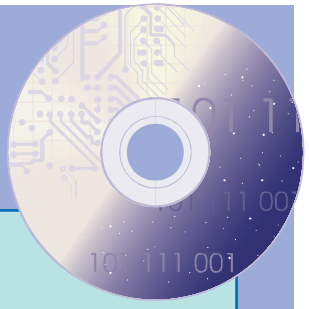
MATH MATTERS

Apply It.



The Math behind IMAGE COMPRESSION

and wavelet based algorithms

Some technical terms used in image compression technology:

Lossy compression, wavelets, fast wavelet transform, wavelet coefficients

Uses and applications:

Reduce a digital image file size while maintaining acceptable image quality

How it works:

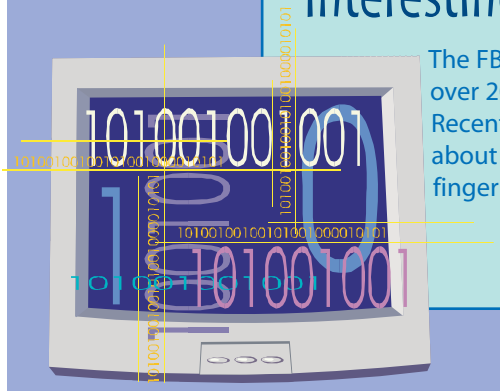
A digital image can be described by listing the color of each pixel. This results in a BIG file. To reduce the size of the file, we can take groups of neighboring pixels that are almost the same color and replace them with an average color. The resulting image is only slightly distorted, and the file is much smaller. This is known as lossy compression.

One of the most successful ways to achieve lossy compression—quickly and while maintaining acceptable quality—is by using a technique that represents an image in terms of special mathematical functions called wavelets. The advantage of wavelets is that they can flexibly describe regions of varying size, shape, and location. A process called "fast wavelet transform" takes an image and computes the coefficients of the wavelets. The averages provide a copy of the original image at lower resolution and the differences provide the wavelet coefficients at that level of resolution.

If we continue this process with the lower-resolution image, eventually we obtain a single overall average and all the differences—the wavelet coefficients—at the various levels of resolution. When neighboring pixels have almost the same value, their difference, and therefore associated wavelet coefficients, are close to zero; throwing away these coefficients will not significantly distort the image. Storing the remaining coefficients along with the overall average value results in a much smaller file.

Conveniently, the image can be reconstructed by running the original transform in reverse. If we start with the overall average and add in the differences for that level of resolution, we can then repeat the process until we have expanded the image to its original size. Some detail will be lost because of the discarded coefficients, but the important features such as edges—where color differences and coefficients are large—will have been preserved.

Interesting Fact:



The FBI began collecting fingerprint cards in 1924, and over the years their collection grew to over 200 million cards. At 10 Megabytes per card, the result was about 2000 Terabytes of data! Recently, the FBI began using wavelets to compress their database. At a compression rate of about 18, a single card is now just 32702 bytes. Thanks to wavelet compression, the FBI's fingerprint database is now just 0.325% of its original size!

The idea for this topic was submitted to the Math Matters: Apply It! contest in January 2007 by the Universitat Politècnica de Catalunya–Barcelona Chapter of SIAM.

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