Newsjournal of the Society for Industrial and Applied Mathematics sinews.siam.org

siam news

Volume 53/ Issue 3
April 2020

A Network Model of Immigration and Coexistence

By Yao-li Chuang, Tom Chou, and Maria R. D'Orsogna

In the summer of 2015, more than one million refugees from the Middle East, Central Asia, and Africa arrived in Europe via dangerous routes across the Mediterranean Sea and the Balkans [7]. German Chancellor Angela Merkel welcomed the newly arrived with an enthusiastic "Wir schaffen das"—"We can do this"—embodying the collective spirit of optimism that pervaded Europe at the time. The vast majority of migrants were fleeing civil wars, brutal dictatorships, or religious persecution; others were seeking better economic opportunities. Preferred destinations among the more prosperous nations included Germany, Sweden, and the U.K., whereas European law and geography placed most of the burden of processing asylum claims on border nations such as Italy, Greece, and Hungary, which were not prepared to cope with such unprecedented numbers of new arrivals.

Measures including the forced return of illegal migrants to Turkey in exchange for economic concessions attempted to stem the flow. Hungary closed its borders, and Italy eventually closed its ports. European lawmakers were unable to devise a clear burden-sharing system among member states; at the same time, refugees and smugglers quickly found and exploited new migrant routes as existing ones saw increased patrolling and border controls. Eventually, the perception of an unmanageable crisis touched the entire continent. Discontent among the general public grew, as did discussions on safety, integration, European identity, secularism, resource availability, and the role of non-governmental organizations. As a result, the issue of migration has dominated elections across Europe over the past few years, and nationalist parties have enjoyed large gains in many countries.

It is within this larger sociopolitical context that many migrants have settled into European cities, each with their own personal story of adaptation, hurdles, discoveries, kindness, and hostility from strangers. Outcomes have thus far been mixed; refugees have successfully integrated in many communities from Italy to Sweden, but in some cases there have been challenges and mistrust. A common observation is that newcomers who do not adapt well—either by circumstance, aversion from natives, lack of resources and/or motivation, etc.—tend to self-segregate and create insular communities [5]. While these enclaves provide immigrants with advantages and a sense of belonging, they may also prevent them from fully integrating into the larger society.

The fateful summer of 2015 presented a most daunting question: Is it possible to integrate vast numbers of asylum seekers in a way that is constructive for natives and migrants alike? This issue is also at the core of our recent mathematical modeling work, wherein we offer a quantitative setting for the study of immigration and coexistence [2]. We consider two communities—"hosts" (N_{h}) and "guests" (N_{g})—as nodes that interact on a social network, both seeking to improve their socioeconomic status. Each node i carries a time-dependent attitude x_i^t towards others and is assigned a utility function U_i^t that depends on its m_i^t connections. Over time, nodes adjust attitudes and reshape links to increase their utility; as a result, the network evolves towards either integration or segregation between hosts and guests. While the utility function follows game theoretic rules, attitudes are assumed to evolve

See Immigration on page 2

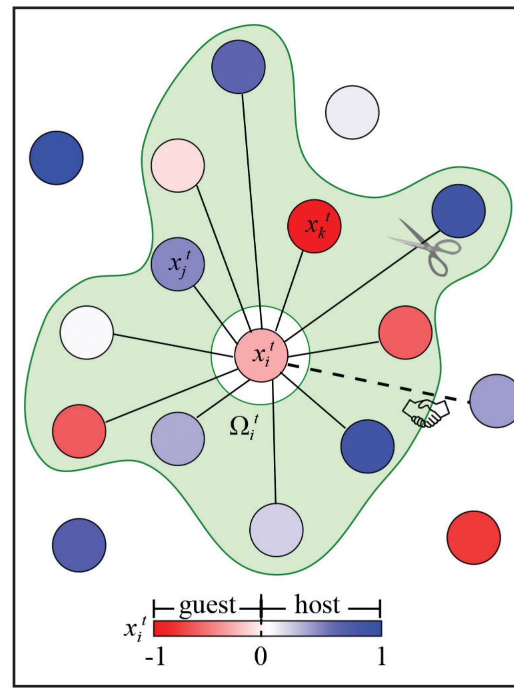

Figure 1. *Each node i is characterized by a variable attitude $-1 \leq x_i^t \leq 1$ at time t. Negative (red) values indicate guests and positive (blue) values represent hosts. The magnitude $|x_i^t|$ represents node i's degree of hostility towards members of the other group. All nodes j, k that are linked to node i represent the green-shaded social circle Ω_i^t of node i at time t. The utility U_i^t of node i depends on its attitude relative to that of its m_i^t connections. Nodes maximize their utility by adjusting their attitudes x_i^t and establishing or severing connections. Figure courtesy of Yao-li Chuang [2].*

The Mathematical Fight for Voting Rights

By Matthew R. Francis

State and local governments will redraw voting districts based on new information following completion of the 2020 U.S. Census. Ideally, this process ensures fair representation. In practice, however, districting often involves gerrymandering: the deliberate planning of districts to dilute the voting power of certain groups in favor of others, which violates the law.

Racial gerrymandering—drawing districts to limit the power of voters of color to select candidates they favor—is a particularly pernicious problem. Section 2 of the Voting Rights Act (VRA) of 1965 specifically prohibits this practice, but that has not stopped authorities from doing it anyway. "A number of court decisions have purposefully asked mathematicians, political scientists, and statisticians to use specific methods to try and understand racial gerrymandering," Matt Barreto, a professor of political science and Chicana/o studies at the University of California, Los Angeles, said.

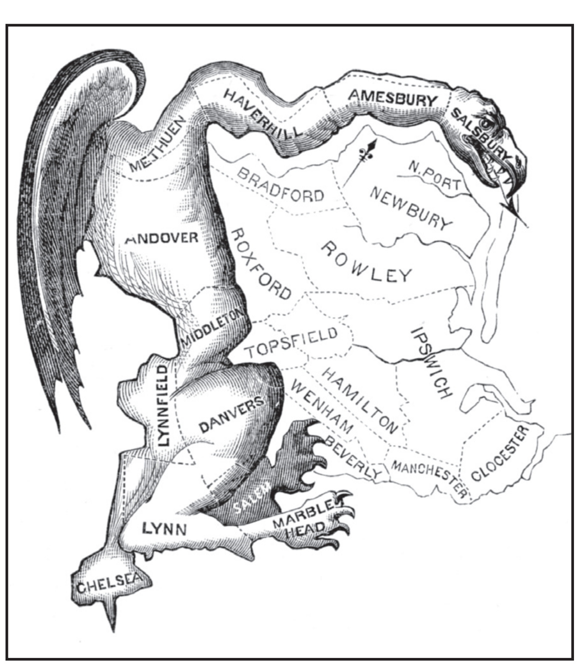

Figure 1. *Cartoonist Elkanah Tisdale's 1812 depiction of Massachusetts Governor Elbridge Gerry's partisan gerrymandering in favor of the Democratic-Republican Party. Public domain image.*

Barreto and his colleagues employ powerful statistical methods and draw on census and other public data to identify gerrymandered districts. Utilizing these tools, mathematicians can test proposed district maps or draw their own, designing them from the ground up to prevent voter dilution.

Since gerrymanderers use the same data to intentionally disenfranchise voters, the question is whether mathematical approaches alone are enough to fight the problem. Just as machine learning algorithms can "learn" racism from their training data,[1] studies show that the results of algorithmic districting can be as bad as deliberate gerrymandering [2]. To put it another way, can math solve problems it did not create?

"Previous efforts that used mathematics were not as accurate, and they did whitewash over some of the black and brown voters living in communities," Barreto said. "By going that extra step and purposefully trying to bring in accurate data on racial and ethnic minorities, we can go back to our trusted mathematical and statistical methods to make sure we're getting accurate counts of people."

Racial Polarization, Racial Gerrymandering

In 1812, cartoonist Elkanah Tisdale noticed that one of the districts created under Massachusetts Governor Elbridge Gerry looked like the mythical fire-monster salamander, so he dubbed it the "Gerry-Mander" (that arguably makes "gerrymandering" the most important legal term ever coined in a cartoon, which pleases me as a frequent comics writer). This original gerrymander is a prime example of partisan gerrymandering because it was created to favor the Democratic-Republican Party over the Federalists (see Figure 1).

Racial gerrymandering has garnered less attention than its partisan counterpart, though the two often go hand in hand. However, racial gerrymandering also happens in effective one-party regions, such as cities where the Democratic Party dominates local politics. In practice, testing for unethical districting involves looking for racially polarized voting patterns — places in which minority voters strongly prefer one candidate over another, but districts are drawn to favor white voter preferences. Chicago—with a history of just two elected African American mayors despite its large black population—is a classic example of this form of gerrymandering.

Consider an imaginary mayoral election with two candidates: Smith, who is preferred by white/Anglo voters, and Herrera, who is preferred by Latinx voters. The city is divided in a such way that Latinx voters never amount to more than 40 percent of the total population in any district, while white voters never comprise fewer than 50 percent — regardless of the city's total racial and ethnic makeup. Racial gerrymandering ensures that Smith always wins over Herrera and Latinx preferences are never represented, which is a violation of the VRA. Perhaps the districting scheme splits apart Latinx-majority neighborhoods and lumps the fragments with white-majority areas; a more equitable and representative division would keep those neighborhoods whole, possibly even allowing for Latinx-plurality districts.

The challenge for mathematicians involves reconstructing racial voting patterns without violating voter privacy, which is protected by law. Barreto and his collaborators use ecological inference (EI), a technique that infers individual behaviors from population-level datasets. Their EI methods involve an iterative Bayesian approach, utilizing publicly available data from petitions, voter records (which merely tabulate if a registered voter cast a ballot), and the census.

See Voting Rights on page 4

[1] https://sinews.siam.org/Details-Page/the-threat-of-ai-comes-from-inside-the-house

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siam news
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Immigration

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through opinion dynamics; the two inform each other in a synergistic way.

Attitudes x_i^t vary between $-1 \le x_i^t \le 0$ for guests and $0 \le x_i^t \le 1$ for hosts; the magnitude $|x_i^t|$ indicates the degree of hostility towards the other group. Thus, $x_i^t \to 0^{\pm}$ characterizes most receptive guests or most hospitable hosts, while $x_i^t = \pm 1$ represents the highest level of xenophobia (see Figure 1, on page 1). The utility U_i^t is given by a pairwise reward—to which each node j linked to i contributes—and by a cost function for maintaining m_i^t connections, such that

$$U_i^t = \sum_{j \in \Omega_i^t} A_{ij} \exp\Big(-\frac{(x_i^t - x_j^t)^2}{2\sigma}\Big) - \exp\Big(\frac{m_i^t}{\alpha}\Big).$$

Here, Ω_i^t is the set of nodes linked to i at time t, so that m_i^t is given by its cardinality: $m_i^t = |\Omega_i^t|$. The pairwise reward depends on the attitude difference $|x_i^t - x_j^t|$ between nodes i and j; a diminishing attitude difference correlates with an increasingly high reward. Therefore, if both i and j are hosts or immigrants, the reward is maximized for $x_i^t = x_j^t$, leading to consensus within the group. But if i and j are from different groups, the reward is optimized only if both nodes adopt more cooperative attitudes: $x_i^t \to 0^-$ and $x_j^t \to 0^+$. The parameter σ controls the reward's sensitivity to attitude differences, the amplitude A_{ij} specifies the maximum possible reward, and the scaling coefficient α governs the cost of maintaining active links. Other models have considered residential segregation between two ethnic groups, with nodes seeking "friendly" neighbors with whom to connect. The most famous of these is the seminal Schelling model of segregation [3, 4, 6]. Our utility function U_i^t adds socioeconomic status as a decision-making factor in the link establishment process.

The dynamics unfold so that connectivities are modified at each time step to maximize utility. Attitudes are changed by imitation, so that

$$x_i^{t+1} = \begin{cases} \min\left(0, x_i^t + \dfrac{x_j^t - x_i^t}{\kappa}\right) & \text{for guests,} \\ \max\left(0, x_i^t + \dfrac{x_j^t - x_i^t}{\kappa}\right) & \text{for hosts,} \end{cases}$$

where κ governs attitude adjustment. Specifically, the timescale for guest cultural adjustment τ_g is given by κ and scaled by the probability of a guest being paired with a host N_h/N, so that $\tau_g \sim \kappa N/N_h$. Similarly, the host cultural adjustment timescale $\tau_h \sim \kappa N/N_g$. Since $N_h \gg N_g$, also $\tau_h \gg \tau_g$; adjustment times for hosts are longer than for guests. These cultural adjustment timescales are compared with the unitary timescale for social link remodeling. Finally, initial conditions represent the way in which guests are originally settled in the community. One extreme case involves a perfectly executed welcoming program that provides refugees with sufficient social ties

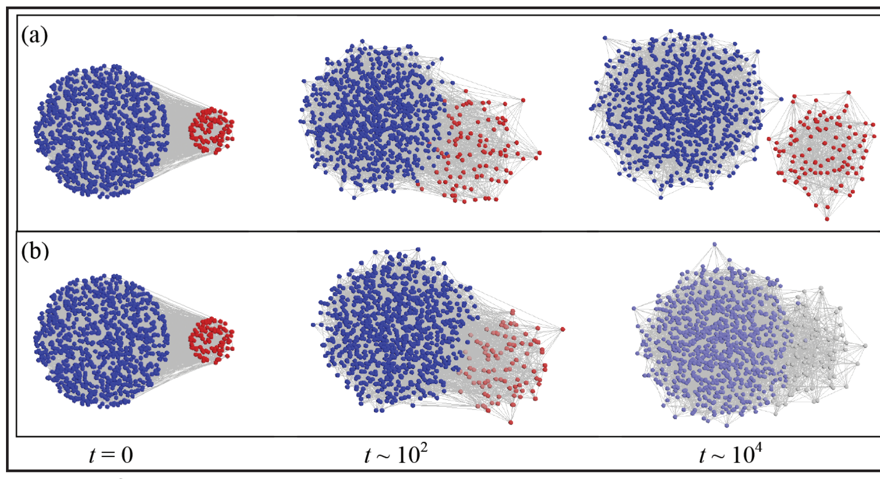

Figure 2. *Simulated dynamics leading to complete segregation (2a) and integration (2b) between guest (red) and host (blue) populations. Initial conditions are randomly connected guest and host nodes with attitudes $x^0_{i,\text{guest}} = -1$ and $x^0_{i,\text{host}} = 1$. Panels 2a and 2b differ only for κ, the attitude adjustment timescale, with $\kappa = 1000$ in 2a, where segregated clusters emerge, and $\kappa = 100$ in 2b, where a connected host-guest cluster arises over time. Figure courtesy of Yao-li Chuang [2].*

to hosts, and where all nodes are randomly connected — regardless of attitudes and utilities. The other extreme case is that of guests who arrive in a completely foreign environment with nonexistent initial resources. Hosts are naturally connected to one another in their own state of equilibrium, and guests are introduced without any links to hosts or each other.

Figure 2 depicts two representative steady-state outcomes. In Figure 2a, hosts and guests segregate and maintain highly hostile attitudes. Any initial cross-group utilities yield low rewards that do not increase over time, so that all ties between hosts and guests are eventually severed. Enclaves emerge when the two separate communities adopt uniform but differing attitudes x_i. In Figure 2b, all nodes develop more cooperative attitudes that increase cross-group rewards, so that hosts and guests remain mixed. Eventually, $x_i^t \to 0$ on all nodes. For both scenarios, $|x_i^t - x_j^t| \to 0$ at steady state, but to which configuration society converges depends on parameter choices and initial conditions.

We find that the main predictor of integration versus segregation is the magnitude of the τ_g, τ_h timescales relative to the unitary network remodeling time. In the case of slow cultural adjustment, immigrant and host communities tend to segregate as accumulation of socioeconomic wealth occurs more efficiently through insular, in-group connections. Conversely, fast cultural adjustment enables the establishment and sustenance of cross-cultural bridges, allowing different groups to reach consensus and maintain active cooperation. This is shown in Figures 2a and 2b, where the only difference is the κ parameter that drives τ_g, τ_h. We also find that a high guest-to-host ratio N_g/N_h increases the likelihood of in-group connections and reduces communication between immigrant and host populations.

One possible approach to avoid segregation is the promotion of cross-group interactions via government incentives, or if newcomers carry or acquire desired skill sets, for example. Note that cultural adjustment does not necessarily mean that either side must abandon their identity; rather, we find that different groups must adopt tolerant attitudes towards one another, engaging in rapport building and acceptance to bridge differences and promote integration [1]. This is the long-term challenge for the future.

Acknowledgments: This work was made possible by support from grant W1911NF-16-1-0165.

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Yao-li Chuang is a former mathematics researcher at the California State University, Northridge (CSUN) and a computational medicine postdoctoral researcher at the University of California, Los Angeles (UCLA). He specializes in dynamical systems, cancer research, system biology, and social sciences. Tom Chou is a professor in the Departments of Computational Medicine and Mathematics at UCLA. His scientific interests include statistical physics, soft matter physics, immunology, hematopoiesis, and computational psychiatry. Maria R. D'Orsogna is a professor of mathematics at CSUN and associate director of the Institute for Pure and Applied Mathematics at UCLA. She works on mathematical modeling of biological, behavioral, and social systems.

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SIAM News Transition

Karthika Swamy Cohen, who oversaw *SIAM News* as managing editor since July 2015, left SIAM last month for a new position. We are grateful for her contributions to SIAM and wish her the best of luck in her future endeavors.

Lina Sorg, who served as the associate editor of *SIAM News* since October 2015, has taken over as managing editor.

Envisioning Tomorrow's Earth: Reflections on the 2020 AAAS Annual Meeting

By Hans Kaper

Attendees of the American Association for the Advancement of Science (AAAS) Annual Meeting are like kids in a candy store: confronted with many choices, unexpected delights, and an ongoing tension between instant gratification and long-term perspective. I have been participating in these meetings since 2018, and I must say that I find them more fascinating with each passing year. Yes, I enjoy and would regret missing the professional meetings of SIAM, the American Geophysical Union, and other societies to which I belong. But AAAS meetings are different. They offer guests a chance to look beyond their silos and provide a unique opportunity to visualize mathematics as part of the larger scientific enterprise.

The theme of the 2020 meeting, which took place this February in Seattle, Wash., was "Envisioning Tomorrow's Earth." What better occasion to reflect on the ways in which science and technology respond to new challenges from both the natural and built world, and conversely, to see how the challenges of today's "real world" can inspire novel mathematics? This theme also coincided with the objectives of the SIAM Activity Group on Mathematics of Planet Earth.[1]

The AAAS is an umbrella organization that represents many sciences. It is historically dominated by the biological and chemical sciences, and a cursory look at *Science*—their flagship magazine—offers ample evidence of this dominance. Rarely does it feature an article explaining a recent result in mathematics, either core or applied. But mathematics and statistics have a seat at the table; it is simply difficult to convince the powers that be of mathematical sciences' relevance to many discoveries in other disciplines. Yet we must keep trying, as it is better to be heard than forgotten.

The 2020 AAAS Annual Meeting had two substantive scientific sessions that focused on mathematical contributions to the public good. One session, organized by Karen Saxe (American Mathematical Society), examined gerrymandering and racial fairness and featured talks by Matt Barreto (University of California, Los Angeles), Jonathan Mattingly (Duke University), and Moon Duchin (Tufts University).[2] The other session, organized by Christiane Rousseau (University of Montreal) and Fred Roberts (Rutgers University), focused on resilience in the digital age, with talks by Amy Luers (Future Earth), Hans Kaper (Georgetown University), and Wayne Getz (University of California, Berkeley). Both sessions were well attended and boasted stimulating question-and-answer dialogues.

Not only does the AAAS Annual Meeting draw participants from academia, government, and the private sector, it also attracts communicators, the press, and numerous think tanks. In fact, the meeting's major thrust is to expose new ideas to a broader community, inform decision-makers, and connect science and society. Not all sessions at the 2020 gathering focused on science; panels and town hall events were devoted to communication issues, workshops highlighted career opportunities, and plenary talks addressed "big picture" ideas.

Two plenary sessions were of particular interest. Bill Gates discussed science policy's influence on public health (and vice versa), which is where the Bill & Melinda Gates Foundation has made significant contributions. And Krysta Svore (Microsoft Corporation) offered an insider's view on the state of the art of quantum information science at Microsoft. In addition, multiple sessions explored Earth's climate system and its many subsystems (ocean, atmosphere, carbon cycle, water cycle, etc.), issues of sustainability (food supply, manufacturing, etc.), extreme events, artificial intelligence, machine learning, and public health.

Bill Gates of the Bill & Melinda Gates Foundation addresses the crowd during his plenary lecture at the 2020 American Association for the Advancement of Science (AAAS) Annual Meeting, which took place this February in Seattle, Wash. Photo courtesy of Robb Cohen Photography & Video.

Most of us realize that a wide gap exists between the scientific research enterprise and the public's appreciation of science, and recognize that it is important to make sustained efforts to bridge this gap. However, determining how to best engage the public and develop meaningful communication channels is not always clear. The social sciences indicate that "show and tell" is not going to close the gap; we need to be more discerning and tailor our message specifically to our audience. We must pay heed to examples of successful communication techniques—many of which were presented at the 2020 AAAS Annual Meeting—so our message is not ignored and lost. To that end, the AAAS has made sizeable investments in training communicators; this is a long-term effort, but the payoff could be significant.

The AAAS Annual Meeting is a great place to learn about mathematical sciences' role in the broad panorama of physical and life sciences. The theme of next year's meeting is "Understanding Dynamic Ecosystems."[3] What better opportunity to showcase the role of applied mathematics? Let's make an effort to highlight our passion for mathematics and demonstrate how we can create a win-win situation for science and society.

Hans Kaper, founding chair of the SIAM Activity Group on Mathematics of Planet Earth and editor-in-chief of SIAM News, is affiliate faculty in the Department of Mathematics and Statistics at Georgetown University.

[1] https://www.siam.org/membership/activity-groups/detail/mathematics-of-planet-earth

[2] See "The Mathematical Fight for Voting Rights" on page 1 for a detailed analysis of Barreto and Duchin's work.

[3] https://meetings.aaas.org/program/meeting-theme/

Symmetry and Polar Decomposition by Mechanics

MATHEMATICAL CURIOSITIES

By Mark Levi

There are several mechanical or geometrical interpretations of the symmetry of a matrix; I would like to describe one that recently occurred to me. It is likely that others have thought of it before, though I did not do a literature search to confirm.

Let us interpret the square $n\times n$ matrix A as a frame of its column vectors $\boldsymbol{a}_k \in \mathbb{R}^n$, thought of as n rigid rods that are welded together and pivot on the origin O. As illustrated in Figure 1, let us connect the tip of the kth column/rod $\boldsymbol{a}_k$ to the tip of the coordinate unit vector $\boldsymbol{e}_k$ by a Hookean spring, i.e., the spring whose tension is directly proportional to its length. All springs have the same Hooke's constant.

Claim 1: *An $n\times n$ $(n\geq 2)$ matrix is symmetric if and only if the aforementioned mechanical system is in equilibrium.*

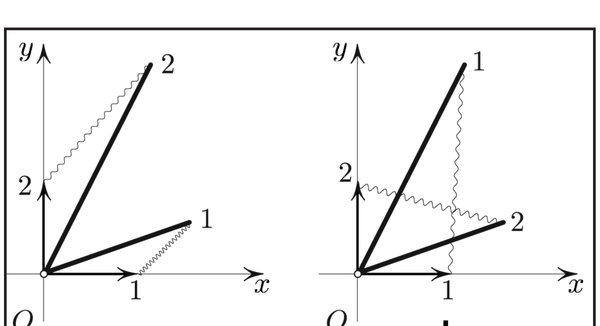

Figure 1. *The kth column vector $\boldsymbol{a}_k$ is connected to the kth coordinate unit vector $\boldsymbol{e}_k$.* **1a.** $\det A>0$. **1b.** $\det A<0$.

Indeed, in an equilibrium state the torque around O in any ij-coordinate plane vanishes; this amounts to $a_{ij}-a_{ji}=0$.

Claim 2: *A matrix is positive-definite if and only if the frame is right-handed and in a stable equilibrium.*

The proof of this claim in dimension 2 is almost purely geometrical. Figure 2 illustrates stable equilibria with the positively-oriented frames. No combination of quadrants other than those in Figure 2 can occur.

It is clear that both eigenvalues are positive in either case. Indeed, with QN denoting the Nth quadrant, we have (see Figure 3)

$$A(Q1)\subset Q1 \quad\text{and}\quad A(Q2)\supset Q2$$

for matrix A in Figure 2a. We use the fact that the matrix maps the basis $\boldsymbol{e}_1,\boldsymbol{e}_2$ to the frame $\boldsymbol{a}_1,\boldsymbol{a}_2$. According to a fixed point theorem, $Q1$ and $Q2$ both contain eigenvectors with positive eigenvalues. We treat the matrix in Figure 2b similarly, with the same conclusion of two positive eigendirections. By contrast, Figure 1b depicts a negatively-oriented frame, and we have $A(Q2)\supset Q4=-Q2$ for the corresponding matrix A; this implies the existence of a negative eigenvalue.

Connection to the Toeplitz Norm

Potential energy of the system in Figure 1 is a mechanical interpretation of the Toeplitz norm $\|A-I\|^2$, up to a constant factor that depends on Hooke's constant. We recall that the Toeplitz norm of a square matrix X is defined as the root of the sum of its elements' squares: $\|X\|^2=\mathrm{tr}(X^TX)$.

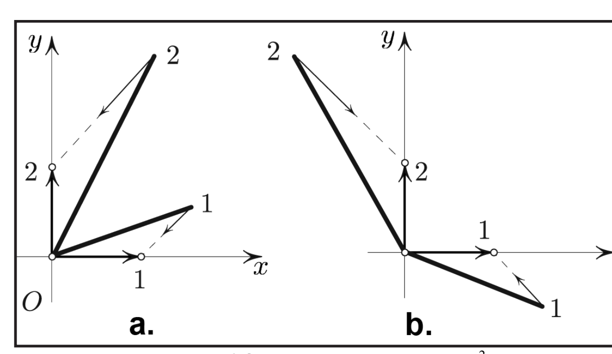

Figure 2. *Illustration of Sylvester's criterion in $\mathbb{R}^2$.*

Polar Decomposition

Given an arbitrary $n\times n$ matrix A that is not necessarily symmetric, let us connect the frame of columns to the springs—as in Figure 1—and then release. After undergoing a rotation $R\in SO(n)$, the frame will settle (assuming some damping) to the orientation of least potential energy; this new frame corresponds to a symmetric matrix S. In short, $S=RA$, i.e., $A=R^{-1}S$, which almost amounts to the polar decomposition of A. The "almost" is due to the fact that S need not be positive definite, as Figure 1b illustrates; one must first compose A with an extra reflection if $\det A<0$, and then carry out the above operation.

Sylvester's Criterion

Sylvester's criterion is a necessary and sufficient condition for the positivity of a symmetric matrix that requires all principal minors to be positive. Minimality of the Toeplitz norm $\|A-I\|^2$ for positive definite 2×2 matrices makes Sylvester's criterion visually transparent. For example, if the Toeplitz norm is minimal for a positively-oriented frame, only two cases of the equilibria shown in Figure 2 can occur; $a_{11}>0$ in both of these cases. Finding a purely visual proof of Sylvester's criterion for $n=3$ in a similar spirit is left as a challenge.

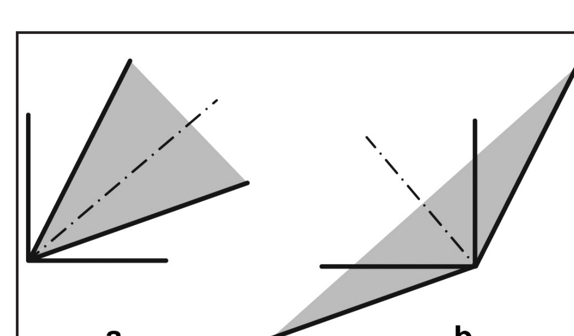

Figure 3. *Two eigendirections (dotted) with positive eigenvalues for the matrix in Figure 2a. Shaded sectors are $A(Q2)$* (**3a**) *and $A(Q1)$* (**3b**).

The figures in this article were provided by the author.

Mark Levi (levi@math.psu.edu) is a professor of mathematics at the Pennsylvania State University.

Voting Rights

Continued from page 1

The census is the only public record that regularly includes racial information. However, it is only updated every 10 years, and citizens may relocate during that period and vote in more than one election in a given year. To infer the race of voters based on registration information, Barreto's group employs a method called Bayesian Improved Surname Geocoding (BISG). This technique uses geographic information to assign a probability that a given surname belongs to one of the major racial/ethnic groups in America—white, black, Asian, Latinx, or other. For instance, my surname "Francis" is more likely to be shared by white people in Iowa but probably belongs to African Americans in New Orleans.

Barreto and his colleagues tested the BISG method using a dataset wherein people self-identified their race. By iteratively improving their Bayesian priors, their model now identifies the race of a particular voter with between 93 and 97 percent accuracy.

In the simplest case—like Herrera v. Smith—a district has two candidates and two distinct racial/ethnic groups (Latinx and Anglo/white). For every precinct i in the district, one must estimate the fraction of each group (β_L^i, β_W^i) that voted for Herrera. The known quantities are the fraction of voters who cast a vote (T_H^i) for Herrera and the Latinx fraction of total voters who participated in this election (X_L^i), estimated using BISG and generally assumed to be independent of the β parameters. Because these quantities are all fractions, the complementary values for white participation is $X_W^i = 1 - X_L^i$ and the vote fraction for Smith is $T_S^i = 1 - T_H^i$.

Unfortunately, even the simplest system does not allow exact solutions, so Harvard University political scientist Gary King and his colleagues proposed the use of tomography graphs by analogy with medical imaging procedures, where one must infer three-dimensional structures from X-rays that pass through the human body [3]. Each precinct is represented by a line that accounts for all possible (β_L^i, β_W^i) parameter values (as given by the linear equation), with the slope and intersect involving known quantities:

$$\beta_W^i = \left(\frac{T_H^i}{1 - X_L^i}\right) - \left(\frac{X_L^i}{1 - X_L^i}\right)\beta_L^i.$$

If the data is clear-cut, the lines on the tomography graph will intersect in a well-defined region (see Figure 2). In this case, a bivariate normal distribution (restricted to $\beta_R^i \in [0,1]$ for $R = \{L, W\}$) yields the likelihood function of the best aggregate values for (β_L, β_W). In contrast, less well-defined data require more complicated analyses.

While this two-candidate, two-race EI model is adequate for some parts of the country, many districts necessitate extended forms of the model. One extension is iterative: separating one racial/ethnic group or candidate at a time and comparing it to the others in aggregate, repeating this process until all groups have been analyzed. Another expansion is the $R \times C$ model, which combines all parameters into a matrix β_{RC}^i, with row R tabulating race/ethnicity and column C tabulating candidate. Barreto and his collaborators developed eiCompare[2]—a freely-available package for the R statistical programming language—to simultaneously calculate the different models' parameters, compare their outcomes, amd provide the best possible EI estimates in real-world elections.

"We're not trying to prove that there's always racially polarized voting," Barreto said. "In some communities there is not, and the data will show us that."

Accounting for Fairness

The U.S. Supreme Court laid out three criteria for demonstrating racial gerrymandering in their 1986 decision on *Thornburg v. Gingles*, including rules for legally proving racial polarization [4]. These "Gingles prongs" are as follows:

1. If there is a minority racial/ethnic group large enough to be a majority in a district
2. If this group votes in cohesive ways, tending to have preferred candidates as a bloc
3. If the white-preferred candidates are almost always able to defeat the minority-preferred candidates despite the first two criteria, then racial gerrymandering is present.

Any redesigned district must therefore account for these conditions to comply with the VRA. To ensure fairness, the court also instructed legislators to consult professional mathematicians and statisticians.

Tufts University mathematician Moon Duchin and her colleagues pair analysis techniques like EI with high-level geometric methods to identify where communities or individual neighborhoods define voting blocs, and generate alternative maps to eliminate gerrymandering. Duchin founded the Metric Geometry and Gerrymandering Group,[3] which provides publicly-available tools to help identify better ways of creating districts. One such tool is Districtr,[4] an interactive online Java program for drawing state-level congressional districts.

But racial gerrymandering is not the only problem that voters of color face. Polling station closures in minority-majority districts, poor polling locations (which are often exacerbated by district shape), arbitrary removal of registered voters, and inclusion of prisons comprise other issues that disproportionately affect minority voters. For instance, Duchin's group was actively involved in the referendum when residents of Lowell, Mass., changed their polling system to ranked-choice voting. This shift provided a parallel way to identify community issues and racial polarization.

Barreto, Duchin, and like-minded researchers also use mathematical methods to break down voting patterns beyond the stereotypical white/African American dichotomy that often dominates national discourse. "There are racial power dynamics inherent in these political systems, which are also sometimes inherent in social sciences and even in math," Barreto said. "We need to make sure that there is a perspective of black and brown scholars who are also very sophisticated statisticians that care about social policy."

Matt Barreto and Moon Duchin presented their work during a session entitled "Gerrymandering and Mathematics: Redistricting the Nation" at the 2020 American Association for the Advancement of Science Annual Meeting, which took place this February in Seattle, Wash.

[2] https://cran.r-project.org/package=eiCompare
[3] http://mggg.org
[4] http://districtr.org

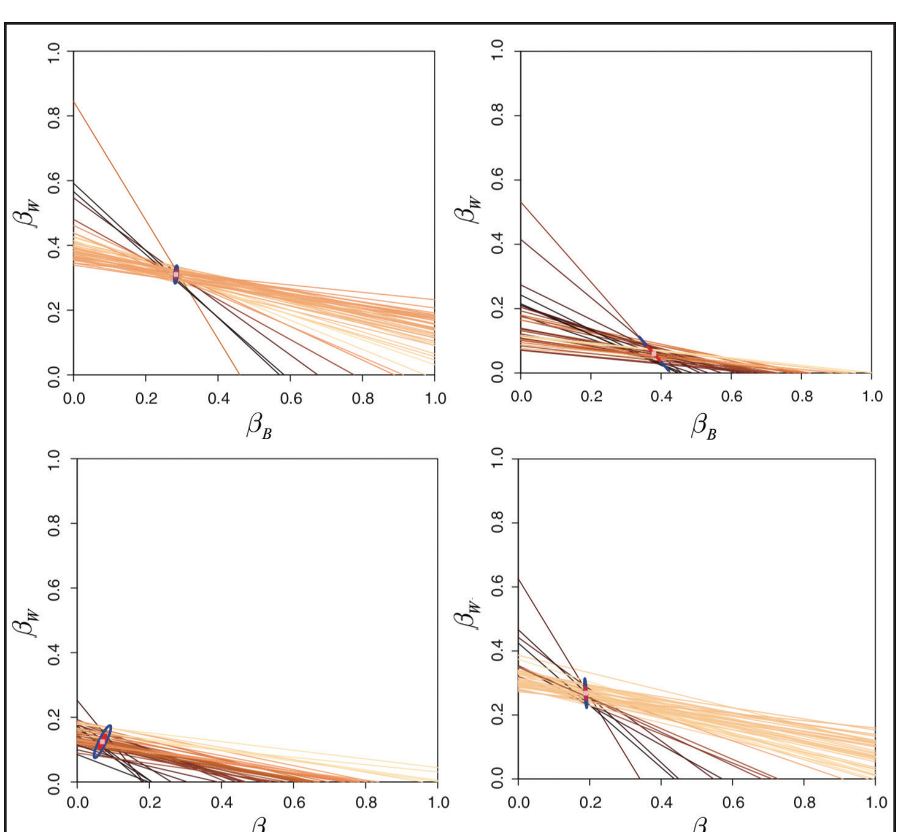

Figure 2. *Each line on these tomography graphs represents all possible fractions of black and white voters (β_B, β_W) who voted for a particular candidate in each precinct. The region where these lines intersect indicates these parameters' "true" values for the entire district. The likelihood function for the parameters is sharply peaked where the overlap region is small, as in this example. Figure adapted from [1].*

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Matthew R. Francis is a physicist, science writer, public speaker, educator, and frequent wearer of jaunty hats. His website is BowlerHatScience.org.

Recognizing the 2020 JPBM Communications Award Recipients

Each year, the Joint Policy Board for Mathematics (JPBM) presents the JPBM Communications Award to reward and encourage communicators who bring mathematical ideas and information to non-mathematical audiences on a sustained basis. Established in 1988, the prize is a collaborative effort between SIAM, the American Mathematical Society, the American Statistical Association, and the Mathematical Association of America (MAA). Both mathematicians and non-mathematicians are eligible for up to two awards of $2,000 annually.

The recipients of the 2020 JPBM Communications Award are Chris Budd and James Tanton. As per the citation, Budd received the prize "for his passionate popularization of mathematics. Through his positions at Gresham College, the Royal Institution of Great Britain, and the Institute of Mathematics and its Applications, his books, and his work with science festivals and schools, he inspires audiences of all ages." Tanton was acknowledged "for global leadership in high school mathematics instruction. Through his "G'Day Math!" online courses, MAA Curriculum Inspirations, numerous textbooks, and the Global Math Project, he is inspiring millions to learn and teach math in wonderful new ways."

Budd and Tanton received their awards during a ceremony at the 2020 Joint Mathematics Meetings, which took place in Denver, Colo., this January.

Read more about the prize and this year's awardees online.[1]

[1] https://sinews.siam.org/Details-Page/christopher-budd-and-james-tanton-to-receive-2020-jpbm-communications-awards

Chris Budd (right) accepts the 2020 Joint Policy Board for Mathematics (JPBM) Communications Award from Mathematical Association of America (MAA) President Michael Dorff. Photo courtesy of Kate Awtrey, Atlanta Convention Photography.

Mathematical Association of America (MAA) President Michael Dorff (left) presents the 2020 Joint Policy Board for Mathematics (JPBM) Communications Award to James Tanton. Photo courtesy of Kate Awtrey, Atlanta Convention Photography.

Mean Field Game Theory: A Tractable Methodology for Large Population Problems

By Peter E. Caines

Mean field game (MFG) theory finds applications in areas such as demand management for domestic users on electrical power grids, algorithmic trading in competitive markets, crowd dynamics, and vaccination strategies. This is because MFG methodology is formulated in terms of tractable infinite population approximations to these problems, each of which involves thousands or even millions of agents, making explicit solutions impossible. A similar situation arises in cell phone networks. Overlapping signals in code division multiple access (CDMA) cell phones can cause poor call quality since such interference may degrade individual signal-to-noise ratios and thus reduce call performance.

Conventional power control algorithms in mobile devices use gradient-type algorithms with bounded step size for transmitted power, which we can approximately represent with the so-called adjustment model for $0 \le t \le T$:

$$dp^i = u_p^i dt + \sigma_p^i dw_p^i, \quad |u_p^i| \le u_{\max},$$

where $1 \le i \le N$, N indicates the number of users, and w_p^i denotes a standard Wiener process. Furthermore, the log-normal model is standard for time-varying attenuation, wherein $e^{\beta^i(t)}$ represents channel gain for the ith agent with respect to the base station at instant t. The product $e^{\beta^i} \times p^i$, where the channel state $\beta(t)$ evolves according to the attenuation dynamics described by a stable uncontrolled stochastic differential equation (SDE), models received power at the base station from the ith agent.

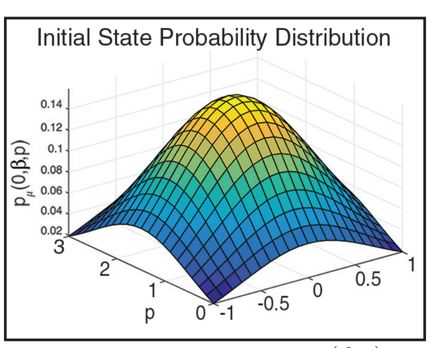

Figure 1. *Initial system mean field $\mu_t(\beta,p)$ density $t=0.00$. Figure courtesy of Mohamad Aziz.*

Using the standard instantaneous quality-of-service ratio for a generic agent $\mathcal{A}_i$ [1], we can model the finite population loss (or performance) function for this agent by

$$J_i(\beta_i,p_i) = E\left[\int_0^T l_i(\beta,p)dt\right] \equiv$$

$$E\left[\int_0^T \left\{ -\frac{e^{\beta^i}p^i}{\frac{1}{N}\Sigma_{j=1}^N p^j e^{\beta^j}+\eta} + p^i \right\} dt\right],$$

where each agent's running cost $l_i(\beta,p)$ involves both its individual transmitted power and its signal-to-noise ratio.

Applying a centralized stochastic control to minimize the sum of the loss of functions $J_i(\beta_i,p_i)$, $1 \le i \le N$, each of which is contingent upon all agent states, is an intractable problem. Furthermore, a decentralized solution in the form of a finite population dynamical game, where every agent attempts to minimize its individual loss $J_i(\beta,p)$, $1 \le i \le N$, is even more complex than the control problem.

While a precise game-theoretic solution might not be possible due to huge complexity, we may employ the classical strategy of passing to an infinite limit, as in the celebrated Boltzmann equation of statistical mechanics and the Navier-Stokes equation of fluid mechanics [5]. In this spirit, MFG theory analyses the existence of Nash equilibria for competitive systems with very large numbers of players by exploiting the relationship between the finite and corresponding infinite limit population problems. A key entity in this formulation is the *mean field*: the probability distribution of the state of a generic agent in the infinite population.

Mean Field Game Theory

The following set of controlled SDEs provides a general framework for MFG theory. For each agent $\mathcal{A}_i$ where $1 \le i \le N$ —with state x_i, control u_i, and Wiener process disturbance w_i (assumed scalar and uniform for simplicity)—this framework incorporates dynamic coupling of the form

$$dx_i(t) = \frac{1}{N}\sum_{j=1}^N f(t,x_i(t),u_i(t),x_j(t))dt + \sigma dw_i(t).$$

In the infinite population limit and for sufficiently smooth f, this yields the controlled McKean-Vlasov equation

$$dx_i(t) = f[x_i(t),u_i(t),\mu_t]dt+\sigma dw_i(t)$$

$$:= \int_R f(x_i(t),u_i(t),z)\mu_t(dz)dt + \sigma dw_i(t),$$

whose solution, given initial conditions, is the process-distribution pair (x,μ). We may similarly pass to the limit in each agent $\mathcal{A}_i$'s performance function, with running costs $l(t,x_i(t),u_i(t),x_j(t))$ averaged over the agents $\mathcal{A}_j$, $1 \le j \le N$; this produces a performance function $J(u_i,\mu)$ with running cost $l[x_i(t),u_i(t),\mu_t]$.

Assuming that the limits exist, we obtain equations for a game-theoretic Nash equilibrium in the infinite population limit. This takes the form of an optimal stochastic control problem between each dynamical agent and the dynamically-evolving infinite population mean field. Specifically, the Hamilton-Jacobi-Bellman (HJB) and Fokker-Planck-Kolmogorov (FPK) MFG equations are as follows:

$$-\frac{\partial V(t,x)}{\partial t} = \inf_{u\in U}\left\{ f[x,u,\mu_t]\frac{\partial V(t,x)}{\partial x} + l[x,u,\mu_t]\right\} + \frac{\sigma^2}{2}\frac{\partial^2 V(t,x)}{\partial x^2}$$

$$\frac{\partial p_\mu(t,x)}{\partial t} = -\frac{\partial\{f[x,u^\circ(t,x),\mu_t]p_\mu(t,x)\}}{\partial x} + \frac{\sigma^2}{2}\frac{\partial^2 p_\mu(t,x)}{\partial x^2} \qquad (t,x)\in[0,T]\times\mathbb{R},$$

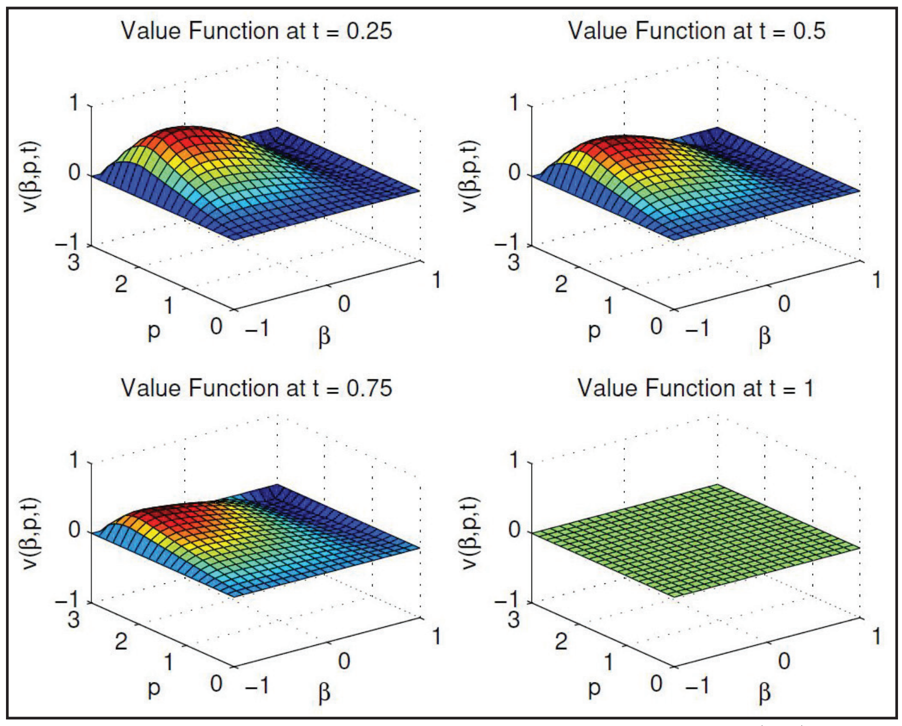

Figure 3. *Evolution of the optimal cost-to-go function from the system state (β,p) over the time interval* [0,1]. *Figure courtesy of Mohamad Aziz.*

plus terminal and initial conditions respectively, where $p_\mu(t,\cdot)$ is the density (assumed to exist) of the linking mean field measure μ_t. And $\varphi(t,x,\mu_t)$ shall denote the infimizer in the HJB equation. The (t,x,μ_t)-dependent optimal control $u^\circ(t,x) = \varphi(t,x,\mu_t)$ is consequently the game-theoretic best response strategy for the generic individual agent with respect to its performance function.

MFG theory was introduced in [6-10] with existence and uniqueness results established in [2, 6, 7], while the related notion of oblivious equilibrium for Markov decision processes appeared in [11]. The solution of the infinite population MFG equations is often tractable (as shown by the examples in this article), whereas the corresponding large population game problem is usually intractable. Consequently, the epsilon-Nash approximation results—for the error incurred when MFG solutions are employed as strategies in the finite population setting—are of theoretical and practical significance [6, 7].

The Code Division Multiple Access Problem

We recall that the mean field in the CDMA problem consists of the distribution of transmitted power and channel attenuation $\mu_t(\beta,p)$ for a generic agent. Beginning with the initial mean field $\mu_0(\beta,p)$ for an infinite population (see Figure 1), the solution to the FPK equation (see Figure 2) depicts the evolution of the equilibrium mean field $\mu_t(\beta,p)$ at four different instants. Figure 3 portrays the evolving value function $V(\beta,p,t)$ that solves the HJB equation, which terminates in the value 0 for all (β,p). For a simulation involving 400 agents, Figure 4 (on page 6) shows a typical agent's sample paths for the respective values of its transmitted power p, value function V, channel attenuation β, and control function u. Figure 2 and the simulation in Figure 4 indicate that the uncontrolled β process—which has stable dynamics—converges to a stationary Gaussian distribution. Figure 3 displays the evolution of the Nash equilibrium of the infinite population, as given by the value function generated by the MFG HJB equation, while Figure 4 (on page 6) depicts the value function over the interval [0,1] for a typical agent in the simulation.

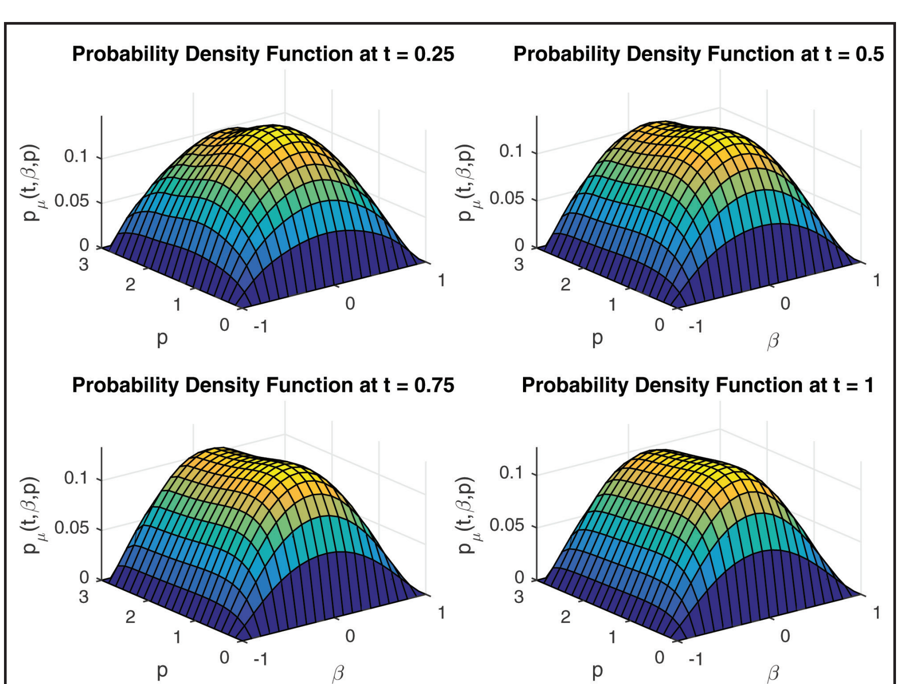

Figure 2. *Evolution of the system state (β,p) mean field density over the time interval* [0,1]. *Figure courtesy of Mohamad Aziz.*

An Optimal Execution MFG Problem

In standard versions of the optimal execution problem, models depict financial traders as balancing price risk from trading slowly with market instability and price impact caused by trading quickly in order to finally maximize their expected wealth.

It is assumed in [4] that a mean field effect of the trading rate of a population of high-frequency traders (HFTs) linearly enters the dynamical equations of a generic minor liquidator trader, yielding

$$dF_i(t) = \Big(\lambda_0\nu_0(t) + \frac{\lambda}{N}\sum_{i=1}^N \nu_i(t)\Big)dt + \sigma dw_i^F(t),$$

$$dZ_i(t) = -S_i(t)dQ_i(t).$$

$Q_i(t)$, $\nu_i(t)$, $S_i(t)$ satisfy first-order linear SDEs (not displayed here) in state and control variables $Q_i(t)$, $\nu_i(t)$, $F_i(t)$, and $u_i(t)$ respectively, where Q_i denotes inventory, ν_i is trading rate, u_i is trading acceleration, F_i denotes fundamental asset price, λ_0, λ_i denote what is called permanent impact, σ is volatility, S_i is execution price, Z_i is cash process, and w_i^F is a Wiener process. The performance function $J_i(\cdot)$ (not fully displayed here), to be minimized by an HFT that aims to liquidate $\mathcal{N}_i$ shares during the interval $[0,T]$, is defined so that the trader tracks a fraction of the market's average selling rate $\nu^N = \frac{1}{N}\Sigma_{i=1}^N \nu_i$ by including the terms $\big(\nu_i(T) - \rho_i \nu^N(T)\big)^2$ and $\int_0^T \rho_i\big(\nu_i(s) - \rho_i\nu^N(s)\big)^2 ds$ in $J_i(u_i,\mu)$. We assume analogous dynamics and performance functions for an acquirer HFT and a single major trader.

We can solve the associated major-minor linear quadratic Gaussian MFG equations in the complete and partial observation

See **Mean Field Game Theory** *on page 6*

Mean Field Game Theory

Continued from page 5

cases. For the latter, the separation principle of stochastic control provides a solution. This yields infinite population minor agent MFG best-response strategies in the form of linear feedback control laws that employ Kalman filter-estimated values for the agent's own state x_i, the major agent's state x_0, and the mean field $\bar{x}$. We hence obtain the following form of a minor agent's best response strategy:

$$\hat{u}_i^*(t) = -R^{-1}\mathbb{B}^T[\Pi\Big(\hat{x}^T_{i|\mathcal{F}_i^y}(t), \hat{x}^T_{0|\mathcal{F}_i^y}(t), \hat{\bar{x}}^T_{|\mathcal{F}_i^y}(t)\Big)^T + s(t)].$$

Employing the MFG solution, Figure 5 shows the corresponding trajectory of a single stochastic agent in an infinite population.

Concluding Thoughts

In this article, we have introduced the basic notions of MFG theory as well as illustrative examples involving cell phone energy management and optimal execution in finance. Other applications of MFG theory include nonlinear control system state estimation, the macroeconomics of growth, systemic risk modeling in banking, optimization of electric vehicle populations in grid charging and battery usage, and domestic electricity demand management on the grid. A recent foundational two-volume MFG monograph also treats many applications [3].

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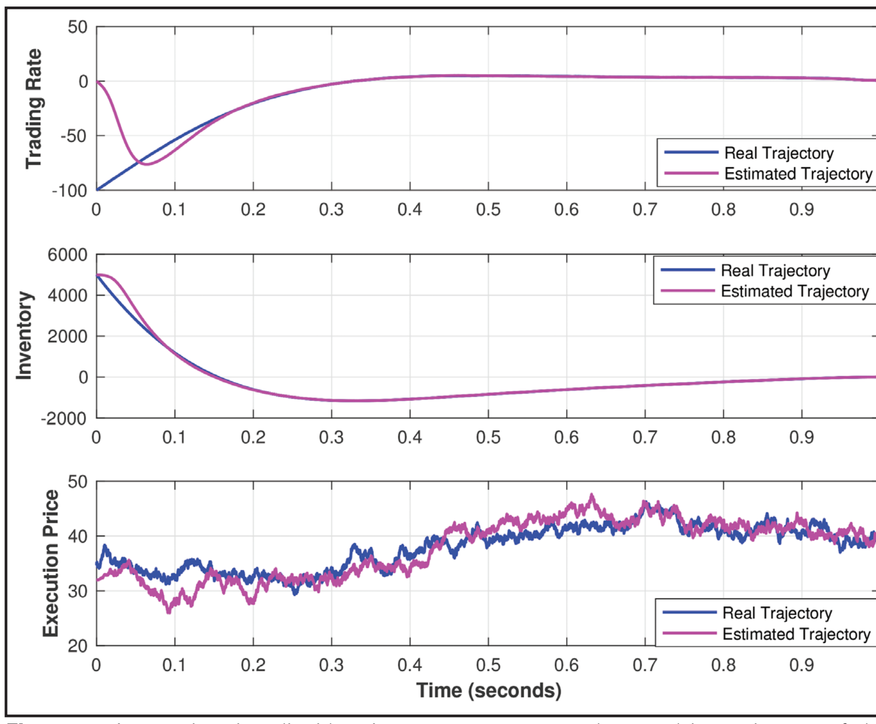


Figure 5. *A generic minor liquidator's state component values and its estimates of their values. Figure courtesy of Dena Firoozi.*

Peter E. Caines is a professor at McGill University. He is the author of Linear Stochastic Systems (part of SIAM's Classics in Applied Mathematics series), and his research interests include hybrid systems, mean field game (MFG) theory and applications, and graphon MFG theory on very large-scale networks.

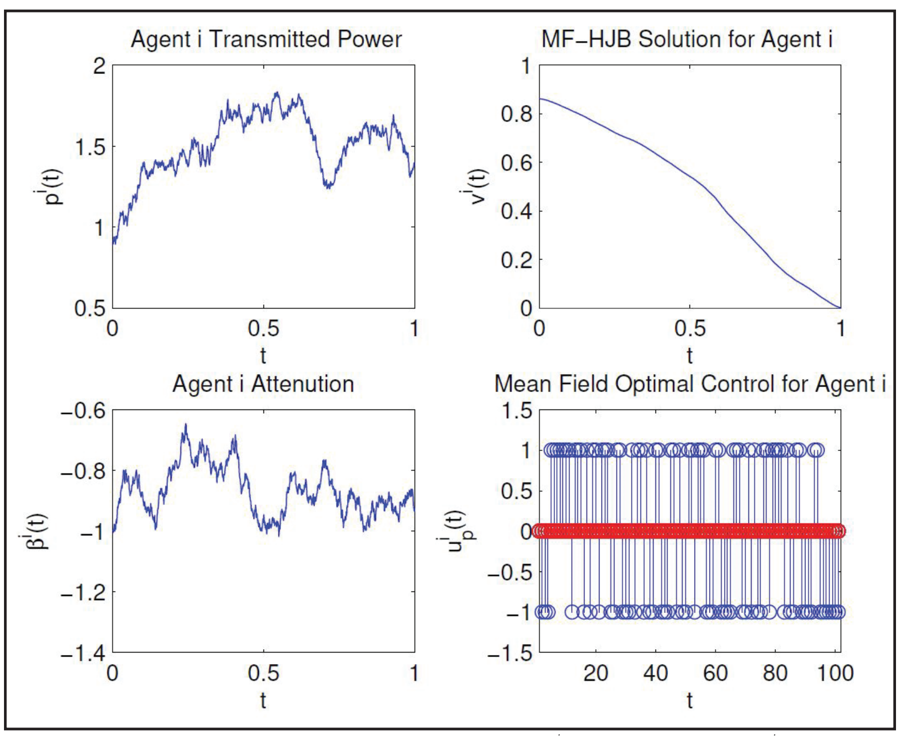


Figure 4. *Trajectories over the interval* $[0,1]$ *of power* $p^i(t)$, *value function* $V^i(t)$, *channel attenuation* $\beta^i(t)$, *and control* $u^i(t)$ *for a generic agent* i. *Figure courtesy of Dena Firoozi.*

Coronavirus Update

SIAM leadership and senior staff have been closely monitoring the outbreak of the SARS-CoV-2 virus and the associated COVID-19 disease. We are deeply committed to the health and well-being of both our members and the general public, and to finding an appropriate compromise between containment of the novel coronavirus and our important mission of advancing industrial and applied mathematics.

Taking these factors into consideration, SIAM has made the difficult decision to cancel or postpone the following meetings:

- **SIAM Conference on Uncertainty Quantification (UQ20)**, originally scheduled to take place March 24-27 in Garching, Germany
- **SIAM Conference on Mathematics of Data Science (MDS20)**, originally scheduled to take place May 5-8 in Cincinnati, Ohio
- **SIAM International Conference on Data Mining (SDM20)**, originally scheduled to take place May 7-9 in Cincinnati, Ohio
- **SIAM Conference on Mathematical Aspects of Materials Science (MS20)**, originally scheduled to take place May 18-22 in Bilbao, Spain
- **SIAM Conference on Optimization (OP20)**, originally scheduled to take place May 26-29 in Hung Hom, Hong Kong
- **SIAM Conference on Discrete Mathematics (DM20)**, originally scheduled to take place June 1-4 in Portland, Ore.
- **SIAM Conference on Mathematics of Planet Earth (MPE20)**, originally scheduled to take place June 8-10 in Garden Grove, Calif.
- **SIAM Conference on the Life Sciences (LS20)**, originally scheduled to take place June 8-11 in Garden Grove, Calif.

This list was updated as of March 19th, when the April issue of *SIAM News* went to press. For the most up-to-date information, please visit **https://go.siam.org/COVID19**. Details about possible rescheduling will be made available in the coming months.

Tears of Wine at PD19

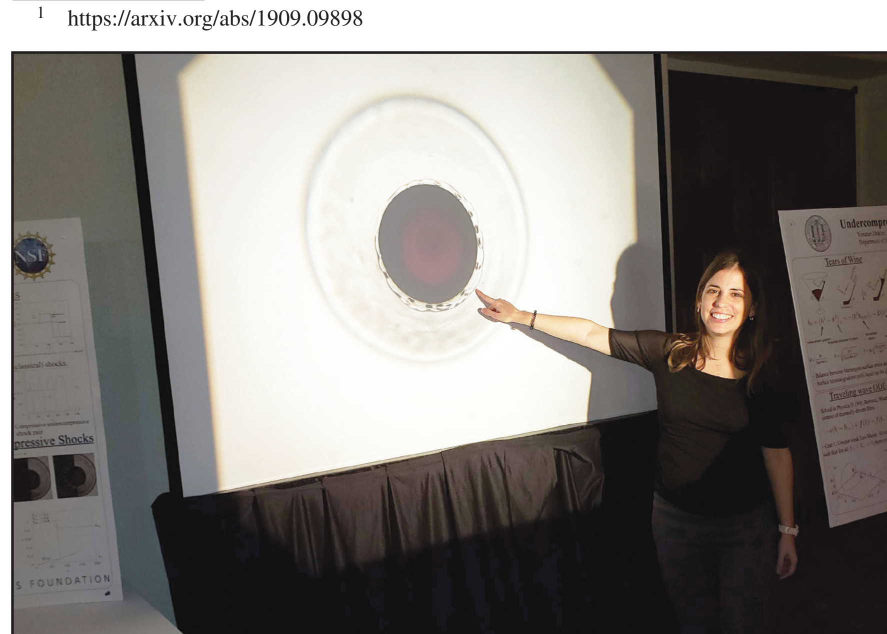
During the 2019 SIAM Conference on Analysis of Partial Differential Equations (PD19), which took place this past December in La Quinta, Calif., Andrea Bertozzi and postdoctoral researcher Claudia Falcon—both of the University of California, Los Angeles (UCLA)—performed a demonstration of "tears of wine." The tears of wine problem is a curious phenomenon that wine drinkers have observed for centuries. In the right setting, one may notice a thin layer of water-ethanol mixture that travels up inclined surfaces—such as wine glasses—against gravity and falls down in the form of "tears."

Demonstration of the famed "tears of wine" problem at the 2019 SIAM Conference on Analysis of Partial Differential Equations, held in La Quinta, Calif., this past December. Photo courtesy of Claudia Falcon.

Bertozzi and Falcon presented a live experiment with a pre-swirled glass, wherein the famous wine tears emerged. They described this particular behavior using a mathematical model that involves a conservation law with a nonconvex flux and higher-order diffusion, due to the bulk surface tension. Such equations have nonclassical "undercompressive" shock solutions, which were the main drivers of the destabilizing front in the demonstration at PD19. Bertozzi and Falcon's work—conducted with UCLA student Yonatan Dukler and postdoctoral researcher Hangjie Ji—will appear in *Physical Review Fluids* and is currently available online.[1] Prior mathematical modeling of this problem addressed the behavior of the meniscus and the film at earlier stages, rather than the wine tears.

— Andrea Bertozzi and Claudia Falcon

[1] https://arxiv.org/abs/1909.09898



Claudia Falcon, a postdoctoral researcher at the University of California, Los Angeles, addresses the "tears of wine" problem at the 2019 SIAM Conference on Analysis of Partial Differential Equations, which took place this past December in La Quinta, Calif. Photo courtesy of Andrea Bertozzi.

Helping Faculty Prepare Students for the Workforce

By Kathleen Kavanagh, Joe Skufca, Ben Galluzzo, and Karen Bliss

Motivated by national initiatives[1] to improve the role of science, technology, engineering, and mathematics (STEM) educators in workforce preparation from K-12 through college, the SIAM Education Committee is focusing on methods to help teachers and faculty better equip their students. A session organized by the committee at the 2020 Joint Mathematics Meetings, which took place this January in Denver, Colo., reflected this focus area. The speakers included Karen Bliss of Virginia Military Institute (VMI) and Ben Galluzzo and Joe Skufca, both of Clarkson University. An industry panel following the talks generated discussion on employer requirements when hiring mathematicians.

In addition to a solid foundation of core mathematics, statistics, and problem-solving abilities, industrial positions also require skills like data literacy, computing, mathematical modeling, interdisciplinary team collaboration, and effective communication. Integrating these skills into new or existing curricula raises challenges for faculty members who may not be experienced in certain areas. For example, while machine learning courses have not historically been part of most undergraduate- or graduate-level coursework, academic departments are now recognizing that a machine learning background increases their students' marketability.

Computing lies at the heart of every future path across all global cultures, including complex societal problems, manufacturing and financial innovation, and even consumer trends. To ensure that high school students are prepared for this future, instructors must be creative and resourceful. To that end, Galluzzo described a National Science Foundation venture that centers on computational literacy using math modeling and R programming in high school courses. The Computing with R for Mathematical Modeling (CodeR4Math) project leverages inherent connections between computational thinking, mathematical modeling practices, and a multitude of representational tools. This combination creates a synergistic solution that allows students to simultaneously develop competencies in both domains. The project is currently generating a collection of facilitated math modeling activities that provide students with "just in time" coding help.

CAREERS IN MATHEMATICAL SCIENCES

Bliss detailed the evolution of a differential equations (DEs) course as part of the transition from a mathematics major to an applied mathematics major at VMI. At the outset, she and her colleagues met with faculty from departments served by the DEs course—like engineering, physics, and chemistry—to determine what they wanted students to gain from the course. This discussion allowed them to pare down the existing pencil-and-paper solution techniques and redirect the course's focus to modeling, specifically the employment of DE utilization to answer questions in different disciplines. Students in Bliss's class explore population models, models of disease spread, and the use of blast attenuating seats (represented by a spring-mass system) in military vehicles to decrease traumatic brain injury. By emphasizing the translation of mathematical models to real-world scenarios, utilizing appropriate technology to obtain solutions, and communicating results via succinct reports, students learn to work in interdisciplinary teams and leverage their math skills to solve important problems.

Skufca overviewed Clarkson's efforts to address emerging workforce needs, as framed in the context of a small, resource-challenged university. Clarkson developed a professional master's degree in data analytics, which is available as an interdisciplinary program across all of the university's departments. Three years after establishing the master's program, Clarkson's Department of Mathematics began offering an undergraduate degree in data science. The flexibility associated with a small university helped achieve both initiatives, which were inspired by industry need.

Skufca then explained how careful curriculum mapping—combined with strong cooperation across disciplines—allowed Clarkson to build these programs without adding many supplementary resources. The directors of both new programs first look to industry to identify relevant skills, then turn to internal academic expertise to determine the fundamental pedagogy that supports, enables, and broadens that specific skillset. A unique component of the data science curriculum is a three-credit "math" course (taught by a mathematics professor) on the ethics of data science and applied mathematics; this is especially pertinent as the interplay of math, industry, and government requires awareness of professional mathematics' impact on society.

Genetha Gray of Salesforce, Carol Woodward of Lawrence Livermore National Laboratory, Pat Quillen of MathWorks, and Aaron Luttman of Pacific Northwest National Laboratory comprised the industry panel that followed the initial presentations. Each panelist began by recounting employers' requirements for hiring mathematicians.

See **Workforce** *on page 8*

[1] https://www.whitehouse.gov/wp-content/uploads/2018/12/STEM-Education-Strategic-Plan-2018.pdf

Professional Opportunities and Announcements

Send copy for classified advertisements and announcements to ***marketing@siam.org****. For rates, deadlines, and ad specifications, visit* ***www.siam.org/advertising****.*

*Students (and others) in search of information about careers in the mathematical sciences can click on "Careers" at the SIAM website (****www.siam.org****) or proceed directly to* ***www.siam.org/careers****.*

A Solution to the $3x + 1$ Problem

I continue to believe that I have solved this very difficult problem. In more than two years, I have received no claims of errors in the first two proofs of the $3x+1$ Conjecture (a proof solves the Problem) from visitors to the paper. Recently, I discovered a third proof of the Conjecture that is only four pages long! No claims of errors so far.

However, no journal will consider my paper because of the Problem's difficulty and the fact that I am not an academic mathematician (my degree is in computer science, and I have spent most of my career as a researcher in the computer industry).

Therefore, I am looking for a mathematician who will help me prepare the paper for submission to a journal, and who will write to the editor stating the mathematician's belief that the paper is worthy of publication.

The paper is called "A Solution to the $3x+1$ Problem," on **occampress.com**.

— *Peter Schorer,* peteschorer@gmail.com

SIAM: The Early Years

By James Crowley

History can reveal a lot about ourselves and our organization — what we stand for and how we reached this point. Two recent coincidences caused me to look back at SIAM's early years and explore how SIAM came to be about mathematics, computing, and their applications. This story is a result of my investigation.

A Chance Occurrence

Not long ago, I met a minister who served a community in which my father lived. When we were introduced, Reverend George Patterson announced something puzzling. "You know, my father was one of the founders of SIAM," he said. At first I was a bit perplexed. I was well aware of Ed Block's notable role in SIAM's founding, but unfortunately my familiarity with the society's early history beyond that point was a bit fuzzy. Patterson and I chatted some more, and when I returned home I did a little homework. Block's "SIAM – Its First Three Years," which published in *SIAM Review* [2], was a natural place to start. The article noted that "The first organizing meeting for the proposed society took place in December 1951 at Drexel Institute of Technology [now Drexel University]. Members of the organizing committee were I.E. Block, Donald B. Houghton, Samuel S. McNeary, Cletus O. Oakley, *George W. Patterson III*, and George Sonneman."

According to Block, the nascent organization sponsored Mina Rees of the Office of Naval Research as its first speaker. Rees delivered a talk entitled "The Role of Mathematics in Government Research." Shortly thereafter, on April 30, 1952, SIAM was incorporated.

The strong integration of mathematicians in industry and government (and industrial mathematics) into SIAM from its outset is certainly interesting. William E. Bradley of Philco was elected as the first president. SIAM had two vice presidents in its initial years: Grace Hopper of the Eckert-Mauchly Division of Remington Rand and George W. Patterson III of the Burroughs Adding Machine Company. Emil Amelotti of Villanova University was the first treasurer, and Block (then at Philco) was the first secretary. The influence of the then-newly emerging computer industry is also apparent within this group. Members of the Board of Trustees included John W. Mauchly,[1] co-founder of the Eckert-Mauchly Division of Remington Rand (with J. Presper Eckert). Mauchly helped create the ENIAC computer and would later become the fourth president of SIAM.

Development of the ENIAC at the University of Pennsylvania's Moore School had a profound impact on both mathematics and the greater Philadelphia area. Organizations raced to embrace this new technology, and companies like Philco and the Burroughs Adding Machine Company—inspired by the region's talent pool—created divisions devoted to the novel computer and hired people with ENIAC development experience.

The newfound focus on computing services—both for military and commercial applications—required the development of new algorithms, and organizations hired mathematicians to accomplish this objective. This provided the basis for a new scholarly society in applied mathematics.

The tradition of appointing and electing officers from industry and/or national laboratories continued over the next five decades. Of SIAM's first 39 presidents, 11 were affiliated with a company or national lab. These organizations included Philco (Bradley), Remington Rand (Mauchly), IBM (Donald Thomsen and Hirsh Cohen), Bell Labs (Brockway McMillan and Margaret Wright), Oak Ridge National Laboratory (Alston Householder), Argonne National Laboratory (Wallace Givens), the Boeing Company and the National Bureau of Standards, now the National Institute of Standards and Technology (Burt Colvin), Los Alamos National Laboratory (Mac Hyman), and MathWorks (Cleve Moler).

Harold Kuhn is credited with expanding SIAM's conference program [4] by eliciting an invitation from the American Mathematical Society to join it—along with the Mathematical Association of America and the Association for Symbolic Logic—for their joint meeting in Pittsburgh, Penn., in December 1954; we now know this conference as the Joint Mathematics Meetings. This was SIAM's first national meeting.

By 1960, SIAM had 2,000 members and counting; today that number exceeds 14,000. By 1976, SIAM had expanded to the point of needing a managing director for its small but growing staff, and the Board of Trustees appointed Block to this position. It was around this time that SIAM really began to take off as a professional society.

A Second Coincidence

Upon return from a trip to Ireland, my wife and I happened to chat with someone who asked me about my work. I responded that I worked with the Society for Industry and Applied Mathematics. "Oh, SIAM!" he exclaimed. "My father was the treasurer of SIAM many years ago." SIAM records indeed confirmed this statement: Richard "Dick" Lamb was SIAM Treasurer from 1965 until 1983.

This discovery inspired yet another journey into SIAM's history, which reinforced my earlier conclusions. My research revealed that Lamb was employed at the Auerbach Corporation's Digital Computing Service for a time, where he probably met Block, who also spent time with Isaac Auerbach. This led me down another fascinating trail of history concerning Auerbach himself, and hints at why SIAM was established in Philadelphia.

In the 1950s, Philadelphia was the Silicon Valley of its time. This was in part due to the ENIAC's development at the University of Pennsylvania, which created a core of experts in the region who left the university after World War II for commercial pursuits in industry. These enterprising individuals helped found both large organizations and smaller technology companies. For example, Eckert and Mauchly's departure inspired the creation of the Eckert-Mauchly Computer Corporation, which became Remington Rand and later gave rise to Unisys. And Isaac Auerbach left the Burroughs Corporation to form his own company, which bore his name. It was at the Auerbach Corporation where Block found Lamb and brought him to SIAM as treasurer.

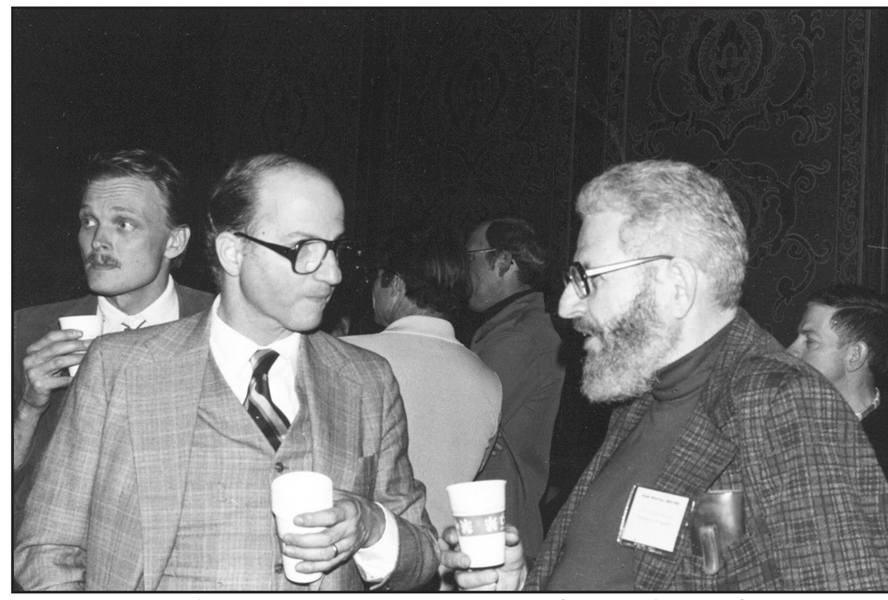
Block spent most of his career in industry. After earning his doctorate in mathematics at Harvard University, he accepted a position at Philco. He then moved to the Burroughs Corporation, where he eventually became a manager at the UNIVAC Engineering Computer Center of the Sperry-Rand Corporation's Remington Rand Division in Philadelphia and served as supervisor of the UNIVAC Division of Sperry Rand's Applied Mathematics Unit. From there, Block joined Auerbach Corporation as technical advisor to the director of the Information Sciences Division.

During this entire period, Block served in various capacities as a volunteer with SIAM: as founding secretary (1952-1955), vice president (1963-1974), Board member (1970-1976), and chairman of SIAM's Publications Committee. At Auerbach, he became vice president of Auerbach Publishers, thus merging his full-time job with his interests in scientific publishing at SIAM [1].

Block was clearly a driving force in SIAM's creation and development. Former SIAM President Bob O'Malley aptly noted that SIAM had been founded "mostly through the efforts of Ed" [3].

The lessons I came away with as a result of my historical expedition were not only an explanation of SIAM's emphases in the early days (which continue to some degree today) to embrace applied and industrial mathematics as well as computing. SIAM offered a place not only for industrial mathematics, but also for mathematicians working in industry and government laboratories. It has always included a strong computing component.

Most of all, we can be thankful for Ed Block, who personified many of these things.



I. Edward Block (left) mingles with attendees at an early SIAM conference. SIAM photo.

References

[1] Bartik, J.J. (2013). *Pioneer Programmer: Jean Jennings Bartik and the Computer that Changed the World*. Kirksville, MO: Truman State University Press.

[2] Block, I.E. (1973). SIAM – its first three years. *SIAM Rev., 15*(2), v-ix.

[3] O'Malley, B. (2008, November 16). Obituaries: George Handelman. *SIAM News, 41*(9).

[4] Society for Industrial and Applied Mathematics (2002). *Looking Back, Looking Ahead: A SIAM History*. Philadelphia, PA: SIAM.

James Crowley is the executive director of SIAM.

[1] https://en.wikipedia.org/wiki/John_Mauchly

Workforce

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Recommended skills included deep knowledge in a specific subject, a background in some realm of science, programming experience with expertise in a compiling language, familiarity with numerical methods, and problem-solving abilities. However, a common theme among all panelists was strong *communication proficiency* — the ability to explain ideas at a high level to experts while also communicating effectively with non-experts.

The audience was interested in types of required writing skills, and inquired about methods that they could use to train students accordingly. Quillen noted that helping students learn succinctness—in an email or storytelling, for example—is crucial. Woodward emphasized the importance of efficiently translating logical arguments into writing while keeping one's audience in mind. One attendee sought advice on building connections with industry associates, identifying and contacting the right partners, and maximizing the likelihood of response. Gray suggested reaching out to public relations personnel since they are often able to best connect individuals with suitable internal contacts. Most importantly, she recommended underscoring the benefits for companies in forming relationships with academia.

Finally, Luttman urged attendees to utilize SIAM resources,[2] and encouraged faculty to take advantage of programs like PIC Math[3] and the Visiting Lecturer Program[4] to help students establish early industry partnerships.

Kathleen Kavanagh is a professor of mathematics at Clarkson University and the Vice President for Education at SIAM. Joe Skufca is a professor of mathematics and chair of the Department of Mathematics at Clarkson. Benjamin Galluzzo is an associate professor of mathematics at Clarkson. Karen Bliss is an associate professor in the Department of Applied Mathematics at Virginia Military Institute.

[2] https://siam.org/students-education/resources

[3] https://math.siam.org/picmath/

[4] https://www.siam.org/students-education/programs-initiatives/siam-visiting-lecturer-program

Aaron Luttman (Pacific Northwest National Laboratory), Pat Quillen (MathWorks), Carol Woodward (Lawrence Livermore National Laboratory), and Genetha Gray (Salesforce) comprised an industry panel at the 2020 Joint Mathematics Meetings, which took place this January in Denver, Colo. Photo courtesy of Kathleen Kavanagh.