Math Matters

Apply It.

The math behind... Catching Fish

Technical terms used:

Differential equation, logistic growth, density dependence, maximum sustainable yield

Uses and applications:

With increasing demand for fish around the world, a better understanding of what drives fish populations to increase and decrease in abundance is needed. Mathematical models are useful tools for understanding these population fluctuations. Analyzing these simple models can allow for fisheries to be harvested sustainably.

How it works:

Understanding the dynamics of fish populations seems like a daunting task. Fish grow, swim around, reproduce, get eaten, eat other organisms, and are captured by humans. We can reduce some of this complexity into basic mathematical descriptions. In the simplest setting, the number of fish in a population next year should equal the number of fish this year, plus births and minus deaths—this is a model for how the population behaves over time. This may sound complicated, but it is really just a matter of bookkeeping.

One mathematical tool for modeling fish populations is a differential equation. A differential equation keeps track of the population growth rate over time. One of the first models used in fisheries biology was the logistic growth equation. This is a differential equation that keeps track of the population growth rate and includes what biologists call density dependence. This simply means that when the population is large, we would expect the growth rate of the population to be reduced because of competition for resources or increased predation.

With the logistic growth equation we can begin exploring questions important to fisheries. For instance, what is the maximum number of fish we can sustainably catch? Or, in fisheries language, what is the maximum sustainable yield? With a bit of mathematics, the logistic growth model tells us that the maximum sustainable yield occurs when we harvest half of the largest population the environment can support. This result allows fishery managers to set limits on how many fish to harvest each year. To make models more realistic, we can add various factors to our logistic equation, such as multiple species, environmental factors, and spatial aspects. We can also model different types of management scenarios. For example, we can project future population sizes for scenarios where fishing is restricted to certain times of the year or with only certain types of fishing gear. These results help provide fishery managers with insight on how to best manage a given fish population.

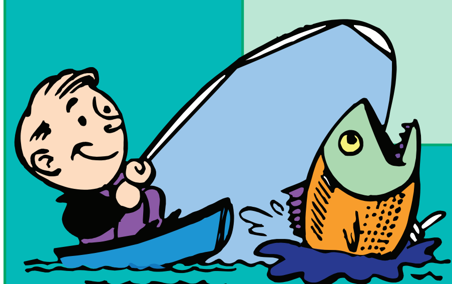
Interesting facts:

Many fisheries in Alaska were severely overfished in the first half of the 20th century. The state of Alaska was able to develop several sustainable fisheries by collecting more data, using this data in mathematical models, setting appropriate fishing limits, and enforcing fisheries rules. Alaska is a great example of how management actions informed by simple mathematical models can ensure sustainable harvest.

References:

B. Worm et al. Rebuilding global fisheries. Science, 325, 578-585 (2009).

R. Hilborn, C. J. Walters, Quantitative Fisheries Stock Assessment: Choice, Dynamics and Uncertainty (Springer, 1992).



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