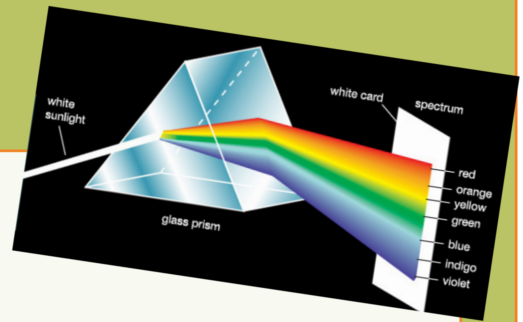
Math Matters

Apply It.

The math behind... Color Outcomes



Technical terms used:

Basis function, linear combination, linear independence, Chebyshev approximation, Chebyshev norm, linear semi-infinite program (LISP)

Uses and applications:

If you are the type of person who values predictability between service promised and service rendered, then the math behind color outcomes is for you. Hair color services are an art, and math helps the color specialist obtain predictable outcomes.

How it works:

A hyperspectral camera measures reflectance over a wide range of wavelengths in a controlled environment [1]. A color specialist selects from hair color stock formulations within a product line. Light interacts with each formulation applied to human hair in a consistent way to produce a spectral response, a mapping from wavelengths between 450 to 1100 nm into percent reflectance from 0 to 100 percent.

We think of a spectral response for a formulation as a basis function. When mixing stock color formulations, we assume that each formulation chemically binds to hair as its relative concentration in the mixture. Finally, we assume the set of spectral responses is linearly independent. We obtain new formulations as linear combinations of stock formulations.

For a specified hair-color, we seek to minimize the difference between the spectral response of the specified hair-color and a linear combination of stock formulation spectral responses in Chebyshev-norm. This is a Chebyshev approximation problem [2]. We further demand the coefficients in the optimal linear combination are all positive and sum to one. The right way to solve a Chebyshev approximation problem is to reformulate it into a linear semi-infinite program (LSIP) [3].

Interesting facts:

Promising research indicates that the hyperspectral imaging camera offers a cost effective and non-invasive means to the early detection of melanoma cancer.

Citations:

[1] L. J. Denes, P. Metes, Y. Li, "Hyperspectral Face Database" (Tech. Rep. CMU-RI-TR-02-25, Carnegie Mellon Univ., Pittsburgh, 2002).

[2] M. Lopez, G. Still, Semi-infinite programming. Eur. J. Oper. Res. 180(2), 491-518 (2007).

[3] R. Hettich, An implementation of a discretization method for semi-infinite programming. Math. Program. 34(3), 354-361 (1986).

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