Analytical Models for Thick and Modified Cylindrical Shell Vibrations

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ENGINEERED
TO OUTRUN

ABB BUSINESS AREAS


ELECTRIFICATION


MOTION

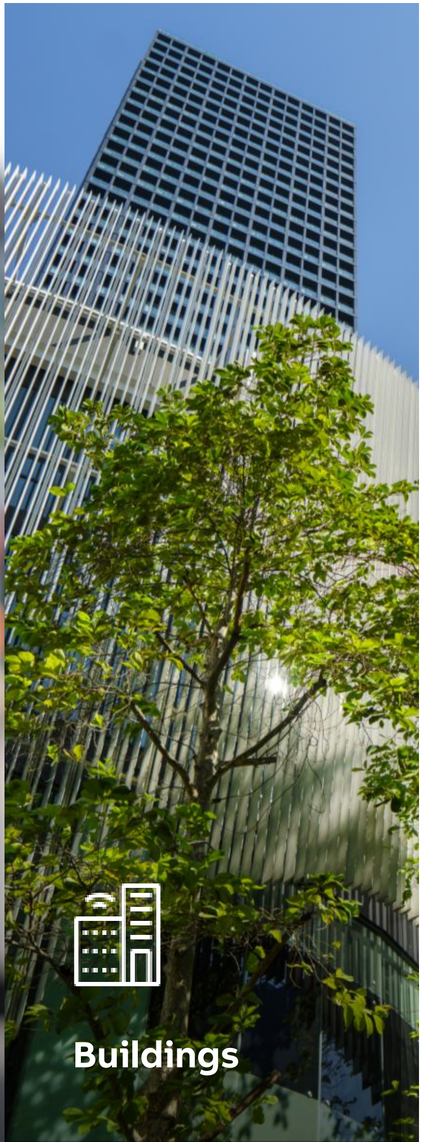

AUTOMATION

Key Sectors of the Economy

Power

Industries

Transport & Infrastructure



Buildings

Power

- Distribution
- Renewables
- Conventional Generation

Industries

- Data Centers
- Oil & Gas
- Mining & Metals
- Water & Wastewater
- Food & Beverage

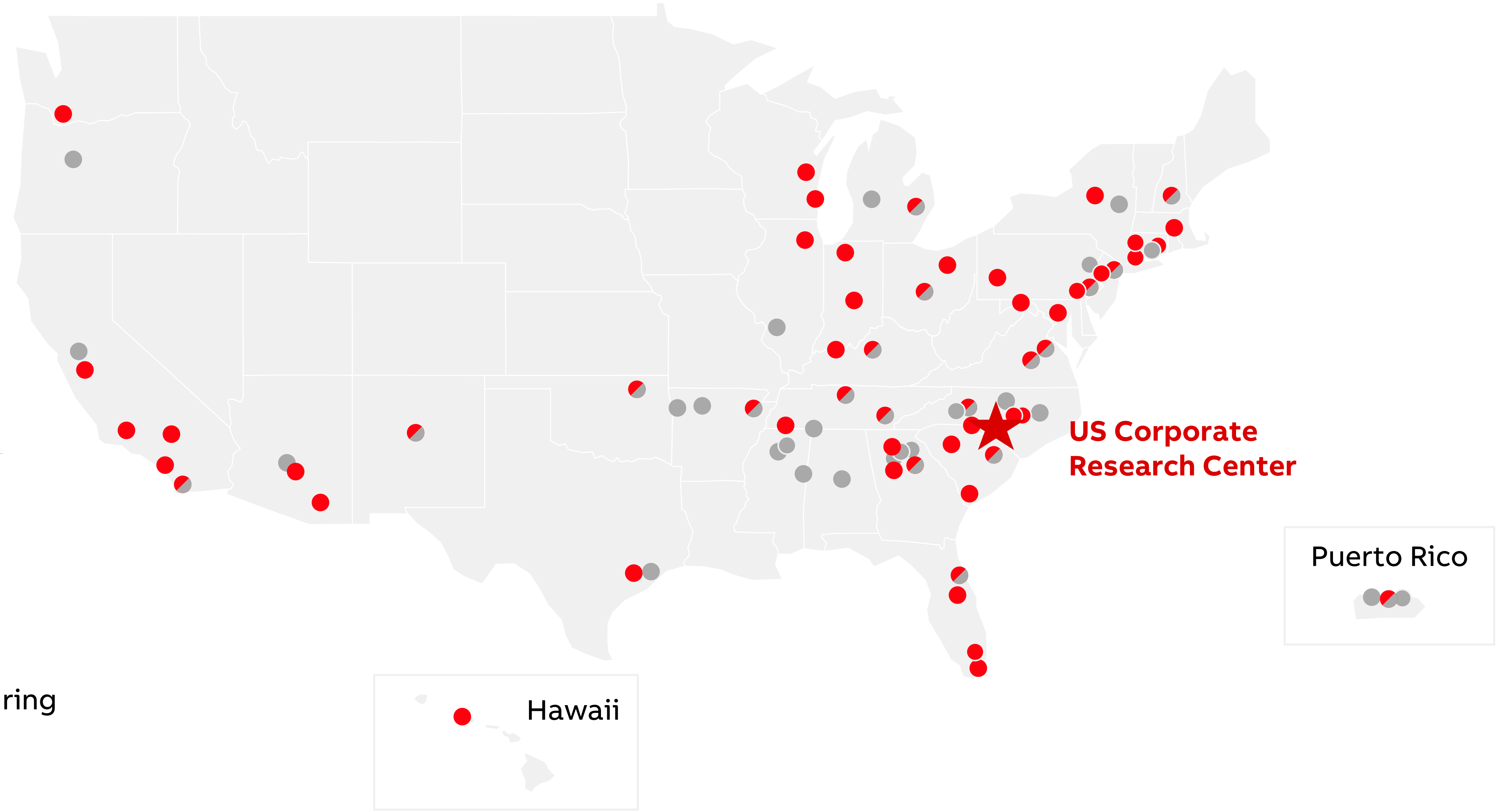
Transport & Infrastructure

- Marine & Ports
- Rail
- Infrastructure

Buildings

- Buildings
- HVACR

ABB United States Footprint



US Corporate Research Center

- ● Office
- ● Manufacturing
- ◐ Office/manufacturing

Pioneering Technology Leadership with Innovative Offerings

2021

ABB Ability™
Condition monitoring for motors, drives and electric powertrains

ABB Access
QR code landing page for product data

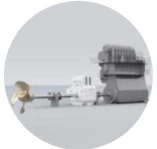
BORDLINE® ESS
Traction and e-mobility battery energy storage

AMXE
Motor offering for e-mobility applications

2022

Energy appraisal
Digital powertrain energy appraisal

DCS880-S
Power converters for hydrogen production



PM shaft generators
PTO/PTI/PTH capability for ships

ACS180
Micro drive extensions to machine builders

2023

Anomaly detection
AI based anomaly detection for LV motors

EC Titanium
Integrated motor drive offering extensions

e530 PTI servo
New member of ABB's servo products for China market

IE5 SynRM Ex
Ultra-premium efficiency motors for explosive atmospheres

2024

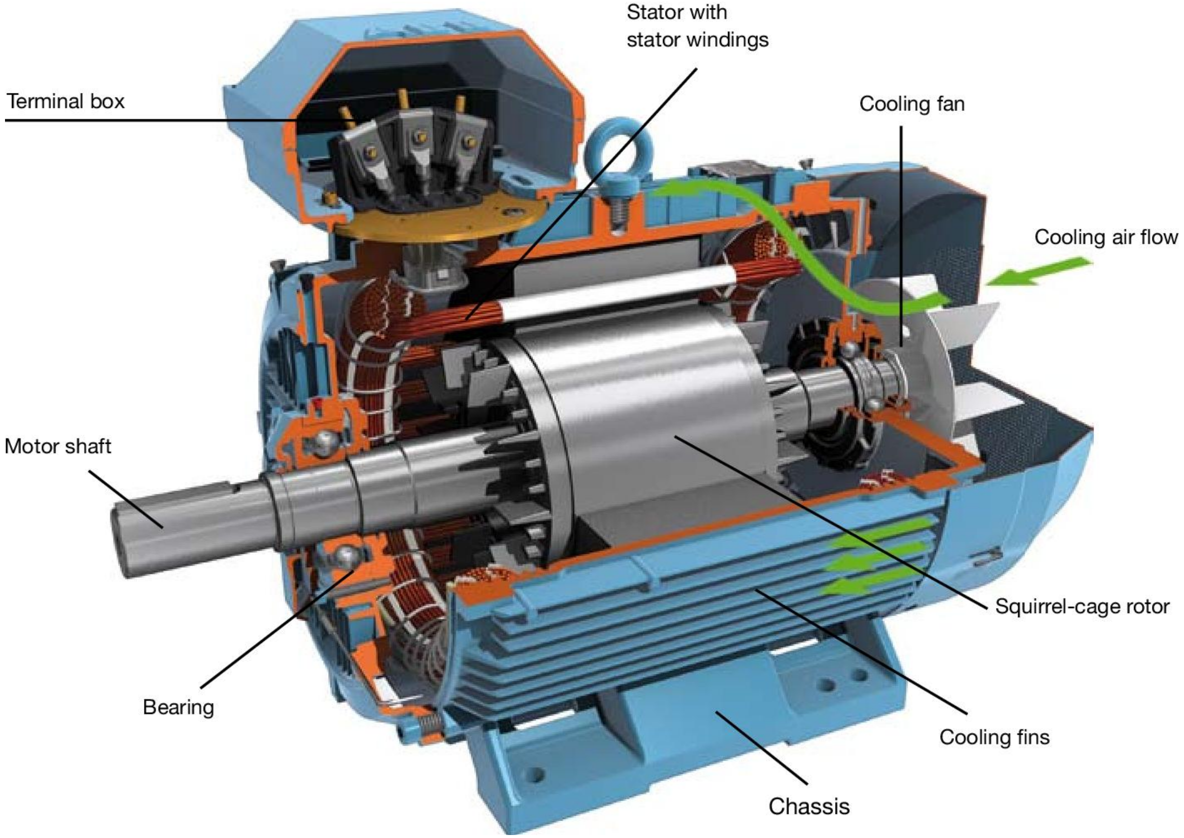
HES580-3L
Motor friendly 3-level e-mobility inverter with high system efficiency

IE5 SynRM
Liquid cooled high efficiency SynRM motors

ACS8080
Next generation air-cooled MV drives

ACS280-04
Cold plate component drives for direct machine integration

Analytical Model for Motor Vibrations



Induction motor example

Motivation

- Enable fast dynamic motor frame analysis for noise sensitive applications
- Flag electromagnetic designs (pole-slot-tooth combinations) that excite structural resonances

Goal & Deliverables

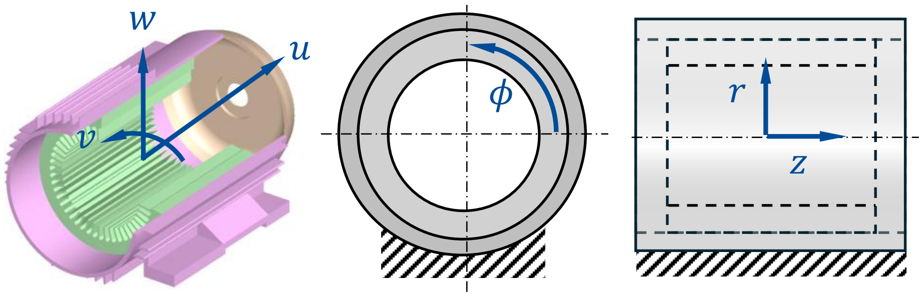
- Extended capabilities of existing model currently used for stators vibrations
- New model to include housing, endcaps, and foot mounts

Research question(s) & Approach

- Modal and harmonic analysis (without a rotor shaft) assuming a rotating sinusoidal force field
- Leverage LaGrange and Rayleigh-Ritz formulations applied to thick-wall cylindrical solids

Existing Modal Model and Limitations

- Currently limited to thick-wall, open cylinder (single-material)
- **Not** considered: Mounting base, endcaps, stator housing, and rotor
- Based on J. Roivainen 2009 PhD Dissertation
- Implemented in C, Python, and Octave/Matlab

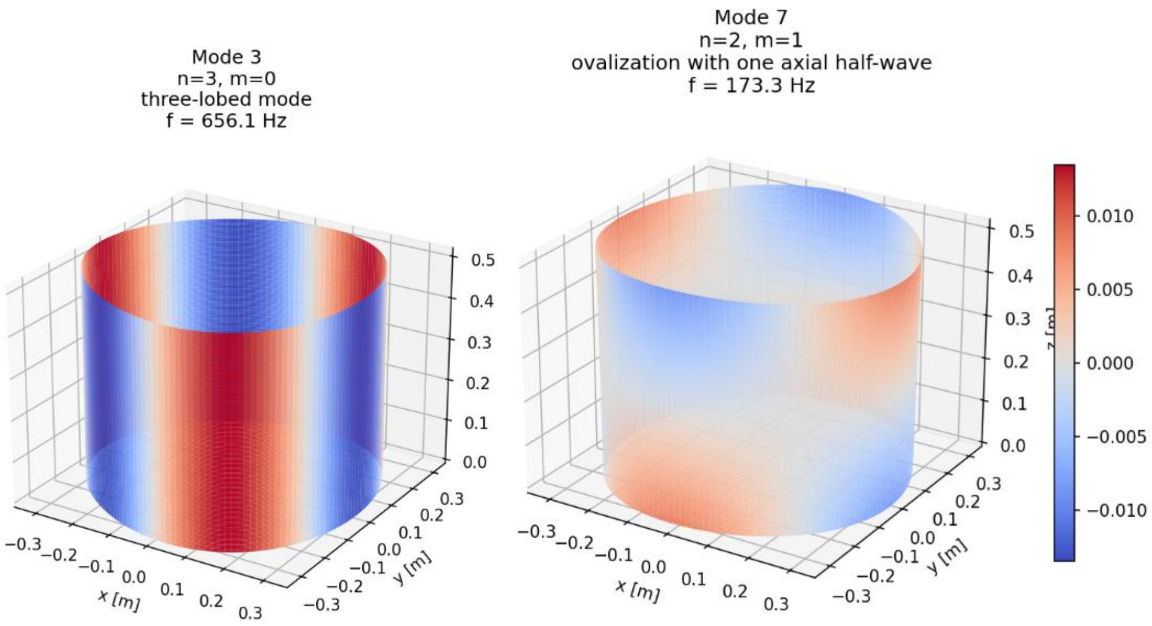


Section, only 1 endcap shown

Mounting base

Conventions

Example modal solutions for continuous, free-free, thick-walled cylinder



Mode 3
n=3, m=0
three-lobed mode
f = 656.1 Hz

Mode 7
n=2, m=1
ovalization with one axial half-wave
f = 173.3 Hz

Assumptions and Starting Points

- Time dependent displacement field is separable into e^{st}, Fourier harmonics in ϕ, and (r, z) dependence

$$\hat{\mathbf{u}}(r, \phi, z, t) = \boldsymbol{\Psi}(r, \phi, z)\,\mathbf{q}(t)$$

- (r, z) dependence is expressed with Ritz basis functions (beam functions or polynomials depending on BCs)
- Form strain and kinetic energy terms Π and T
- *Reduced* LaGrange equation yields mass and stiffness quadratic forms

$$\mathbf{M}\,\ddot{\mathbf{q}}(t) + \mathbf{K}\,\mathbf{q}(t) = \mathbf{0}$$

- Assume harmonic motion:

$$\mathbf{q}(t) = \boldsymbol{\phi}\,e^{st}$$

- Solve for generalized eigenvalues and mode shapes

Energies and Constitutive Material Law

Strain Energy $\Pi = \frac{1}{2}\int_V \boldsymbol{\varepsilon}^T \mathbf{C}\, \boldsymbol{\varepsilon}\, dV$ $\boldsymbol{\sigma} = \mathbf{C}\, \varepsilon$ $\mathbf{C} = \mathbf{S}^{-1}$

$$\mathbf{S} = \begin{bmatrix} \frac{1}{E_z^*} & -\frac{\nu_{z\phi}}{E_z^*} & -\frac{\nu_{zr}}{E_z^*} & 0 & 0 & 0 \\ -\frac{\nu_{\phi z}}{E_\phi^*} & \frac{1}{E_\phi^*} & -\frac{\nu_{\phi r}}{E_\phi^*} & 0 & 0 & 0 \\ -\frac{\nu_{rz}}{E_r^*} & -\frac{\nu_{r\phi}}{E_r^*} & \frac{1}{E_r^*} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{\phi r}^*} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{zr}^*} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{z\phi}^*} \end{bmatrix}$$

$$\begin{aligned} \varepsilon_z &= \frac{\partial u}{\partial z} \\ \varepsilon_\phi &= \frac{1}{r}\frac{\partial v}{\partial \phi} + \frac{w}{r} \\ \varepsilon_r &= \frac{\partial w}{\partial r} \\ \gamma_{\phi r} &= \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial w}{\partial \phi} \\ \gamma_{zr} &= \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} \\ \gamma_{z\phi} &= \frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial u}{\partial \phi} \end{aligned}$$

$$\boldsymbol{\varepsilon}(\mathbf{x},t) = \mathcal{B}\hat{\mathbf{u}}(\mathbf{x},t) = \underbrace{\mathcal{B}\boldsymbol{\Psi}(\mathbf{x})}_{\mathbf{B}(\mathbf{x})}\mathbf{q}(t)$$

$$\boxed{\Pi = \tfrac{1}{2}\int_V (\mathbf{B}\mathbf{q})^T \mathbf{C}(\mathbf{B}\mathbf{q})\, dV = \tfrac{1}{2}\mathbf{q}^T \left[\int_V \mathbf{B}^T\mathbf{C}\mathbf{B}\, dV\right]\mathbf{q} = \tfrac{1}{2}\mathbf{q}^T\mathbf{K}\mathbf{q}}$$

Kinetic Energy $T = \frac{1}{2}\int_V \rho(u_t^2 + v_t^2 + w_t^2)dV$

Simpler, only need to find time derivatives and multiply with scalar ρ

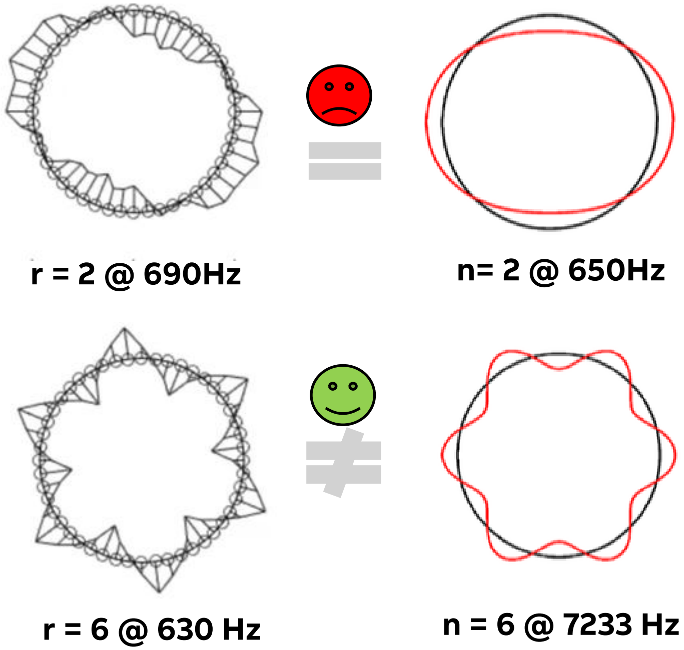
$$\boxed{T = \tfrac{1}{2}\int_V \rho\,(\boldsymbol{\Psi}\dot{\mathbf{q}})^T(\boldsymbol{\Psi}\dot{\mathbf{q}})\, dV = \tfrac{1}{2}\dot{\mathbf{q}}^T\left[\int_V \rho\,\boldsymbol{\Psi}^T\boldsymbol{\Psi}\, dV\right]\dot{\mathbf{q}} = \tfrac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}\dot{\mathbf{q}}}$$

Quadratic Forms of Strain and Kinetic Energies

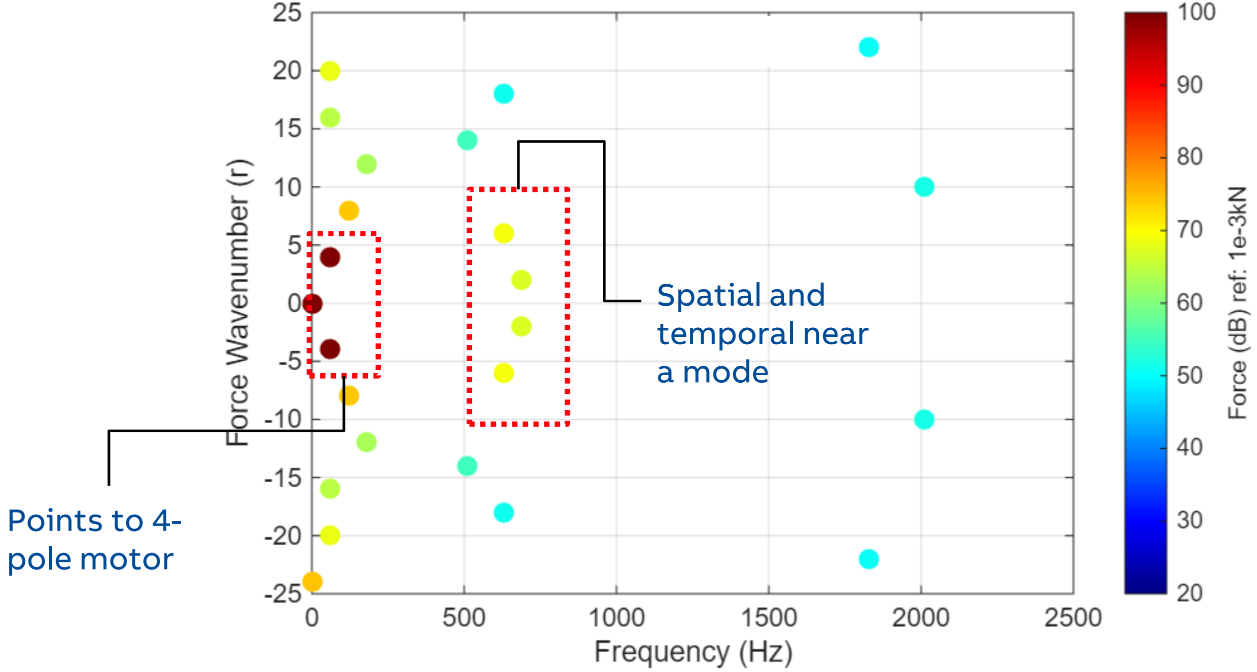
Quantity	Strain energy Π	Kinetic energy T
Physical integrand	$\frac{1}{2}\,\boldsymbol{\varepsilon}^T\mathbf{C}\,\boldsymbol{\varepsilon}$	$\frac{1}{2}\,\rho\,\dot{\mathbf{u}}^T\dot{\mathbf{u}}$
Operator on $\mathbf{u}$	$\boldsymbol{\varepsilon} = \mathbf{B}\mathbf{q}$ (spatial derivatives)	$\dot{\mathbf{u}} = \boldsymbol{\Psi}\dot{\mathbf{q}}$ (time derivative only)
Middle "material" factor	$\mathbf{C}$ (elasticity tensor)	ρ (scalar density)
Result	$\mathbf{K} = \int_V \mathbf{B}^T\mathbf{C}\,\mathbf{B}\,dV$	$\mathbf{M} = \int_V \rho\,\boldsymbol{\Psi}^T\boldsymbol{\Psi}\,dV$
Quadratic form	$\frac{1}{2}\,\mathbf{q}^T\mathbf{K}\,\mathbf{q}$	$\frac{1}{2}\,\dot{\mathbf{q}}^T\mathbf{M}\,\dot{\mathbf{q}}$

Excitation Forces and Unit Wave Response

- Radial forces dominant, tangential forces negligible
- Temporal and spatial characteristics depend on speed and design
- Worst case: Force temporal and spatial characteristics near mode
- Unit wave response (UWR) found from modal superposition



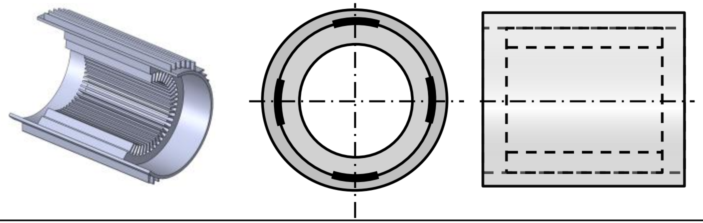
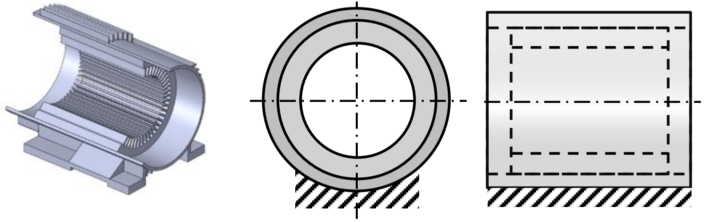
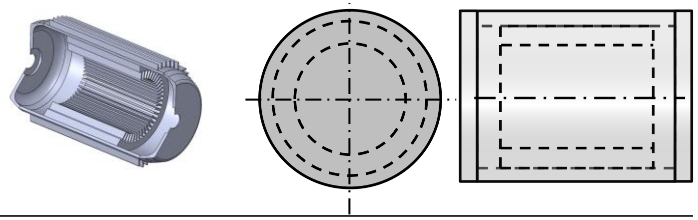
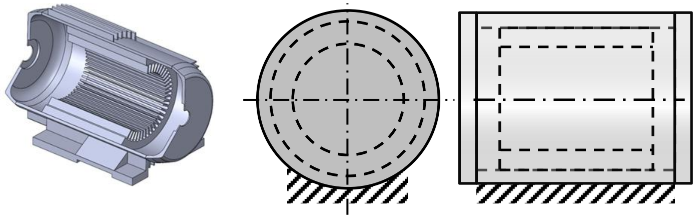
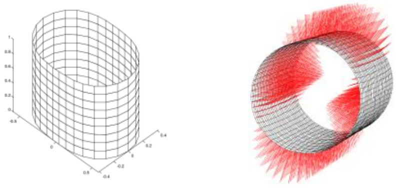
Example radial airgap force for $f_S = 30\ \mathrm{Hz}$



Spatial and temporal near a mode

Points to 4-pole motor

Expected Model Capabilities

#	Characteristics	Diagrammatic Sketch
1	Modal analysis with stator housing of a different material (“nested” cylinders)	
2	Modal analysis with mounting base (a) With endcaps (b)	 
3	Modal analysis, combining stator housing, mounting base, and endcaps	
4	Harmonic response analysis	

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Janne Roivainen, *Unit-Wave Response-Based Modeling of Electromechanical Noise and Vibration of Electrical Machines*, Doctoral Dissertation, Helsinki University of Technology, 2009. https://aaltodoc.aalto.fi/items/a721c2ba-d89b-4b36-adb7-4cf869cd5400

Ramses Girgis, *Theoretical and Experimental Investigations on Resonant Frequencies and Natural Response of Stators of Electrical Machines*, Doctoral Dissertation, University of Saskatchewan, 1978. https://harvest.usask.ca/items/638a7c32-a026-4d0a-948d-fff76e1a4160

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Appendix

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ABB IN THE UNITED STATES

Investment & Growth

Country headquarters
Cary, North Carolina

About ~17,000
people in the U.S.

$96+ billion in U.S. market opportunities

U.S. is approximately **29% of ABB revenue**

Entered U.S. in 1925 when BBC formed ABBEC (American Brown Boveri Electric Corp.)

$14 billion invested 15 acquisitions & CAPEX since 2010

Nearly 40 manufacturing, distribution, and operational facilities, including 10 major R&D centers

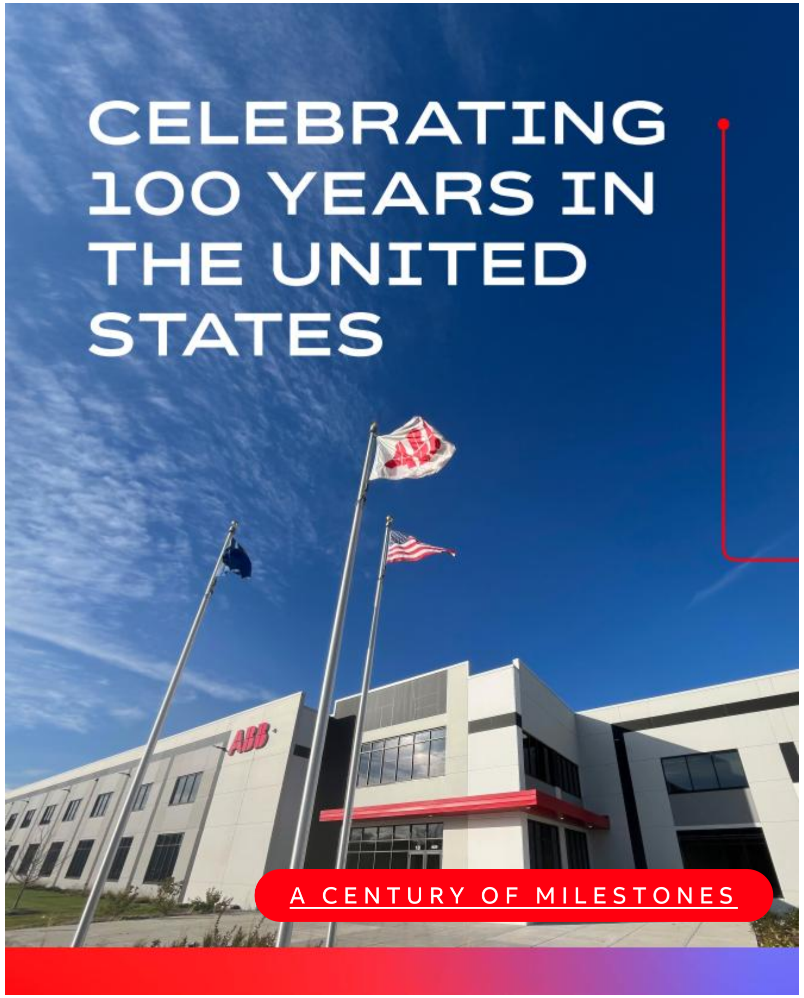
75-80% of products are made in the US.

~14,000 U.S. customers
in all 50 States

100% renewable electricity used in all manufacturing operations

9,000+ distributor locations

More than $32 million in charitable contributions to U.S. communities since 2016



Motion Business Area

Revenues	$7.8 bn
Employees	~22 k
Global market position	No. 1

Divisions

- Drive Products
- System Drives
- Motion Services
- NEMA Motors
- IEC LV Motors
- Large Motors & Generators
- Traction

ABB Motion, a global leader in motors and drives, is at the core of accelerating a more productive and sustainable future. We innovate and push the boundaries of technology to contribute to energy efficient, decarbonizing and circular solutions for customers, industries and societies.

With our digitally enabled drives, motors and services we support our customers and partners to achieve better performance, safety and reliability.

Building on over 140 years of domain expertise in electric powertrains, we help the world’s industries outrun – leaner and cleaner, by delivering motor-driven solutions for a wide range of applications in all industrial segments.

FY 2023 results.


ABB