensure that nominations are made, or the prize could be lost. Ideally, one would hope for two to three times the minimum number of nominations. Additionally, the nominations for a healthy prize should reflect the pool of eligible candidates, in terms of subject area, gender, affiliation (university, lab, or industry), geography, and underrepresented groups. Ensuring diversity of the nominations helps to produce diversity in prize winners.

Prize committees, which select winners, are appointed by the president, or for SIAM Activity Group (SIAG) prizes, by the SIAG with the approval of the vice president at large. In my experience, colleagues are generous in volunteering their time to serve on prize committees and regard it as an honor to be asked.

Administering SIAM's 14 major prizes and 25 SIAG prizes is a significant task. It involves issuing prize calls; forming selection committees; collecting nominations (including rollovers from previous cycles) and checking eligibilities; communicating this information to the committees (hundreds of files are in play in any given year); getting recommendations approved; and notifying winners. SIAM recently invested in prize management software to streamline these processes. The software provides a better way for selection committees to access nominations, offers easy reporting of relevant statistics, and will ultimately save staff time.

The vice president at large—currently Ilse Ipsen—and the Major Awards Committee oversee the SIAM Prize Program. One thing I learned during my time as vice president at large (2010-2013) is that there is always work to do on prizes: tweaking specifications, considering proposals for new prizes, and so on. This year, we have been revising conflict of interest guidelines for prize selection committees in order to clarify what constitutes a conflict and how to handle one. The new guidelines will go to the Council for approval at the 2017 Annual Meeting in Pittsburgh, Pa., this July.

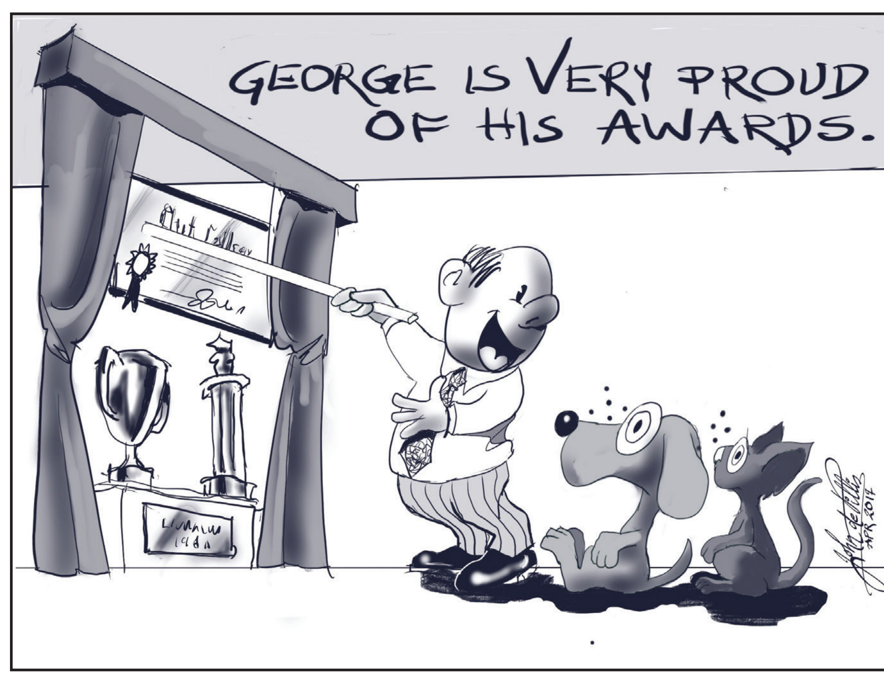
Some prizes have an associated lecture, which provides an opportunity for a conference audience to hear the winner speak about his or her work. A prime example is the John von Neumann Lecture, SIAM's premier award. At the 2016 Annual Meeting in Boston, Mass., last year's winner, Donald Knuth, gave a spellbinding lecture[2] on the satisfiability problem. Recordings of many of these prize lectures are available on SIAM Presents,[3] which also contains recorded plenary invited talks and minisymposia from select SIAM conferences — most recently from the 2017 SIAM Conference on Computational Science and Engineering. I encourage readers to make use of this excellent resource.

*Nicholas Higham is the Richardson Professor of Applied Mathematics at the University of Manchester. He is the current president of SIAM.*

[1] https://www.siam.org/prizes/policy.php

[2] https://www.pathlms.com/siam/courses/3028/sections/4140

[3] https://www.siam.org/meetings/presents.php




*Cartoon created by mathematician John de Pillis.*

# Deep Learning Effects in Detroit Auto and Beyond

LETTER TO THE EDITOR

I'm a SIAM member and a Ph.D. computational multibody dynamicist working at Detroit Auto and truck transportation. I also worked for a company that made a modeling and simulation software called Adams, fueled and solved by former SIAM President C. William Gear's stiff differential equation integration algorithm.

I was happy to read Michael Elad's review of deep learning and the growing application of success versus governing equation-controlled results. In the auto world, I call this "business" engineering versus "science" engineering. Harvard MBA CEOs tend to support the former over the latter, and costs are the controlling direction. Will this driven intention of making huge wealth for a small amount of people result in overturning scientific rigor and precise process? And do humans learn anything here, or does that matter anymore, given the advent of robots and autonomous "vehicles of knowledge"? I know a lot of low academic types who favor the business engineering approach and would gladly accept the answer from a neural network algorithm. I remember this methodology using feedback control devices, and I'm sure the concepts are being arranged and addressed for future autonomous vehicle developments.

Yet, I'm sad to see how science is being battered, if not ignored, in the matter. But it's also like the mathematics of finite element analysis, where too many degrees of freedom are being reduced using a variety of projection methods to remove all the computational noise, getting to the essential mechanics equations that yield the critical solutions with great accuracy. In this case, the large computational problem is being slimmed down. Will this happen in deep learning, with its excessive data collection and analysis approach?

Very good article provided here by asking more about the philosophy of human value in what transpires on the planet. I mean at what point do humans, even the very rich ones, become inconsequential debris in the way of much deeper thinkers that are not human? Deep learning could be writing a new science fiction to fact-scripted plot.

— Al Kovacs, *South Lyon, MI*

*Read Michael Elad's article, "Deep, Deep Trouble,"[1] in the May issue of SIAM News.*

[1] https://sinews.siam.org/Details-Page/deep-deep-trouble

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## Deep Learning

*Continued from page 1*

Despite the immense success of machine learning in other fields, there is very little published research on its application to finance, and almost none on deep learning.

### Applications of Deep Learning in Finance

Today, stocks are frequently traded via electronic exchanges (e.g., NASDAQ and NYSE Arca). Traders continuously submit, cancel, and execute buy and sell orders in the exchange's *limit order book*. Market events are often reported at the nanosecond granularity, and therefore the limit order book data generated over time is very large (terabytes to petabytes). Deep learning can model key quantities, such as the probability distribution of future price movements given the current state of supply and demand in the market. An example is presented in [11].

The limit order book represents the known supply and demand for a stock at different price levels at any particular point in time. It consists of all existing orders at all prices. The "bids" are the buy orders and the "asks" are the sell orders. The best ask price is the lowest sell order, while the best bid price is the highest buy order. The mid-price (the average of the best bid and best ask prices) is often called the "price" of the stock. However, it is an artificial quantity, since one cannot buy or sell a stock at the mid-price. The best ask price and best bid price are the actual prices at which one can respectively buy or sell the stock.

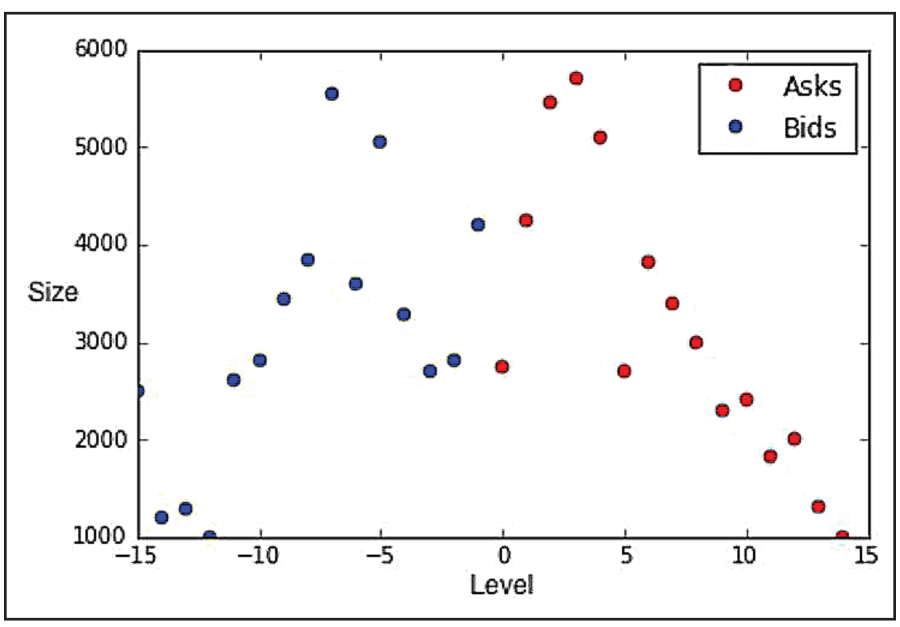


**Figure 1.** *Bid and ask sizes for the first 15 bid and ask prices for Microsoft. Level 0 is the best ask price. Image courtesy of [11].*

Figure 1 shows an example of the limit order book for Microsoft. The *size at level k* is the number of shares available in the limit order book to be bought/sold at *k* discrete price levels from the best ask price. Over time, the limit order book (and the best ask and best bid prices) will evolve due to new limit orders, cancellations, and market orders.

A recent paper [11] trains deep learning models to predict the movement of the best ask and best bid prices. It uses a dataset of 489 stocks and tests on a three month out-of-sample period. The histogram in Figure 2 shows an example of a neural network's performance gain compared to a logistic regression model across the 489 stocks. Figure 2 presents the results for predicting the next price move of the best ask price. See [11] for details.

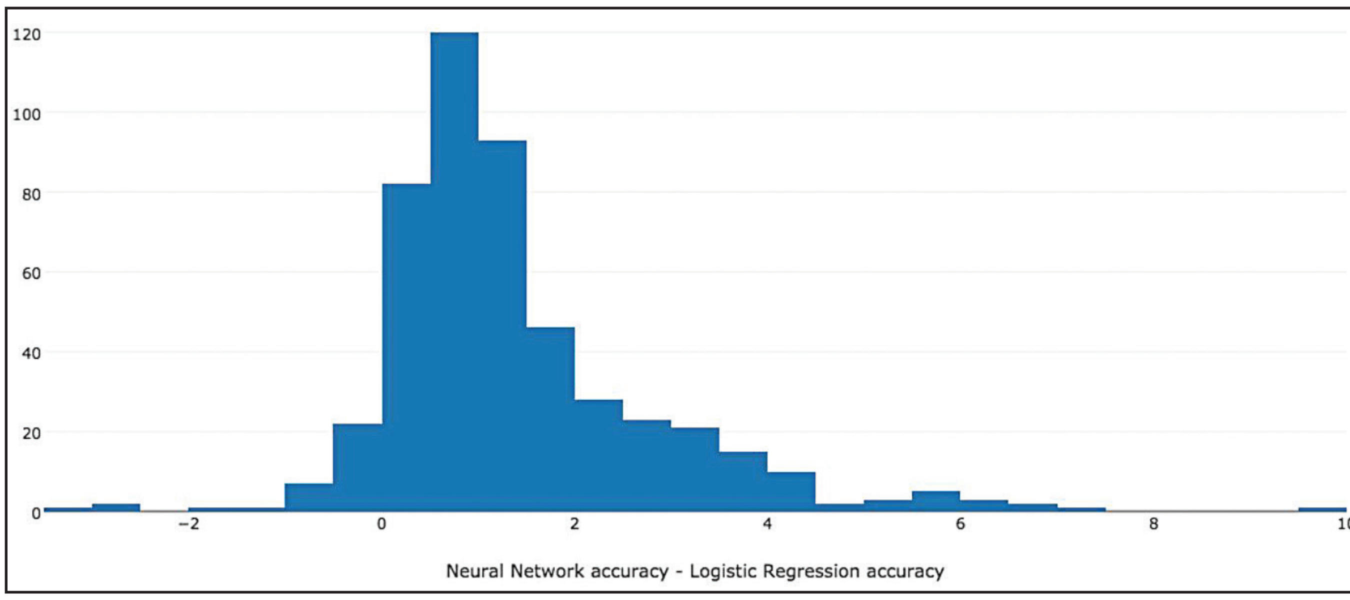


**Figure 2.** *Increase in out-of-sample accuracy of neural network over logistic regression. Accuracies are measured in percentages. Results are depicted for the marginal distribution of the best ask price at the time of the next price move. Image courtesy of [11].*

Another promising application of deep learning is for modeling loan risk. [6] develops and tests a deep learning model for mortgage risk on data from tens of millions of loans. Besides loan-level predictions, the model can be used to select mortgage investment portfolios. Other examples of deep learning in finance include [2] and [8].

### Mathematical Analysis in Machine Learning and Deep Learning

There are also opportunities for the mathematical analysis of machine learning and deep learning algorithms (not necessarily specific to finance). Although successful in practice, deep learning methods are often ad hoc and therefore present an exciting opportunity for more rigorous analysis. A recent example is [9], where Stéphane Mallat develops a mathematical approach to understand the success of deep convolution neural networks. A convolution neural network is a type of neural network architecture that has proven incredibly successful at image classification. Examples closer to the field of mathematical finance include [1], [4-6], and [13].

[13] studies stochastic gradient descent in continuous time (SGDCT). SGDCT provides a computationally efficient method for statistical learning of continuous-time models, which are widely used in finance, engineering, and science. The algorithm follows a (noisy) descent direction along a continuous stream of data. [13] proves convergence of the continuous-time stochastic gradient descent algorithm. The analysis relies upon describing the parameter behavior for large times using a type of Poisson partial differential equation. Besides model estimation, SGDCT can also be used for the optimization of high-dimensional continuous-time models, such as American options. High-dimensional American options have been a longstanding computational challenge in finance. [13] successfully combines SGDCT with a deep neural network to solve American options in up to 100 dimensions.

### Future Opportunities

There are a broad range of opportunities for (1) the development of new deep learning models and methods for financial applications and (2) mathematical analysis of these machine learning approaches. It is the hope of this article to highlight some of these opportunities and encourage future research in these areas.

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# SIAM Committee on Science Policy Discusses Priorities in Current Political Climate

*By Karthika Swamy Cohen, Miriam Quintal, and Eliana Perlmutter*

A key objective of SIAM is to promote the value of applied mathematics research and ensure its application in solving real-world problems. To this end, the SIAM Committee on Science Policy monitors developments in the federal and state governments that are of interest to SIAM and its members.

The committee meets biannually in Washington, D.C., and held its spring meeting in April to discuss SIAM policy goals and advocate for these priorities in Congress. Representatives from relevant offices at the National Science Foundation (NSF), Department of Energy (DOE), and Department of Defense (DoD) offered updates on their respective agencies. Additionally, the committee met with staff in the offices of key members of Congress and congressional committees to highlight the value of federal investment in applied mathematics and scientific computing at the NSF, DOE, DoD, and the National Institutes of Health (NIH).

Lewis-Burke Associates, the organization that supports SIAM's government affairs in Washington, hosted the meeting. They kicked off the session with an overview of the latest updates from Washington. Representatives from the firm discussed the likelihood of an omnibus bill to fund government agencies and programs for the last five months of fiscal year (FY) 2017, which has since been passed by Congress and signed into law (Public Law 115-31).

Despite the cuts proposed by the Trump administration, the omnibus increases federal investments in many research areas. Congress appropriated \$7.472 billion—essentially flat funding—for the NSF and \$5.392 billion for the DOE Office of Science, an increase of \$42 million, or 0.8 percent, over the FY 2016 enacted level. Advanced scientific computing research at the DOE will receive an increase of \$26 million over FY 2016 levels, for a total of \$647 million for FY 2017. Basic research at the DoD was allocated \$2.3 billion, a 1.4 percent decrease from the FY 2016 level. The NIH was appropriated \$34 billion, an increase of \$2 billion, or 6.2 percent, above the FY 2016 enacted level. In the end, the administration relented on its top priorities, such as funds for a border wall, and increased defense spending to avoid a government shutdown.

The president's full budget proposal for FY 2018, anticipated the week of May 22nd (after this issue went to press), is expected to mirror the skinny budget released by the administration in March. For projected increases in national security, the budget would make steep cuts to federal research agencies, targeting programs focused primarily on research, the environment, international assistance, housing, and loans and grants. The proposal is expected to diminish DOE research programs by \$3.1 billion and NIH funding by \$5.8 billion. NSF and DoD research programs have not been discussed publicly by the administration, and the budget

*See* **Science Policy** *on page 6*

## Electricity Demand

*Continued from page 1*

own interest. Of note is the 2016 Nobel Prize in economic sciences, which was awarded to Bengt Holmström and Oliver Hart for their crucial contribution to the theory of contract in continuous time.

We show here how contract theory concepts and methods can help successfully design efficient demand-response programs. We concentrate on a situation in which the principal is a power producer and the agent is a consumer. The power producer has to satisfy the consumer's electricity demand for the following day. The key variable is the deviation from the predicted or baseline electricity demand; we denote it with $X_t$, where $t$ is an instant of the next day. This deviation procures a utility to the consumer $f(X_t)$. Moreover, the consumer can act on this deviation by reducing her mean consumption and volatility. In mathematical terms, this means that the process $X_t$ satisfies the following stochastic differential equation:

$$dX_t^{a,b} = -\sum_i a_{i,s}ds + \sigma^i \sqrt{b_{i,s}}\, dW_s^i, \tag{1}$$

where $a_i$ represents the agent's effort on the mean consumption of electricity usage $i$, and $b_i$ denotes the effort on the volatility of usage $i$. A positive $a_i$ represents a reduction of demand on usage $i$, while $b_i < 1$ indicates a reduction of the volatility of usage $i$. These efforts induce a cost represented by the function $c(a,b)$.

For the producer, the deviation $X_t$ generates an extra cost $(X>0)$ or an economy $(X<0)$, denoted $g(X)$. Moreover, variation of the deviation $X_t$ over time also induces costs on the producer, whose flexibility capacity is limited. Figure 1 (on page 1) illustrates this point. Both consumptions are equal in energy, but the blue consumer presents a quadratic variation $\langle X \rangle := \sum_t \left(X_{t+1} - X_t\right)^2$ of 650 while the red has a quadratic variation of only 12. These variations incur a cost on the producer, whose generation plants are not flexible enough to follow such erratic behavior. We suppose that this cost is proportional to the quadratic variation with constant $h$.

The producer needs to find an incentive scheme, denoted $\xi$, that will prompt the consumer to reduce the average consumption and its variation. But the producer has no knowledge of what is happening in the house, being able to merely observe the total consumption. The contract $\xi$ can only depend on the observed values of $X$ and not on the efforts $a, b$. When exposed to $\xi$, the consumer aims to maximise the expected utility $f(X_t)$ minus the cost of effort $c(a,b)$ plus the payment $\xi$. Namely, its purpose is to solve

$$\sup_{a,b} \mathbb{E}\Big[\xi + \int_0^T \Big(f(X_s^{a,b}) - c(a,b)\Big)ds\Big]. \tag{2}$$

The optimal controls of consumer $a^\star(\xi)$ and $b^\star(\xi)$ are functions of the contract scheme $\xi$.

On the other hand, the objective of the producer is to maximise his own utility. In mathematical terms, the producer is the following objective function:

$$\sup_\xi \mathbb{E}\Big[U\Big(-\xi - \int_0^T g(X_t^\star)ds - h\langle X_t^\star \rangle_T\Big)\Big], \tag{3}$$

where $X_t^\star := X_s^{a^\star(\xi), b^\star(\xi)}$ represents the consumer optimal electricity deviation induced by contract $\xi$. The producer first has to determine the optimal responses $a^\star(\xi)$, $b^\star(\xi)$ of the consumer for any given $\xi$, and then generate his own optimization knowing these responses. Contract $\xi$'s dependence on the observation of the whole trajectory $X$ complicates the problem. The producer will use all the available information contained in the variations of $X$ to determine whether—and to what extent—the agent is making an effort, or if the observations are just subject to random outcomes.

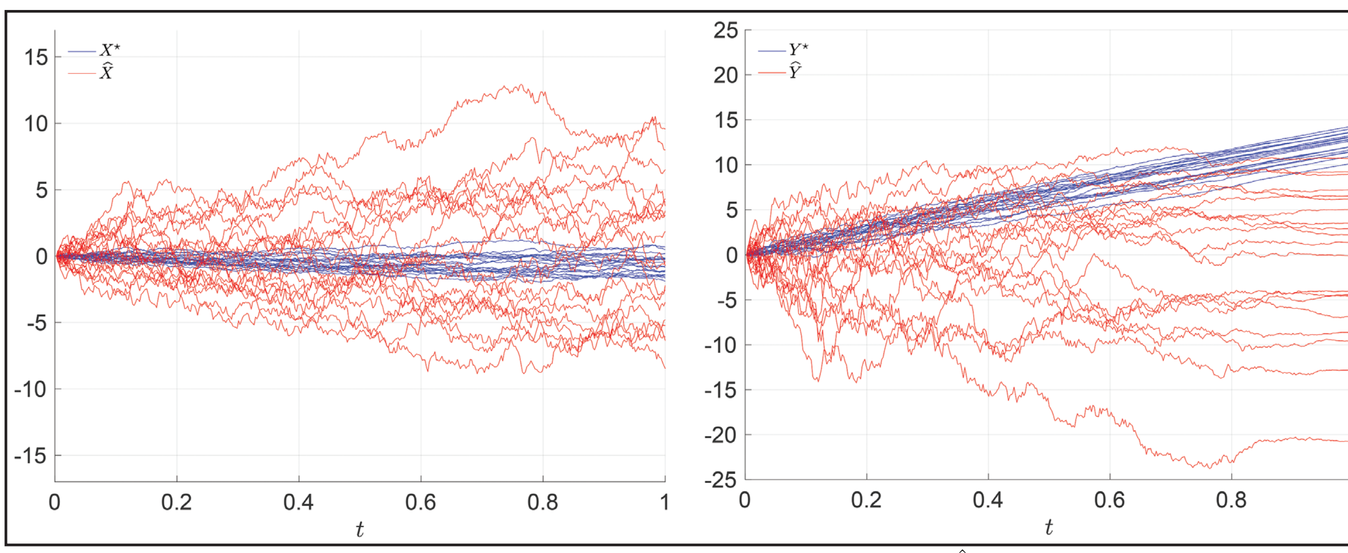


**Figure 2. Left.** *Electricity deviation $X^\star$ of a rational consumer compared to a passive consumer $\hat{X}$.* **Right.** *Associated payment $Y^\star$ for the rational consumer and $\hat{Y}$ for the passive consumer. Image credit: René Aïd, Dylan Possamaï, and Nizar Touzi.*

### Optimal Contract and Numerical Simulation

Possamaï and Touzi, along with Jakša Cvitanić, provide a general methodology to solve (3). They show that the optimal contract can be written as

$$\xi = Y_t^{Z,\Gamma} := Y_0 + \int_0^t Z_s dX_s + \frac{1}{2}\Gamma_s d\langle X \rangle_s - \Big(H(Z_s, \Gamma_s) + f(X_s)\Big)ds, \tag{4}$$

where $Z$ and $\Gamma$ are two stochastic processes to be chosen by the principal, and $H(z,\gamma)$ is the Hamiltonian of the agent:

$$H(z,\gamma) := \sup_{a,b}\Big\{-\sum_i a_i z + \frac{1}{2}\sum_i \sigma_i^2 b_i^2 \gamma - c(a,b)\Big\}.$$

This result brings the problem of the principal back to a standard stochastic control problem, where the objective function is given by (3), (4) replaces $\xi$, the controls are the two processes $Z$ and $\Gamma$, and the dynamics are given by (2) for $X$ and (4) for $Y$. The principal's problem thus admits a nonlinear partial differential equation representation whose solution is approachable by numerical techniques.

We illustrate the resulting interaction between a producer and a consumer with numerical simulations on a one-day period. Figure 2 represents the consumption (left) and the payment (right) to a rational consumer (blue) who has signed and applied the contract, and a passive consumer (red) who has signed the contract but not applied it. We notice that the rational consumer reduces both the deviation and its volatility when compared to the passive consumer, meaning that the contract produces the desired outcome. Moreover, the rational consumer receives a positive payment in all cases, while the passive consumer receives no payments and may face a penalty. As time goes by, the expected payment for the passive consumer becomes less and less volatile as the producer observes the consumption and infers that the consumer is making no effort.

### Perspectives

The model presented in this article is the first step towards a practical implementation of a demand-response contract at a large scale. Indeed, one should consider the case when a large number of agents must be controlled. Possamaï, Romuald Elie, and Thibaut Mastrolia have already investigated the instance of a single principal and a large number of agents, indicating that demand-response could possibly extend to more realistic situations. Despite its infancy, our model opens the door to the social engineering of power systems. Indeed, behavioral sciences are necessary to provide realistic yet tractable models of response function to price signals for a large population of consumers. Demand-response is on the list of required technologies to achieve an efficient zero-carbon electric system, and we hope to contribute to its development.

*This article is based on René Aïd's presentation of his joint work with Dylan Possamaï and Nizar Touzi at the 2016 SIAM Conference on Financial Mathematics and Engineering, held last fall in Austin, Texas.*

### Further Reading


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# Nonsmooth Dynamical Systems in Neuroscience

*By Wilten Nicola and Sue Ann Campbell*

Large-scale models of the human brain, which help researchers understand humans' rich plethora of potential behaviours, consist of millions of individual neurons coupled into large-scale networks. The Blue Brain Project,[1] a European initiative that utilizes state-of-the-art computing resources, models large circuits of the brain in exquisite detail [6]. The Semantic Pointer Architecture Unified Network (SPAUN),[2] a large functional network of 2.5 million model neurons, can even solve problems from the standardized IQ test [2].

[1] http://bluebrain.epfl.ch/
[2] http://nengo.ca/build-a-brain/spaunvideos/

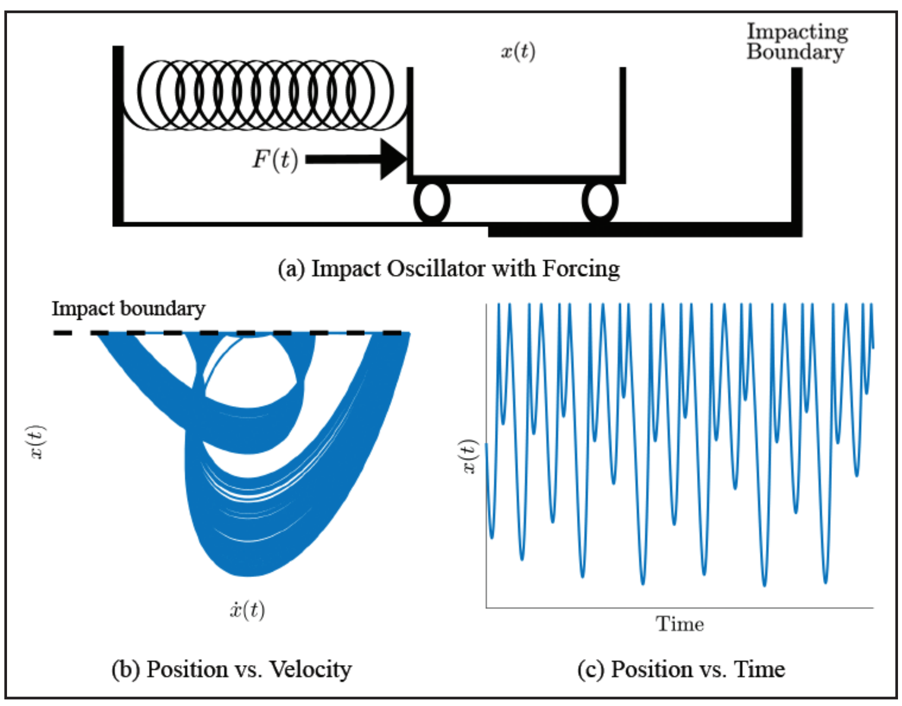


**Figure 1.** *Impact oscillator.* **1a.** *The simple impact oscillator is derived from a cart attached to a spring with an external forcing term $F(t)$. The equations of motion for the position $x(t)$ and velocity $\dot{x}(t)$ of the cart form a nonsmooth system when accounting for interactions with the boundary. The impact oscillator can display behaviours that are not present for the system without an impacting boundary, such as chaotic dynamics.* **1b.** *An example attractor.* **1c.** *Trajectory on the attractor shown in 1b. Image credit: Wilten Nicola.*

While existing simulations replicate certain behaviours or experimental observations and thus provide an important step in understanding brain function, exhaustive parameter searches, additional simulations, and costly computing resources are necessary for real insight. A functional understanding of the behaviour of these large-scale models is needed to comprehend the human brain's internal dynamics.

Fortunately, the macroscopic behaviour of large networks of identical (or even heterogeneous) subunits is often low-dimensional. We may derive a model, commonly called a mean-field or firing rate model, for macroscopic behaviour from the large-scale model. The derivation of mean-field equations frequently leads to models with discontinuities [3, 7], which arise because—at the most fundamental level—neurons have two states: quiescence (off) and firing (on). Analysis of the mean-field model can yield predictions and insights into the large-scale network that simulations alone would not provide. Prediction of the emergence of network-induced behaviour such as bursting, a special type of neuronal oscillation, is one such example. We may also derive mean-field models from first principles using knowledge of the underlying system or experimental data [10]. This approach helped to create a large-scale brain model that focusses on biologically-realistic connectivity [4]. In the mean-field approach, firing rate models represent the behaviour of a whole population of neurons, and can also be used as simplified models for individual neurons [3].

Firing rate models have led to applications both within and beyond neuroscience. For example, we can use firing rate equations to design networks of more complicated model neurons that display arbitrary dynamics [8]. This is the fundamental idea behind the brain simulator SPAUN; it presents a powerful potential application in the field of neuromorphic research, which aims to create engineered physical systems that process information in a manner similar to the human brain. Furthermore, networks of nonsmooth firing rate equations have surged in popularity with the growth of machine learning. In particular, deep learning uses networks of piecewise-smooth continuous nonlinearities to train large networks to solve what were once intractable machine learning problems. Examples include AlphaGo's recent victory over world Go champion Lee Sedol [9].

The presence of nonsmooth nonlinearities in the models is the common theme of these applications. The analysis of nonsmooth dynamical systems has recently undergone rapid expansion due to the prevalence of phenomena in mechanical and electrical systems most easily explained by the assumption of switching behaviour in the underlying model [1]. The time is ripe to use these tools to understand the nonsmooth dynamical systems that arise in neuroscience and related applications.

## Nonsmooth Dynamical Systems

An example of a nonsmooth dynamical system is the simple impacting system given by the following model:

$$\dot{x} \;=\; -2\eta x - y + F(t) \qquad (1)$$

$$\dot{y} \;=\; x \qquad (2)$$

$$\begin{aligned} x(t^-) \;&=\; b, \quad \Rightarrow \\ x(t^+) = b, \quad & y(t^+) = -ry(t^-), \end{aligned} \qquad (3)$$

where $F(t) = \sin(\omega t)$ is a simple periodic forcing function for an otherwise linear oscillator. Each time the oscillator impacts the surface $x = b$, the velocity changes discontinuously from $y(t^-)$ to $-r\,y(t^-)$. If we were to remove the impacting boundary, the resulting system could only display oscillations. However, an impacting boundary radically alters the dynamical repertoire that this system can display, and even allows the system to exhibit chaotic behaviours for the right parameter regimes (see Figure 1).

In the aforementioned example, the differential equation describing the system is smooth, but the impulses cause a discontinuous solution of the dynamical system. A dynamical system can also be nonsmooth if the right-hand side of the system itself is discontinuous. Such systems are defined via a set of differential equations with smooth right-hand sides, $F_i$, each valid on a different subset, $S_i$, of the phase space. We represent this mathematically as

$$\begin{aligned} \dot{x} = F_i(x,\mu), \quad & x \in S_i, \\ i = 1,\ldots,n, \quad & x \in \mathbb{R}^n,\ \mu \in \mathbb{R}^p, \end{aligned} \qquad (4)$$

where we assume that the boundary $\Sigma_{ij}$ between regions $S_i$ and $S_j$, commonly known as the switching manifold or discontinuity boundary [1], is a smooth

*See* **Neuroscience** *on page 7*

# Beyond UQ: Dealing with Deep Uncertainty

*By Hans Kaper*

Most *SIAM News* readers are familiar with the concept of uncertainty quantification (UQ). But when I received an invitation to participate in a workshop on "Decision Making Under Deep Uncertainty," I was unsure of what made "deep uncertainty" special.

The workshop, organized by the Society for Decision Making Under Deep Uncertainty (DMDU) and officially the society's 2016 Annual Meeting, was hosted by the World Bank in Washington, D.C., last November. It was preceded by a day of training for novices (like me) who wanted to learn more about the concepts and tools of DMDU.

DMDU differs from UQ. It plays an important role in the development of policy and business strategy under an extreme degree of uncertainty, i.e., when multiple plausible alternatives exist that cannot be ranked in terms of their perceived likelihood. This incapacity for ranking may be due to a lack of data, or a lack of knowledge of the mechanisms or functional relationships that govern the behavior of the system under consideration. In the worst-case scenario, all we know is that we are dealing with unknowns. But—and this is the essential element in DMDU—this ignorance is factored into the decision-making process. Think of a so-called "black swan" event — an event that lies outside the realm of regular expectation and is explainable only after the fact. Such events are more common than we think, and their impact can be catastrophic. One example is the level 9.0 earthquake that hit Japan in 2011. A tsunami and a nuclear catastrophe followed, which then led to supply chain disruptions (of automobile parts, for example) around the world.

UQ's premise is that uncertainty can be reduced, e.g., by gathering more information. With DMDU, probabilities are fundamentally unknowable and unpredictable. Yet decision-makers must make decisions under this level of uncertainty, and these decisions often concern major infrastructure projects that have long life spans and require significant investments. Consider the World Bank and infrastructure projects in developing countries.

When faced with uncertainty, decision-makers generally emphasize one of the following:

• *Resistance*: plan for the worst conceivable case

• *Resilience*: develop a strategy that results in quick recovery after an unanticipated event

• *Robustness*: develop and implement a policy that will perform reasonably well in all conceivable situations.

A policy based on resilience does not account for black swan events and may therefore be costly, while one focused on recovery may lead to possibly significant short-term losses. A robust policy, on the other hand, yields outcomes that are *satisfactory* across a wide range of scenarios, according to some predetermined assessment criteria. This is in contrast to an *optimal* policy, which may achieve the best results among all possible plans but carries no guarantee of doing so beyond a narrowly defined set of circumstances.

Robust strategies are appropriate when uncertainty is deep or decision-makers face a rich array of options. Instead of attempting to characterize uncertainty in terms of probabilities, as is done in UQ, deep uncertainty explores the possible effects of different assumptions about future values of the uncertain variables for the decisions actually at hand. As one of the workshop speakers noted, if something seems worth doing, it is worth doing first superficially. An exploratory approach might reveal options and provide an initial assessment of pathways for future considerations.

Workshop participants presented several case studies concerning major infrastructure projects where decision-makers faced deep uncertainty. The projects ranged from water and energy systems planning, flood risk management, infrastructure development, transportation networks, forest management, and public health to policy negotiations and security cooperation. The most important drivers of uncertainty in these projects were found to be climate change, rising sea levels, population growth, technology breakthroughs, and social choices.

A pitfall in all of DMDU is so-called "presentism" — a bias toward the present that too often results in "regrets" when prior commitments block decision paths. Another disadvantage, especially in the design of dynamic adaptive strategies, is a focus on the wrong indicators to monitor progress. Presenters gave examples where indicators did not align with a project's overall objectives, resulting in unintended consequences.

The workshop program also featured hands-on sessions about hypothetical decision-making problems and a fascinating plenary TED Talk by Andrew Revkin titled "Conveying Wicked Climate and Energy Realities in an Uncertain Communication Climate." Revkin was trained as a biologist but made a career shift to investigative journalism. He writes the Dot Earth blog[1] about science and environmental issues for *The New York Times* and recently moved to ProPublica, where he focuses on how countries and companies are—and are not—responding to climate change.

The workshop left me with some food for thought. Did I learn any new mathematics? No, but I discovered that DMDU touches on several areas of interest to the SIAM community, namely UQ and the mathematics of planet Earth. Thus, here is a new opportunity for collaboration with our community, with potentially significant benefits for society.

More information about the Society for Decision Making Under Deep Uncertainty can be found on their website.[2]

**Acknowledgments:** Some of the material in this article is based on the document "Deep Uncertainty" by Warren E. Walker, Robert J. Lempert, and Jan H. Kwakkel (2016), distributed at the "Decision Making Under Deep Uncertainty" workshop.

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[1] https://dotearth.blogs.nytimes.com/
[2] http://www.deepuncertainty.org

## Science Policy

*Continued from page 4*

blueprint provides no details on the science and technology programs at these agencies.

Congress has the power of the purse to control funding levels in appropriations bills, and is likely to reject the deep cuts proposed by the administration. For example, the administration's budget blueprint proposes cutting the NIH by close to $6 billion, but NIH funding has bipartisan support and received a $2 billion increase in the omnibus. The most likely appropriations scenario for FY 2018 is a full-year continuing resolution (CR), which would continue to fund the government at FY 2017 levels. If Congress is able to pass appropriations bills for the next fiscal year instead of a CR, evidence indicates that such bills would be considerably different from the president's budget request to reflect congressional priorities.

Barbara Helland, associate director of Advanced Scientific Computing Research at the DOE Office of Science, updated the committee on DOE priorities and changes. She talked at length about the Exascale Computing Project (ECP), emphasizing the project's role in providing strategic leadership and a foundation for capable exascale systems. The ECP identifies and supports research ventures to expedite applications, software, hardware platforms, and architectures imperative to the development of a capable national exascale ecosystem to support key DOE missions and contribute to the nation's economic competitiveness.

Michael Vogelius, director of the Division of Mathematical Sciences (DMS) at the NSF, reported on activities in his division. As Vogelius is nearing the end of his term as division director, he urged the mathematical sciences community to suggest replacement candidates. The job posting is available on USAJOBS.[1]

Vogelius described four types of solicitations comprised in the DMS Workforce Program: Mathematical Sciences Postdoctoral Research Fellowships, Enriched Doctoral Training in the Mathematical Sciences, Research Experiences for Undergraduates Sites, and Research Training Groups in the Mathematical Sciences. He emphasized the division's commitment to continuing the mathematics institutes, underscored the importance of mathematics in biology, and expressed interest in making mathematical biology more quantitative. He added that the DMS is working with the NSF Directorate for Biological Sciences (BIO) on a new initiative to support collaborations between biologists and mathematical scientists to advance the NSF "Rules of Life" big idea on predicting phenotype from genotype and environmental inputs. Vogelius also stated his interest in involving data science more heavily in the DMS portfolio, and noted that the new Transdisciplinary Research in Principles of Data Science program enhances collaboration on fundamental data science among mathematicians, theoretical computer scientists, and statisticians.

As part of SIAM's effort to connect with the international applied math community, Michael Günther, the European Consortium for Mathematics in Industry (ECMI) representative of the European Service Network of Mathematics for Industry and Innovation (EU-MATHS-IN), spoke to the committee. EU-MATHS-IN aims to leverage the impact of mathematics on innovations in key technologies by fostering communication among stakeholders in Europe. Günther explained that EU-MATHS-IN is a unique network of networks with members in each European country. The ECMI—which is itself a network of 97 institutional members in 22 European countries and Israel—focuses on using excellent science and industrial leadership to address societal challenges.

Günther referred to a Deloitte report,[2] which assesses the economic impact of mathematics on the Dutch economy at the request of the board of Platform Wiskunde Nederland, an organization representing the Dutch mathematics community. The Dutch report found that the mathematical sciences yield significant value for the economy, supporting a quarter of the Dutch national income and 26% of all jobs in the Netherlands, and contributing to 30% of gross value added. It also determined that almost a million Dutch employees use mathematical sciences, with many occupations requiring math as part of daily work routines. The report concluded that a coordinated effort is needed to enhance cooperation between mathematics and business/society.

Frederica Darema, director of the Air Force Office of Scientific Research (AFOSR), updated committee members on her office, framing the AFOSR as the "NSF for the Department of Defense." She described the agency's "find, fund, forward" mission, explaining that the AFOSR aims to identify breakthrough research opportunities, foster evolution of all basic research for AFOSR needs, and transition technologies to the DoD and industry. Program ideas are shaped by program managers, professional societies, and the community, she said. Darema spoke of the 36 programs in the basic research division and 20 programs in the international office. The current emphasis includes programs on computational biology, quantum biology, and quantum computing.

On the second day of the meeting, members of the committee met with staff from key congressional offices to convey support for robust federal investment in applied mathematics and computational science. Discussion included the importance of basic research through the NSF, the DoD, and the NIH, as well as the role of applied mathematics and computational sciences in strengthening national security and preserving U.S. leadership in biomedical research, energy science, and computing capabilities.

*Interested in learning more about the future of science and math under the Trump administration? Attend a minisymposium on Thursday, July 13th at the SIAM Annual Meeting, during which speakers from the National Science Foundation, Department of Energy, and Department of Defense will analyze the current administration's plans for implementation of national priorities and the potential contributions of major mathematics funding agencies.[3] Lewis-Burke will also be present to answer questions about the Trump administration and moderate the session.*

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[1] https://www.usajobs.gov/GetJob/ViewDetails/468213600?org=NSF

[2] http://euro-math-soc.eu/system/files/uploads/DeloitteNL.pdf

[3] http://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=62405

## Neuroscience

*Continued from page 5*

$(n-1)$ dimensional manifold. Unlike more standard differential equations, nonsmooth differential equations can display different degrees of "nonsmoothness." We classify the systems by the right-hand side's smoothness, or equivalently, the solutions' smoothness as they pass through the switching manifold. In Filippov systems, the solutions are continuous but have discontinuous time derivatives; in piecewise smooth-continuous (PWSC) systems, the discontinuity arises in the second order or higher time derivatives [1].

Nonsmooth systems described by (4) often display novel and exotic dynamical behaviours and bifurcations that the underlying smooth systems (which are separated by the switching manifold) cannot display themselves. While the current literature classifying and describing these nonsmooth dynamical behaviours is extensive [1], much work remains to generate a general bifurcation theory of nonsmooth dynamical systems that is as applicable and illuminating as the theory for smooth ones.

### Nonsmooth Dynamics and Bifurcations in Neural Models

Depending on the underlying model, the dynamics of a single neuron can be quite complicated. Thus, the analysis of a network of neurons' dynamical repertoire quickly becomes intractable. However, we can formally derive a low-dimensional system of differential equations describing the network behaviour in the large network or thermodynamic limit [3, 7, 10]. Due to their low dimensionality, these equations—called a mean-field model or firing rate model—are often much more tractable than the original network of neurons. Derivation of such a model from the original network is usually predictive of the behaviour of a sufficiently large network of model neurons. The bifurcation that causes the neuron to change from quiescence (a steady state equilibrium) to spiking (an oscillatory behaviour) plays a key role in these low-dimensional approximations, and can be represented by firing rate function $f(I)$, which gives the frequency of the oscillation as a function of the current input into the neuron. When these curves are often nonsmooth (see Figures 2a and 2c), the resulting mean field or firing rate model is a nonsmooth dynamical system.

Consider a generic network of type I model neurons, i.e., neurons that undergo a saddle-node on invariant circle (SNIC) bifurcation in their transition from quiescence to firing. Under appropriate conditions, we can formally show that the resulting mean-field model is approximately given by:

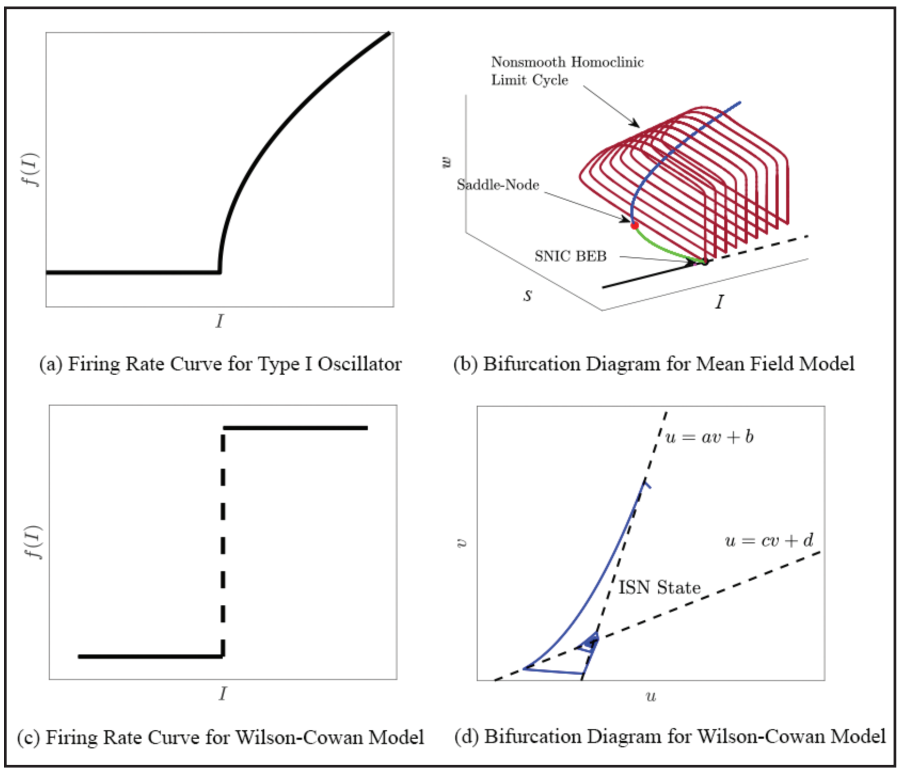
$$\tau_s \dot{s} = -s + \lambda_s F(s,w) \qquad (5)$$

$$\tau_w \dot{w} = -w + \lambda_w F(s,w) \qquad (6)$$

$$F(s,w) = \begin{cases} \sqrt{H(s,w)} & H(s,w) > 0 \\ 0 & H(s,w) < 0, \end{cases} \qquad (7)$$

where $H(s,w) = \alpha_{00} + \alpha_{10}s + \alpha_{01}w + \alpha_{20}s^2$, with $\alpha_{ij}$ determined by the parameters of the original model neurons [7]. The variables $s$ and $w$ correspond to network average values of currents in the neurons. In particular, $s$ corresponds to excitatory synaptic coupling and $w$ to inhibitory self-coupling, commonly referred to as spike frequency adaptation.

The system of equations governed by (5)-(6) is PWSC. A pair of local, nonsmooth, codimension 2 bifurcation points largely determines the entire bifurcation structure of this system. These points correspond to the collision of a saddle-node and Hopf bifurcation equilibrium point with switching manifold, $H(s,w) = 0$, respectively.




(a) Firing Rate Curve for Type I Oscillator (b) Bifurcation Diagram for Mean Field Model

(c) Firing Rate Curve for Wilson-Cowan Model (d) Bifurcation Diagram for Wilson-Cowan Model

**Figure 2.** *Nonsmooth neural model examples.* **2a.** *The firing rate curve for type-I excitability often has a square-root type discontinuity. This leads to mean-field equations that are piecewise smooth-continuous (PWSC).* **2b.** *The resulting mean-field systems display novel nonsmooth bifurcations, such as a saddle node on invariant circle (SNIC) boundary equilibrium bifurcation where a nonsmooth fold bifurcation collides with a nonsmooth limit cycle.* **2c.** *A Heaviside function often approximates the firing rate curve for the Wilson-Cowan equations. This transforms the Wilson-Cowan equations into a Filippov system.* **2d.** *The Filippov system features an intersection of two switching manifolds, which creates a new equilibrium in the plane. This equilibrium corresponds to the inhibitory stabilized network state in the related biological neural network. Image credit: Wilten Nicola.*

As such, they act as organizing centers from which smooth saddle-node and subcritical Andronov-Hopf bifurcations and several nonsmooth, codimension 1 bifurcations emerge. This system also exhibits interesting nonsmooth limit cycle bifurcations, such as when a limit cycle becomes homoclinic with a boundary equilibrium bifurcation point [7] (see Figure 2b).

We can also derive a nonsmooth mean-field system using a Heaviside step function for the nonlinearities in the classic Wilson-Cowan model [10], leading to the following equations [5]:

$$u' = -u + f(\alpha_0 + u + \alpha_1 v) \qquad (8)$$

$$\tau v' = -v + f(\beta_0 + u + \beta_1 v) \qquad (9)$$

$$f(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0\,. \end{cases} \qquad (10)$$

Unlike system (5)-(6), which only has a single switching manifold, equations (8)-(9) have a pair of switching manifolds ($\alpha_0 + u + \alpha_1 v = 0$ and $\beta_0 + u + \beta_1 v = 0$). Here we interpret the variables $u$ and $v$ as the firing rates of two groups of neurons in the network (excitatory and inhibitory neurons, respectively).

System (8)-(9) is a Filippov system. In one of the first analyses of a system with multiple switching manifolds, [5] demonstrates the existence of a nonsmooth analogue of the Hopf bifurcation occurring at the intersection points of these two switching manifolds for critical values of time constant $\tau = \tau_{HB}$. Furthermore, the authors illustrate the existence of nonsmooth analogues to the homoclinic and SNIC bifurcations.

Besides demonstrating interesting and novel nonsmooth bifurcations, the analysis of systems (5)-(6) and (8)-(9) yields substantial insights into the behaviour of the underlying neuron networks. We derived system (5)-(6) from a network of model neurons; simulations of the large network model [7] directly demonstrated the nonsmooth bifurcations. For example, the emergence of a nonsmooth limit cycle in the mean field model predicts when bursting will emerge in the network. System (8)-(9) demonstrates a link between the pseudo-focus occurring at the intersection of two switching manifolds and the inhibitory stabilized network (ISN) state [5] (see Figures 2c and 2d). In this state, feedback inhibition balances strong recurrent excitation to maintain stability. As we vary the parameters in the model equations, the ISN state loses stability via the nonsmooth Hopf analogue. Thus, nonsmooth analysis of these systems can yield substantial insights into the functioning of networks of neurons while simultaneously expanding the theory of nonsmooth dynamical systems.

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# Marching for Science

*By Hans Kaper and Hans Engler*

On April 22, science went public in a major way. For the first time, thousands of scientists gathered in Washington, D.C., and transformed Earth Day into a tribute to science.

The concept of a March for Science[1] originated in social media discussions, similarly to the Women's March on Washington in January. Support grew rapidly. The American Association for the Advancement of Science (AAAS) partnered with the march, which evolved into a mass movement within the scientific community in the U.S. and around the world. Numerous professional organizations, including SIAM, endorsed the goals of the march: *to call for science that upholds the common good and for political leaders and policy makers to enact evidence-based policies in the public interest*. What started locally grew into a global phenomenon, with events held in more than 600 cities on six continents. The following are some of our impressions of the events that took place in Washington, D.C.

At an early morning pre-rally at AAAS headquarters, AAAS CEO Rush Holt—former Democratic congressman from N.J. who holds a Ph.D. in physics—introduced representatives of professional societies involved in the march's organization, such as the American Geophysical Union, Oceanographic Society, American Physical Society, and Optical Society, among others. He also acknowledged numerous noteworthy attendees, including William Phillips (National Institute of Standards and Technology, 1997 Nobel Laureate in Physics), Shirley Malcolm (director of Education and Human Resources, AAAS), and Congressman Bill Foster (D-IL, the only Ph.D. scientist remaining in the U.S. Congress).

The rally itself kicked off at 10 a.m. at the Washington Monument on the National Mall. The four-hour program featured an impressive lineup of prominent speakers representing a broad range of ages, backgrounds, and expertise, including Denis Hayes (co-founder of the first Earth Day in 1970), Bill Nye the Science Guy, and YouTube star Tyler DeWitt. The program concluded with a Twitter message from none other than Pope Francis.

The collective events offered several powerful take-home messages, some of which we outline here. Science is at a critical juncture. The scientific method is under attack, and individuals and interests with the power to do real harm are threatening the very idea of evidence, logic, and reason. We, as members of the scientific community, must go public and explain our work to general audiences. It behooves us to make people aware of our research and its benefits to society. Scientists should talk *to* people, instead of at them. We should not complain about slashed funding if we cannot tell taxpayers why science matters.

Following the rally, the crowd marched down Constitution Avenue, from the Washington Monument to the foot of Capitol Hill. While a rally is more or less a static affair, participants of the march demonstrated solidarity — vocally by chanting in unison or in waves along the parade route, and visually by showing off their signs. Here are some of our favorites:

- Science, not silence
- Oceans are rising, and so are we
- There is no Planet B
- In peer review we trust
- Make America think again
- Alternative facts are not statistically significant
- Science cures alternative facts
- Empirical data trump imperial alt-facts
- No beer without science

Favorable weather forecasts earlier in the week took a turn for the worse, and march day turned out to be soggy and wet. Still, most people endured and kept their spirits high. Although we were all soaked at the end of the day, it felt good to have been part of this unprecedented event.

[1] https://satellites.marchforscience.com/

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# Counting Crowds at the National Mathematics Festival

*By Blake Reichmuth, Ratna Khatri, Rachel Neville, and Suzanne Weekes*

The National Mathematics Festival, held on April 22 in Washington, D.C., brought an incredible mixture of education, entertainment, mathematics, and science to the nation's capital. The festival was organized by the Mathematical Sciences Research Institute, in cooperation with the Institute for Advanced Study and the National Museum of Mathematics. Events included more than a dozen short films, games and activities for younger audiences, geometric balloon bending, and over 50 presentations ranging from standup mathematics comedy and magic to quantitative analysis and hurricane storm surge modeling. However, the long duration of the event, use of multiple entrances at the venue, and lack of tracking due to free admission made estimating the number of individual attendees quite a challenge.

The total count of attendees at this year's festival was an estimated ***M*** unique individuals.[1] The team settled on this estimate using a step-by-step process. Volunteers took cell phone pictures of crowds from various vantage points with the help of 67-inch selfie sticks. They snapped photos of attendees in assigned rooms, hallways, and concourses every hour for the duration of the festival. After these pictures were collected and heads counted, the total number of heads counted, i.e., "people hours" — an approximation to $\int_0^T \#\mathrm{people}(t)\,dt$ was calculated to be 20,350.

The team then set to work determining the number of unique individuals attending the event by filtering out people accounted for in earlier tallies. During their off hours, SIAM volunteers polled the crowd to get a sense of how long attendees expected to stay. This polling information on attendance duration offered insights into the number of people who would appear more than once in the hourly photos.

Student volunteers at the National Mathematics Festival snapped photos of crowds to estimate the total number of attendees. Photo used with permission.

From left to right: Chong Wang (George Washington University), team chair Rachel Neville (Colorado State University), and Ratna Khatri (George Mason University) comprised the Festivals Working Group of the SIAM Education Committee at the National Mathematics Festival. Photo credit: Steven Neville.

The Festivals Working Group of the SIAM Education Committee, comprising of three graduate student leaders of SIAM student chapters—all pursuing Ph.D.s in applied mathematics—were tasked with the responsibility of orchestrating the crowd counting. Team chair Rachel Neville (Colorado State University), Ratna Khatri (George Mason University), and Chong Wang (George Washington University) recruited graduate and undergraduate student volunteers from their three respective schools, as well as Shippensburg University. Volunteers stood out in the crowd thanks to special SIAM buttons pinned to their National Math Festival shirts.

Let us consider the process in more detail. Take the "hour 1" count of the number of people in attendance. The distribution of predicted attendance times allows us to estimate the number of these people who would show up in each of the later photos. The number of *new* people in the second round of photos is the "hour 2" head count minus the number of heads we estimate were already present in the first count, i.e., those folks who indicated they were staying at the festival for more than an hour. The number of new people at "hour 3" is the head count at hour 3 minus both those people who arrived in time for the hour 2 photos and stayed at the festival for longer than an hour, and the hour 1 people who stayed at least two hours. And so on. The number of people who attended the festival is the total of all the new people present in the hourly photos.

In mathematical terms, let $h_k$ be the number of individuals that have joined the crowd between hour $k-1$ and hour $k$, and let's define $\tilde{h}_k = max(0, h_k)$. Let $H_k$ be the total number of people counted at hour $k$. Then $h_0 = 0$, and $h_1 = H_1$ is the total number of attendees that entered the festival from the time the festival began to the moment the first pictures were taken. Therefore, $h_2 = H_2 - \delta_1 \tilde{h}_1$, where $\delta_k$ is the fraction of people that plan to stay more than $k$ hours, according to our polling of attendance times. Next, $h_3 = H_3 - (\delta_2 \tilde{h}_1 + \delta_1 \tilde{h}_2)$ is the approximate number of new attendees between the second and third hour of the festival. In general, $h_n = H_n - \Sigma_{i=1}^{n-1} \delta_{n-i} \tilde{h}_i$ and the total number of unique individuals who attended the festival, $h^* = \Sigma \tilde{h}_i$. We assume that $\delta_k$ is not time-dependent, whereas in reality the fraction of people who plan to stay two+ hours will certainly be smaller near closing time than in the early hours of the festival. While our process is only an estimate, we do believe it provides a good start to determining the number of attendees at this year's National Math Festival.

If you happen to be one of those individuals who attended and enjoyed the event, we hope you were able to keep your balloon-edged cube or tetrahedron safe from sharp corners, like those of your business card tetrahedra. Or perhaps you were able to meet famous mathematicians and science media experts. Either way, the festival will return in two years packed with math fun for all ages.


Special SIAM buttons identified student crowd-counting volunteers. Photo credit: Ratna Khatri.

[1] Since crowd counts cause so much controversy these days, we will leave this as just ***M***.

*Blake Reichmuth is a master's student in mathematical sciences at George Mason University. He is a member of the AMS, the MAA, and the AWM and SIAM student chapters. Ratna Khatri is an applied mathematics Ph.D. student at George Mason University. She is an active member of the university's AWM and SIAM student chapters. Rachel Neville will receive her Ph.D. in applied topology from Colorado State University this June, and will begin her position as the Hanno Rund Postdoctoral Research Associate at the University of Arizona in the fall. She is chair of the Festivals Working Group of the SIAM Education Committee. Suzanne Weekes is professor of mathematical sciences at Worcester Polytechnic Institute. She is chair of the SIAM Education Committee.*

# Snapshots from Earth Day in Washington, D.C.


Approximately 150,000 people attended the March for Science in Washington, D.C., on Earth Day, April 22. The event began with speeches, teach-ins, and rallies, and concluded with a march from the Washington Monument to Capitol Hill. Photo credit: Nicholas Higham.


Attendees of the March for Science braved the rain during the afternoon trek down the National Mall. Photo credit: Nicholas Higham.


March for Science participants displayed creative signs conveying the relevance and significance of scientific research to the public interest. Photo credit: Hans Kaper.


Participants of the March for Science came out in full force to defend the role of science in policy and society. Photo credit: Nicholas Higham.

Attendees of the National Math Festival, held in Washington, D.C., on April 22, enjoyed magic tricks that utilized math. Photo credit: Nicholas Higham.

National Mathematics Festival guests celebrated math with geometric sculptures. Photo credit: Nicholas Higham.


Children used tri-string wands to make large bubbles at the National Math Festival. Photo credit: Nicholas Higham.

A hands-on 'obstacle' course at the National Math Festival required participants to shoot giant vortex smoke rings out of trash cans. Photo credit: Nicholas Higham.

# The Business of Hedges, Bets, and Blackjack

## *The Life and Career of Edward Thorp*

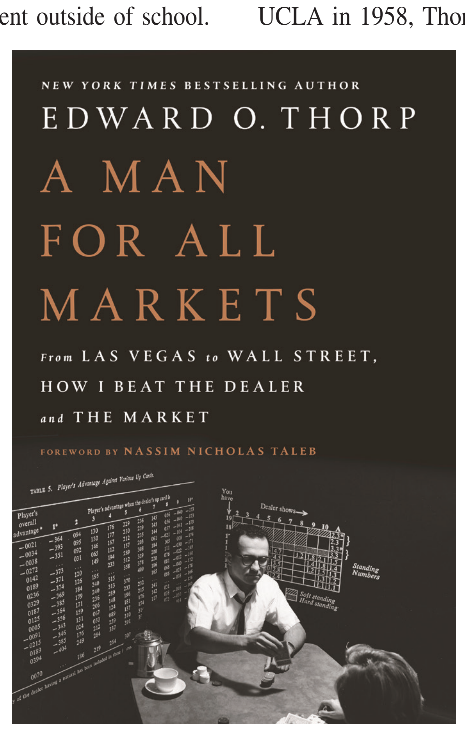
**A Man for All Markets: From Las Vegas to Wall Street, How I Beat the Dealer and the Market.** *By Edward O. Thorp. Random House, New York, NY, January 2017. 416 pages, $30.00.*

BOOK REVIEW
*By James Case*

The basic details of Edward O. Thorp's legendary career are well known. As a postdoctoral instructor at the Massachusetts Institute of Technology (MIT), he programmed the school's brand-new IBM 704 computer—in the then-brand-new FORTRAN language—to calculate the odds of winning a hand of blackjack dealt from an incomplete deck. Following coverage of the feat in both *Sports Illustrated* and *Life* magazine, Thorp's book *Beat the Dealer* became a *New York Times* best seller, heightening interest in casino blackjack for years to come. Not long thereafter, in collaboration with Claude Shannon, Thorp developed a "wearable computer" that enabled the pair—working as a team—to beat the house at roulette. Rather than pursue that opportunity, however, the two parted ways, focusing separately on Wall Street: the biggest casino of all. In 2014, Celebrity Net Worth[1] placed Thorp at number 36 on its list of 50 richest hedge fund managers, with a fortune estimated at $800 million.

*A Man for All Markets* describes Thorp's struggle, as a child of the Great Depression, to attend college, much less forge an academic career. Learning first to count and then to read, young Edward was devouring classics such as *Treasure Island*, *Gulliver's Travels*, and *King Arthur and the Knights of the Round Table* by age six. When challenged by a stranger amused to find one so young in possession of Charles Dickens' *A Child's History of England*, the prodigy responded by naming every English monarch from King Arthur through Queen Victoria.

The Thorp family moved from Chicago to Lomita, Calif., on the eve of World War II, in search of factory work. With both of his parents working long hours, Thorp was obliged to find his own amusement outside of school. Increasingly, he found it in science.




*A Man for All Markets: From Las Vegas to Wall Street, How I Beat the Dealer and the Market. By Edward O. Thorp. Courtesy of Random House.*

Thorp won California's competitive high school physics exam, entitling him to first pick among the scholarships offered that year by in-state institutions. He started school at the University of California, Berkeley as a chemistry major in the fall of 1950, but switched to physics after one semester and soon transferred to the University of California, Los Angeles (UCLA) to be closer to home.

Despite receiving a scholarship to Columbia University, Thorp remained at UCLA upon graduation due to financial reasons, quickly completing his masters and the course work for a Ph.D. He married his undergraduate classmate Vivian Sinetar and embarked on a thesis regarding the shell structure of atomic nuclei, which required a number of advanced math courses. Ultimately, this prompted a switch to mathematics, and a thesis pertaining to compact operators on Banach spaces.

Following the completion of his thesis at UCLA in 1958, Thorp worked there as an instructor for a year. Because Las Vegas offered packages that included cheap lodging, meals, and A-list entertainment, he and Vivian decided to vacation there during Christmas break. Having long believed it possible to develop a winning strategy for roulette, Thorp planned to use the visit to conduct feasibility studies. Before leaving, however, he learned of a recently published strategy for blackjack. Devised by four army mathematicians led by Roger Baldwin using desktop calculating machines during World War II, it reduced the odds favoring the house to a mere 0.62, substantially lower than any other game in town. In the process of losing $8.50 of his $10.00 stake while following an abbreviated form of the Baldwin strategy, Thorp became hooked on blackjack.

Back in Los Angeles, Thorp located the journal article describing the strategy and experienced an immediate "ah-ha" moment. He realized that all the probabilities in the article had been calculated as if each hand was dealt from a fresh, randomly-shuffled deck, whereas casino hands were typically dealt from depleted decks, devoid of the most recently played cards. Depending on which cards were missing, a depleted deck could be more or less favorable to the house than a fresh deck. And by betting heavily when the deck is in the players' favor, and lightly (if at all) when it isn't, an individual can expect to win in the long run. The only remaining questions were which depleted decks were favorable to whom, and how to recognize the promising ones in real time.

Arriving at MIT in the fall of 1959, Thorp was pleased to discover that the school's IBM 704 could carry out a thousand man-years of routine calculations in a mere ten minutes of run time. Yet his work was still frustrating; results took two to three days after a job (deck of punched cards) was placed in the queue. Thorp's first discovery, made by avoiding some of the approximations used by the Baldwin team, was that their strategy actually reduced[2] the house advantage to a mere 0.21! His second was that the 48-card deck missing

*See* **Bets and Blackjack** *on page 12*

[1] www.thecelebworth.com

[2] Around 1980, as computers became powerful enough to require no approximations, researchers discovered that the Baldwin strategy actually placed the house at a 0.13 *dis*advantage, even without counting cards!

# Foucault's Pendulum with a Twist

MATHEMATICAL CURIOSITIES

By Mark Levi

While teaching a mechanics course, I stumbled upon the following amusing observation. It is well known that small-amplitude trajectories of a pendulum are approximately ellipses (see Figure 1). Indeed, the linearized equations for the $(x, y)$ coordinates of the bob are

$$\ddot{x} = -\omega^2 x$$
$$\ddot{y} = -\omega^2 y, \qquad (1)$$

or

$$\ddot{\mathbf{r}} = -\omega^2 \mathbf{r}, \ \ \mathbf{r} = \langle x, y \rangle,$$

with $\omega^2 = g/L$; $g$ is the gravitational acceleration and $L$ is the length of the pendulum. The general solution is $x = a\cos\omega t + b\sin\omega t$, $y = c\cos\omega t + d\sin\omega t$. This is a parametric representation of an ellipse centered at the origin. Indeed, $(x, y)$ is the image of the unit circle $(\cos\omega t, \sin\omega t)$ under the linear map whose matrix has elements $a, b, c, d$. Figure 2 illustrates the three types of motions.

Now, let's put ourselves in a frame centered at the origin and rotating with angular velocity $\omega$, where $\omega$ is the same as above: the frequency of the pendulum. *How will the elliptical motions of the pendulum look in this new frame?*

Perhaps surprisingly, the answer is *circular, and with a constant speed*. Moreover, the angular velocity of these circular motions is $2\omega$, twice the frequency of the pendulum (see Figure 3). The circle passes through the origin precisely if the angular momentum is zero. And the circle in the rotating frame is centered at the origin if the ellipse in the inertial frame is a circle.

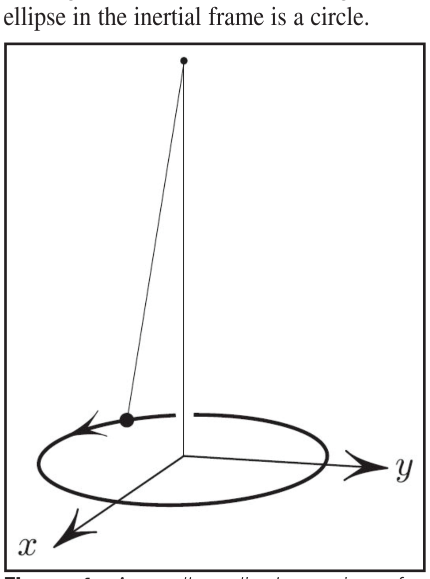



**Figure 1.** *A small-amplitude motion of a spherical pendulum.*

Here are two explanations of this phenomenon, from two different angles.

**Explanation 1.** This explanation is based on the observation that *any solution of* $\ddot{\mathbf{r}} = -\omega^2 \mathbf{r}$ *is a combination of two circular motions*, one counterclockwise and the other clockwise. In complex notation $\mathbf{r} = x + iy$, the general solution is

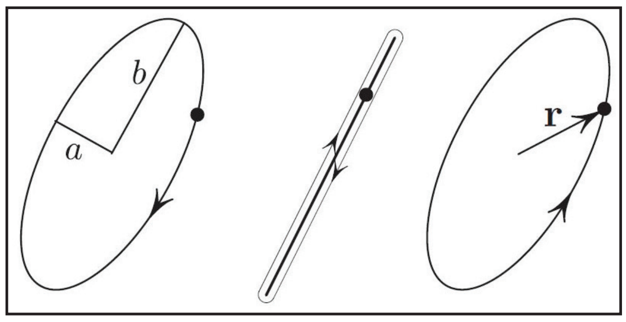



**Figure 2.** *Elliptical trajectories of (1) with negative, zero, and positive angular momentum respectively.*

$$\mathbf{r} = Ae^{i\omega t} + Be^{-i\omega t},$$

where $A$ and $B$ are arbitrary complex constants. It follows that the solution in the rotating frame is

$$A + Be^{-2i\omega t},$$

as claimed.

**Explanation 2.** In addition to the restoring force, the bob in the rotating frame perceives two additional inertial (fictitious) forces acting on it: the centrifugal force $\omega^2 \mathbf{R}$, where $\mathbf{R}$ is the bob's position expressed in the rotating frame, and the Coriolis force $-2i\omega\dot{\mathbf{R}}$. The apparent acceleration is thus the sum of the two inertial forces and the forces of the spring:

$$\ddot{\mathbf{R}} = -\omega^2 \mathbf{R} + \omega^2 \mathbf{R} - 2i\omega\dot{\mathbf{R}} = -2i\omega\dot{\mathbf{R}} \qquad (2)$$

(formally, one obtains (2) by substituting $\mathbf{r} = e^{i\omega t}\mathbf{R}$ into $\ddot{\mathbf{r}} = -\omega^2 \mathbf{r}$ and simplifying). Note that the centrifugal force cancels the restoring force!

According to (2), the particle in the rotating frame is subject to the force normal to its velocity, same as the Lorentz force on a charged particle in the magnetic field of magnitude $2\omega$ and normal to the plane. This demonstrates that the trajectories are circles (just as the trajectories of a charged particle are in the constant magnetic field perpendicular to the plane of the particle's motion).

Interestingly, passage to the rotating frame replaces the Hookean force by the Coriolis force, as just indicated.

The above equivalence is reversible; we conclude that the particle in a constant magnetic field, viewed in an appropriately rotating frame, behaves exactly as the planar harmonic oscillator (1).

I end with a tongue-in-cheek application to the Foucault pendulum, mounted over the North Pole. Wishing to match the pendulum's frequency to Earth's angular velocity, we choose the length $L$ to satisfy

$$\sqrt{g/L} = \frac{2\pi}{24 \cdot 3{,}600};$$

this gives $L \approx 1{,}176$ miles. A Foucault pendulum of this length, mounted over the North Pole, will execute circular motions of the type shown in Figure 3, making one revolution in 12 hours — provided that we raise the suspension point by, say, 100 miles to take the bob out of the atmosphere and prevent viscous drag. On a more realistic note, to observe this effect on a carousel making one revolution in six seconds, the length of the pendulum must be around 10 meters.

A more detailed discussion of this problem can be found in [1].

*The figures in this article were provided by the author.*

### References


[1] Levi, M. (2014). *Classical Mechanics with Calculus of Variations and Optimal Control: an Intuitive Introduction*. In *Student Mathematical Library* (Book 69). Providence, RI: American Mathematical Society.



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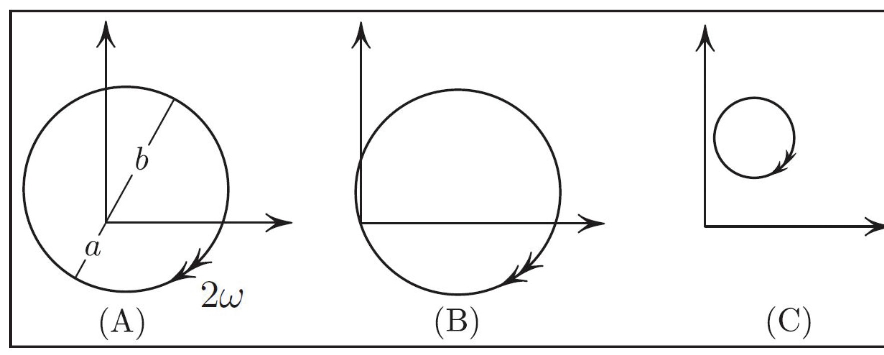


**Figure 3.** *Trajectories in the frame rotating counterclockwise with angular velocity* $\omega$, *with negative (A), zero (B), and positive (C) angular momenta. The lengths* $a$ *and* $b$ *in (A) are the semiaxes of the ellipse in Figure 2.*

# James H. Wilkinson Prize

Lek-Heng Lim has been awarded the James H. Wilkinson Prize in Numerical Analysis and Scientific Computing. He was educated at Stanford University, the University of Cambridge, Cornell University, and the National University of Singapore. Lim was a Morrey Visiting Assistant Professor at the University of California, Berkeley and thereafter an assistant professor at the University of Chicago. He has received an AFOSR Young Investigator Award, a DARPA Young Faculty Award, an NSF Faculty Early Career Award, a Smale Prize from the Foundations of Computational Mathematics Society, and a Director's Fellowship from DARPA.

*Lek-Heng Lim, recipient of the James H. Wilkinson Prize in Numerical Analysis and Scientific Computing.*

"Looking at the list of past winners, I am deeply honored and humbled that the prize committee decided I belong on this list," Lim said. "SIAM President Nick Higham's email informing me of the prize came as a very pleasant surprise. I am extremely grateful to the people who kindly nominated me."

Lim went on to describe an aspect of his research related to real-world application. "In joint work with Thomas Schultz, an exceptionally talented computer/neuroscientist in Bonn, we figured out a way to get high-resolution 3D images of neural fibers in the human brain from diffusion MRI measurements," Lim said. "Among other things, mapping major bundles of neural fibers in the human brain is vitally important in neurosurgical planning."

Lim will present the James H. Wilkinson Prize in Numerical Analysis and Scientific Computing Lecture, "Tensors in Computational Mathematics," on Wednesday, July 12, from 3:00-3:30 p.m. in the David Lawrence Convention Center (room Spirit of Pittsburgh A – 3rd Floor) as part of the SIAM Annual Meeting to be held in Pittsburgh, Pa., from July 10-14. He will receive his prize at the Prizes and Awards Luncheon on Tuesday, July 11 at the Allegheny Ballroom of the Westin Hotel. All other major SIAM prizes will also be awarded at the luncheon.

# Cause and Dynamics of a Silent Epidemic: Confronting the Dropout Crisis and Keeping Children in Schools

## *A Mathematical Analogy with an Infectious Diseases Approach*

*By Anuj Mubayi*

In 2014, the U.S. high school graduation rate was 83.2 percent. In 2013, the average four-year cohort dropout rate for all Chicago high schools was 26.2%. Researchers from Arizona State University (ASU), Northeastern Illinois University (NEIU), and the University of Texas at Arlington (UTA) developed a data-driven mathematical model to study the influence of student environment on high school dropout patterns. This modeling study includes a 2013 survey—designed by researchers using a sampled Chicago high school, which is particularly vulnerable to student dropouts—to parameterize the model, making it unique among these types of investigations.

Bechir Amdouni, a high school teacher and graduate student at NEIU, and I (formerly of NEIU and now a professor at ASU) spearheaded the research project after visiting a local high school in South Chicago and witnessing the seriousness of the dropout problem firsthand. Upon interacting with students and faculty at the high school, we realized that academic achievement shaped by peer influence and parental guidance might offer significant support to vulnerable students; such influences may be affecting dropout rates in this community. Marlio Paredes, a dynamical systems expert at the University of Puerto Rico-Cayey, and Christopher Kribs, a specialist in mathematical education and mathematical epidemiology from UTA, joined us to formally design the study.

### Methods and Analysis

Despite the complexity of dropout dynamics, previous studies have argued that the U.S. government's policies are not supported by mechanistic-based temporal research; thus, dropout rates in many parts of the country are rising rapidly. Our study aims to identify driving mechanisms and develop, analyze, and test a mathematical model to better understand high school dropout dynamics; this is followed by analysis, calibration, and simulation. Our dynamic model uses an analogy from susceptible-infected-recovered-type infectious disease models via a set of nonlinear differential equations. Bifurcation analysis identified two tipping point quantities: a threshold that evaluates the generation point of the critical mass of academically vulnerable students in a school, and a threshold that captures conditions under which the number of failing students becomes large enough to increase dropout rates. The resulting model, based on students' academic performance in core courses, can imitate four different situations:

(i) A 'healthy' school, in which all students perform very well academically

(ii) An institution where some students fail core courses but academic failure does not cause dropouts

(iii) A school with low dropout rates

(iv) A school with extremely high dropout rates

Following model formulation, we developed and administered a precise survey instrument to a group of ninth through twelfth grade students in a Chicago public school. We used data about the school's enrollment and attempted to identify factors that correlated with the establishment and maintenance of high dropout levels.

### Findings and Implications

Our research team studied the effects of multiple mechanisms—including effective teaching, school demographic factors, peer influences on and off campus, parental influences, and student academic performances—on the dynamics of dropout rates. We found that parental involvement and peer interactions have the highest impact, and hence decided to further study their impact on student outcome.

Students' academic achievement is most directly related to the level of parental involvement, or lack thereof. Survey analysis revealed that over half of the dropouts did not live with their parents, reinforcing the potential effect of social, economic, and emotional environments on students' educational development.

In our sample, more than 50% of students were in frequent contact with individuals who are of the mindset that attending school is a waste of time. Preemptively identifying vulnerable students and increasing parental involvement lowers the number of disaffected friends, thus raising the question of how to monitor or effectively restrict "good" students from mixing with failing students or dropouts. However, if negative social interactions (social mixing) increase beyond a certain threshold, the impact of parental involvement becomes less significant (see Figure 1). And if intervention is left until students are actively failing at school, attempts at parental guidance are futile.

The study thus suggests that peer interaction, like parental guidance, is critical to the development of higher dropout rates. However, separating students based on negative peer interactions and behaviors towards one another raises many practical issues regarding curriculum. For example, if there is a major shortage of skilled faculty in similar schools, who will teach these separated groups of students?

Chicago public schools use metrics, such as dropout rates, to evaluate school performance. If this metric is low, schools may lose funding or even face closure (50 schools closed in the U.S. in 2013). Therefore, teachers feel a continuous pressure to not fail students, which raises other issues. Do students passing with very low grades deserve to pass, or are their passing grades a result of pressure of a potential school closing? If they graduate, do they enroll in a college or university? Have they truly gained quality education in high school?

Numerous factors contribute to dropout rates, making its investigation quite challenging. This study serves as a starting point to begin to understand these complexities, though some limitations exist. Future studies must consider higher sample sizes, a greater number of schools per sample, data stratification based on race and ethnicity, and construction and analysis of dynamical models that capture peer influences depending on class year, age group, social context, and neighborhood.

### Summary

The factors we consider in this analysis are different from those typically investigated in existing dropout-related studies. Moreover, unlike cross-sectional and longitudinal approaches, our research focuses particularly on the dynamics of dropout rates, likely the most effective way to identify critical factors and design a lasting intervention. In summary, while parental influence can deter student dropout up to a certain point, the amount of time vulnerable students spend with friends who have already dropped out is also significant. We included general positive and negative trends as part of the study hypotheses, but the ways in which they interact and limit each other are complex. The model—while still a gross oversimplification of human interaction—allowed us to capture some of that complexity.

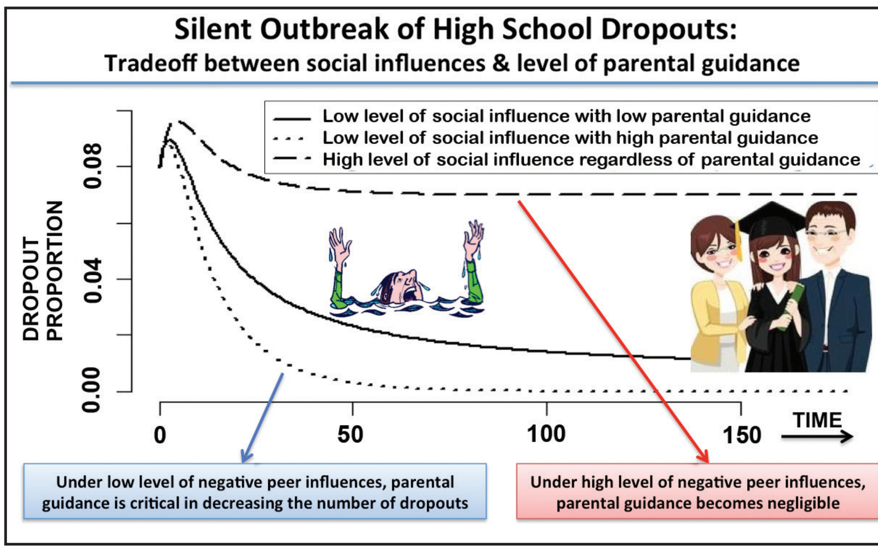


**Figure 1.** *Parental guidance and peer influence are most influential on student dropoff rates. Although preemptive parental involvement can keep students in school, that involvement loses significance as the quantity of negative social interactions increases. Image adapted from [1].*

### References

[1] Amdouni, B., Paredes, M., Kribs, C., & Mubayi, A. (2017). Why do students quit school? Implications from a dynamical modelling study. *Proceedings of the Royal Society A, 473*(2197).

*Anuj Mubayi is an assistant professor of applied mathematics in the School of Human Evolution and Social Change as well as the Simon A. Levin Mathematical Computational and Modeling Science Center at Arizona State University. He is co-director of the Mathematical and Theoretical Biology Institute's*[1] *summer training program for undergraduate students. The program aims to improve dropout rates for college students, specifically underrepresented minorities, and encourage and train them for the challenges of graduate-level research in biology and applied mathematics.*

[1] https://mtbi.asu.edu/

## Bets and Blackjack

*Continued from page 10*

all four aces gave the house a 2.72 percent advantage, far greater than the 0.21 of a fresh deck. This realization showed that the odds can shift significantly after only four cards have been played. By the fall of 1960, Thorp completed his calculations, used the results to develop a variety of more or less complicated winning strategies, made a "proof-of-concept" visit to Las Vegas, and prepared to publish his findings.

To reach the largest possible audience, he asked Claude Shannon to submit his paper to the *Proceedings of the National Academy of Sciences*. After a searching cross examination, Shannon agreed to do so, and directed Thorp to a paper by John Kelly containing a now-famous (if still somewhat controversial) formula for choosing the fraction of one's current endowment to wager on a sequence of promising, though risky, propositions.

With his knack for detecting statistically advantageous wagers and the Kelly rule for exploiting them, Thorp was ready for an assault on the financial markets. In collaboration with Sheen Kassouf, an economist at the University of California, Irvine (where Thorp taught math and finance between 1965 and 1982), he wrote a book called *Beat the Market*, explaining a few of the ways to do exactly that. In 1969, at the suggestion of Warren Buffett, Thorp formed a hedge fund called Convertible Hedge Associates (later renamed Princeton Newport Partners, or PNP), with offices in Newport Beach, Calif., and Princeton, N.J. The West Coast operation, managed by Thorp, identified promising trades and communicated them to the East Coast division, which made the trades and sought additional investors.

In early 1973, Thorp received a preprint of a paper by Myron Scholes and Fischer Black that rigorously derived a formula for evaluating certain options about which Thorp presumed no one else knew. Having deduced it by what he calls "plausible mathematical reasoning," Thorp had been using the now-famous Black-Scholes formula since 1967. But once it entered the public domain, he realized that he would have to develop new tools to stay ahead of the competition.

In December 1987, operatives of the International Revenue Service, the Federal Bureau of Investigation, and the U.S. Postal Service raided the Princeton office of PNP. The five top people were indicted and tried on 64 charges of stock manipulation, stock parking, and tax, mail, and wire fraud. All were convicted on multiple counts. As a result, Thorp elected to close the operation down. At no time was he, or anyone else in the West Coast office, accused of any wrongdoing.

Thorp is able to estimate the amount of money he might have made, had PNP remained in business, merely by reflecting that a market-neutral hedge fund operation—the Citadel Investment Group—was built on the PNP model by former hedge fund manager Frank Meyer and investment prodigy Ken Griffin. Beginning with a few million dollars in 1990, when Thorp became the group's first limited partner, it has produced annualized gains of about 20% through 2015, when Griffin's net worth was estimated at $5.6 billion. During much of that time, Thorp continued to act as an informal consultant and benevolent guru to a variety of investment startups.

Thorp dwells at length on the criminal behavior he has encountered over the years, from plagiarism in academic life to dirty tricks in casinos to white collar crimes reported in the *Wall Street Journal*. Neither the powers in academic life, the Nevada state gaming commission, nor the U.S. Securities and Exchange Commission seems prepared to confront the malfeasance that surrounds them. Protect yourself, he warns. No one else is likely to do it for you.

Upon closing PNP's doors in 1987, Thorp paused to reflect on the nature of time. How you spend it, he decided, is what counts in the long run, and there are tradeoffs to be made between time, health, and wealth. What seems to please Thorp most is the knowledge that, simply by thinking about problems of interest, he has been able to change the way people choose to live their lives and occupy their time.

*James Case writes from Baltimore, Maryland.*