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**IP1****Crossing Numbers: Dead or Alive?**

The crossing number of a graph is the minimum possible number of edge crossings in a drawing of the graph in the plane. This notion goes back to Turns Brick Factory Problem from the 1940s, but over the years different authors have adopted slightly different definitions. About twenty-five years ago, Gza Tth and I wrote a paper titled Which crossing number is it, anyway? in which we distinguished several competing notions of crossing numbers and analyzed their relationships. Since then, the study of crossing numbers has grown into a substantial area of research in combinatorics and theoretical computer science, with more than a thousand papers devoted to various aspects of the topic. Despite this flurry of activity and the introduction of dozens of new variants, many of the most intriguing problems remain unsolved. In this talk, after a whirlwind survey of the field, I will report on some recent progress on crossing numbers, including joint work with Jacob Fox and Andrew Suk.

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**IP2****Improved Bounds for Sarkozys Theorem**

We discuss recent improved bounds for Sarkozys theorem which concerns the largest set of  $[N]$  with no square common difference. A key tool in our work is the development of an arithmetic analog of global hypercontractivity as developed by Keevash, Lifshitz, Long and Minzer. Based on joint work w. Green.

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**IP3****Graphs with High Chromatic Number**

The classical theorem of Brooks tells us that if a graph  $G$  has no colouring with its maximum degree  $\Delta \geq 3$  colours, then it contains a complete subgraph with  $\Delta + 1$  vertices. Does a similar phenomenon occur when the number of colours is slightly smaller than  $\Delta$ ? Even the next case is unknown: in 1977, Borodin and Kostochka famously conjectured that if  $\Delta \geq 9$  and  $G$  has no  $(\Delta - 1)$ -colouring then it contains a complete subgraph with  $\Delta(G)$  vertices. We discuss various results and questions around this conjecture.

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**IP4****Clustering Coefficients of Graphs**

Many real world networks possess the so-called small world phenomenon where every node is relatively close to every other node and have a large clustering coefficient, i.e., friends of friends are likely friends. The task of learning an adequate similarity measure on various feature spaces often involves graphs with high clustering coefficients. We investigate the clustering effect in sparse clustering graphs by examining the structural and spectral properties as well as

quasirandom classes for strongly regular clustering graphs. In addition, we consider random graph models for clustering graphs that can be use to analyze the behavior of complex networks.

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**IP5****Random Regular Graphs and the Kim-Vu Sandwich Conjecture**

The random regular graph  $G_d(n)$  is selected uniformly at random from all  $d$ -regular graphs on a fixed set of  $n$  vertices. Compared to the binomial random graph  $G(n, p)$ , the lack of independence between the appearance of the edges has made the random regular graph in practice usually much harder to study. In 2004, Kim and Vu conjectured that when  $d$  is much larger than  $\log n$  it is possible to ‘sandwich’ the random regular graph  $G_d(n)$  between two binomial random graphs with a similar edge density, allowing properties of the random regular graph to be inferred from those of the binomial random graph. I will discuss a recent proof of this conjecture, building on work of Gao, Isaev and McKay who proved the conjecture for  $d$  at least  $(\log n)^4$ . This is joint work with Natalie Behague and Daniel Ilkovic.

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**IP6****A Survey of Coarse Graph Theory**

Many graphs and metric spaces have a much simpler ‘coarse’ structure in which distances are only slightly distorted. This idea is captured by the notion of quasi-isometries. For example, string graphs and complete Riemannian planes are both quasi-isometric to planar graphs. The blossoming area of coarse graph theory aims to find the simplest graphs that various classes of graphs and metric spaces are quasi-isometric to. We give a survey of coarse graph theory and highlight applications to various other areas of geometry, graph algorithms, and geometric group theory.

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**IP7****On the Structure of Sets of Bounded VC2-dimension in Elementary Abelian  $p$ -groups**

We begin by presenting work of the author and Julia Wolf from 2021 showing that any subset of an elementary abelian  $p$ -group of bounded VC2-dimension is well approximated by a union of atoms of a quadratic factor of bounded complexity. This result relies on a general quadratic arithmetic regularity lemma of Green and Tao, and consequently, yields bounds on the linear and quadratic complexities of the factor which are of tower-type. We then present more recent work which improves the bound on the quadratic complexity to logarithmic (joint with Julia Wolf), and work which improves the bound on the linear complexity to triple exponential (joint with Hannah

Sheats).

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### IP8

#### Refined Absorption and Applications to Probabilistic and Extremal Design Theory

The study of combinatorial designs has a rich history spanning nearly two centuries. In a recent breakthrough, the notorious Existence Conjecture for Combinatorial Designs dating back to the 1800s was proved in full by Keevash via the method of randomized algebraic constructions. Subsequently Glock, Khn, Lo, and Osthus provided an alternate purely combinatorial proof of the Existence Conjecture via the method of iterative absorption. We introduce a novel method of "refined absorption" for designs and use it to provide a new alternate proof of the Existence Conjecture. The method can also be applied in a black-box fashion to many other design theory problems; in this talk we survey some recent results including high girth designs, the high minimum degree setting, and finding sufficiently spread distributions on designs. This is based on joint work with Luke Postle and various subsets of Cicely (Cece) Henderson, Tom Kelly, and Thomas Lesgourgues.

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### SP1

#### 2026 Dnes Knig Prize Lecture: High-Girth Steiner Triple Systems

We prove a 1973 conjecture due to Erdos on the existence of Steiner triple systems with arbitrarily high girth. To do so, we develop tools to track the evolution of constraints in random processes, with specific interest in the iterative absorption of Glock, Khn, Lo, and Osthus, and develop efficient absorbers for Steiner systems. These directions have seen further exciting progress in recent years, and we hope to provide a concrete view of the high-level ideas in this area. Joint work of the prize recipients with Matthew Kwan and Michael Simkin.

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### CP1

#### Counting Defective Parking Functions of Fixed Maximum Preference

Defective parking functions are a variant of classical parking functions in which one tracks the number of cars that fail to park; this number is called the defect. In this work we study the interaction between defect and maximum preference in nondecreasing defective parking functions. Using a modified Dyck path reflection argument, we enumerate nondecreasing defective parking functions of length  $n$  with defect  $d$  and fixed maximum preference  $a_n$ . We show that the resulting counts  $M(n, d, a_n)$  are given by

entries of Catalan's triangle, specifically

$$M(n, d, a_n) = T(n + d, a_n - d),$$

where  $T(i, j)$  denotes the  $(i, j)$ th Catalan triangle entry. This provides a refined Catalan type enumeration of defective parking functions that records exact maximum preference.

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### CP1

#### Minuscule Coxeter Dressians

Matroidal subdivisions of hypersimplices are well studied and have been demonstrated to have deep connections throughout mathematics. These subdivisions are governed by a tropical prevariety known as the Dressian, which contains the tropicalization of the Grassmannian. Matroids are in fact special cases of Coxeter matroids in type A associated with minuscule representations. In general, Coxeter matroids admit polyhedral encodings. We study a class of tropical prevarieties which we call Coxeter Dressians. They arise from equations capturing a generalization of valuated symmetric basis exchange, which is well studied for usual matroids. In particular we show that the subdivisions of Coxeter matroid polytopes induced by points in these prevarieties consist of cells which are again Coxeter matroidal. We study these prevarieties for Coxeter matroids in types B, C, D, and E. For reasonable examples, we directly compute these prevarieties using the OSCAR software system. Our work generalizes the results mentioned above for usual matroids.

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### CP1

#### Balanced Weighted Motzkin Paths: Pearson Structure and Saddlepoint Asymptotics

We study weighted Motzkin paths whose up, level, and down step multiplicities depend affinely on the current height. In the balanced case, the associated exponential generating function satisfies a first-order Pearson-type partial differential equation. Solving this equation by the method of characteristics yields explicit closed-form expressions for the generating functions in all drift regimes, classified by the discriminant of the characteristic system. The solution exhibits a moving algebraic singularity, which governs both the local and global asymptotic behavior of the paths. From this structure we derive a Gaussian central window for the terminal-height distribution, as well as an explicit limit cumulant generating function and an  $n$ -speed

large deviation principle. For finite  $n$ , we apply Daniels' lattice saddlepoint approximation, obtaining a single uniform formula that remains accurate from the center to the tails. In quadratic regimes, this approximation achieves a uniform interior relative error of order  $O(n^{-1})$ . The results reveal a novel link between the geometry of Pearson-type distributions and uniform saddlepoint methods, and extend naturally to other weighted lattice path models and to tridiagonal recurrences.

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## CP1

### The Poincaré Polynomial of a Finite Coxeter Folding

Folding subgroups give a way to realize non-simply-laced Coxeter groups as subgroups of simply-laced Coxeter groups through an admissible set partition of the generating set of the group. We study how folding subgroups are distributed length-wise by calculating the length generating function of the subgroup with respect to the length of the ambient group,  $U_{\tilde{W}}^W(q) := \sum_{w \in \tilde{W}} q^{\ell(\phi(w))}$ , where  $\phi : \tilde{W} \rightarrow W$  is the corresponding embedding map. For simply-laced Coxeter groups of type  $A$  and  $D$ , let  $\ell_A$  and  $\ell_D$  denote their corresponding length functions. Then

$$\begin{aligned} \bullet \quad U_{B_n}^{A_{2n-1}}(q) &= \sum_{w \in B_n} q^{\ell_A(\phi(w))} = [2]_q [3]_{-q} [4]_q \cdots [2n-1]_{-q} [2n]_q \\ \bullet \quad U_{B_n}^{A_{2n}}(q) &= \sum_{w \in B_n} q^{\ell_A(\phi(w))} = [2]_q [3]_{-q} [4]_q \cdots [2n-1]_{-q} [2n]_q [2n+1]_{-q} \\ \bullet \quad U_{B_n}^{D_{n+1}}(q) &= \sum_{w \in B_n} q^{\ell_D(\phi(w))} = [2]_q [3]_{-q} [4]_q \prod_{k=3}^n ([2k+1]_q - q^k) \\ \bullet \quad U_{I_2(2n)}^{A_{2n-1}}(q) &= \sum_{w \in I_2(2n)} q^{\ell_A(\phi(w))} = [2]_{q^{n-1}} [2]_{q^n} [n]_{q^{2n-1}} \\ \bullet \quad U_{I_2(2n+1)}^{A_{2n}}(q) &= \sum_{w \in I_2(2n+1)} q^{\ell_A(\phi(w))} = [2]_{q^n} [2n+1]_{q^n} \end{aligned}$$

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## CP2

### Vertical Compressibility of Random Skip Lists

A skip list is a probabilistic data structure for dictionary operations designed to be a fast alternative to balanced trees. We can associate to any skip list a random binary tree whose edges run horizontally or vertically. This tree is generated by a sequence of i.i.d. geometric random variables  $G_1, G_2, \dots, G_n$  which determine the heights of the vertical branches. It turns out that skip list operations can be carried out most efficiently when the set of the  $G_i$  is coprime. We call such trees *vertically compressed*. In this talk, we will show that random skip list trees are vertically compressed with high probability. Specifically, we will derive the exact distribution of the greatest common divisor of  $G_1, G_2, \dots, G_n$  and a rate for the almost sure convergence to 1. We will also borrow techniques from analytic combinatorics to enumerate the number of vertically

compressed skip list trees of a prescribed size and to obtain limit laws for various path length statistics over these trees.

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## CP2

### Entropy-Based Comparison of Classical and Quantum Random Number Generators

This work presents a unified, entropy-based framework for comparing classical pseudorandom number generators (PRNGs) and quantum random number generators (QRNGs), with a particular focus on understanding how different randomness-generation mechanisms influence statistical quality and computational efficiency. The study evaluates representative classical generators, including the Mersenne Twister and an expander-based random-walk PRNG, alongside quantum approaches such as the standard Hadamard-gate QRNG and a newly proposed quantum-walkbased QRNG. The paper introduces a quantum-walk QRNG that leverages discrete-time quantum walks on graphs. By exploiting superposition, interference, and tensor-product graph decompositions, this construction generalizes classical walk-based PRNGs. All generators are evaluated under a common statistical methodology grounded in information-theoretic metrics including Shannon entropy, min-entropy, entropy deficit, Kullback-Leibler divergence, Jensen-Shannon divergence, and total-variation distance. Runtime and sampling efficiency are also measured to capture practical performance trade-offs. The work clarifies the conceptual and practical distinctions between algorithmic randomness and physically generated quantum randomness. It demonstrates that walk-induced mixing, classical or quantum, is a powerful principle for randomness generation and it positions quantum walks as a promising foundation for future high-assurance QRNG designs.

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## CP2

### The Problem of Locating Objects in Networks: Recent Developments

In order to locate objects moving through networks, sensors are placed at certain nodes of a network which measure distance to objects in the network. This set of sensors locates objects in a network if the vector of distances of each object to the sensors is distinct. A natural problem is to determine the minimum number of sensors required to locate objects at any node of the network. For this purpose,

a network is represented by a graph  $G$ , the sensors by the set  $S \subseteq V(G)$ . A vertex (i.e. a sensor)  $s \in S$  distinguishes a pair of vertices  $x, y \in V(G)$  if  $d(x, s) \neq d(y, s)$ . The set  $S$  is a resolving set of  $G$  if for every pair of vertices there exists a sensor  $s \in S$  which distinguishes it. The cardinality of a smallest resolving set  $S$  is the metric dimension  $\dim(G)$ . Finding the metric dimension of a graph is NP-hard, so we establish methods enabling faster calculation for certain families of graphs, such as unicyclic graphs and cacti. We also consider making a resolving set tolerant to the failure of one sensor. The set  $S$  is fault-tolerant if  $S \setminus \{s\}$  remains a resolving set for any  $s \in S$ . We show that in the worst case the cardinality of a smallest fault-tolerant resolving set can be exponential in  $\dim(G)$ . This is generalized to the case when each pair of vertices of  $G$  is distinguished by at least  $k$  sensors. A variation, the weak  $k$ -metric dimension, is also considered.

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### CP3

#### How Many Coin Tosses Would You Need until You Get $n$ Heads Or $m$ Tails?

In the classical negative binomial distribution, the expected number of coin tosses required to obtain  $n$  Heads from a coin with the probability of getting Heads  $p$  is  $n/p$ . We study a natural generalization in which the process stops when reaching either  $n$  Heads or  $m$  Tails. Although this setup is related to the classical ‘‘Problem of points,’’ prior work by Fermat and Pascal has focused on the probability of getting  $n$  Heads vs getting  $m$  Tails, rather than on the duration of the game. Using probabilistic techniques, together with symbolic computation, we derive probability generating functions, closed-form expressions, and asymptotic formulas for moments of the stopping time to reach either  $n$  Heads or  $m$  Tails. In the symmetric case  $n = m$ , we show that the expected duration with a fair coin is a polynomial in  $p$  whose coefficients are Catalan numbers. Joint work with Svante Janson and Doron Zeilberger.

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### CP3

#### Structured and Punctured Nullstellensätze

A Nullstellensatz is a theorem providing information on polynomials that vanish on a certain set: David Hilberts Nullstellensatz (1893) is a cornerstone of algebraic geometry, and Noga Alons Combinatorial Nullstellensatz (1999) is a powerful tool in the Polynomial Method, a technique

used in combinatorics. Alons Theorem excludes that a polynomial vanishing on a grid contains a monomial with certain properties. This theorem has been generalized in several directions, two of which we will consider in detail: Terence Tao and Van H. Vu (2006), Uwe Schauz (2008) and M. Lasón (2010) exclude more monomials, and recently, Bogdan Nica (2023) improved the result for grids with additional symmetries in their side edges. Simeon Ball and Oriol Serra (2009) incorporated the multiplicity of zeros and gave Nullstellensätze for punctured grids, which are sets of the form  $X \setminus Y$  with both  $X, Y$  grids. We generalize some of these results; in particular, we provide a common generalization to the results of Schauz and Nica. To this end, we establish that during multivariate polynomial division, certain monomials are unaffected. This also allows us to generalize Pete L. Clarks proof of the nonzero counting theorem by Alon and Füredi to punctured grids.

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### CP3

#### Lower bounds for multivariate independence polynomials and their generalisations

In statistical physics, the multivariate hard-core model describes a system of particles, each of which receives its own fugacity. In graph-theoretic language, the partition function of the model translates to the *multivariate independence polynomial*, i.e., the multiaffine generalisation of the independence polynomial, defined by  $Z_G(\lambda_1, \dots, \lambda_n) := \sum_{I \in \mathcal{I}(G)} \prod_{v \in I} \lambda_v$ , where  $\mathcal{I}(G)$  denotes the set of all independent sets in a graph  $G$  on  $[n] := \{1, 2, \dots, n\}$ . We prove that for every simple graph  $G$  on  $[n]$  and  $\lambda_1, \dots, \lambda_n \geq 0$ ,

$$Z_G(\lambda_1, \dots, \lambda_n) \geq \prod_{i=1}^n (1 + (d_i + 1)\lambda_i)^{1/(d_i+1)},$$

where  $d_1, \dots, d_n$  is the degree sequence of  $G$ . This generalises a result of Sah, Sawhney, Stoner, and Zhao, who proved the univariate case  $\lambda_1 = \dots = \lambda_n = \lambda$ . We further conjecture that our inequality should generalise to other antiferromagnetic models and give some evidence in support of it. In particular, for  $\lambda_i, \mu_i \geq 0$ ,  $1 \leq i \leq n$ , we obtain a stronger inequality

$$\sum_{\substack{I, J \in \mathcal{I}(G) \\ I \cap J = \emptyset}} \prod_{v \in I} \lambda_v \prod_{u \in J} \mu_u \geq \prod_{i=1}^n (1 + (d_i + 1)(\lambda_i + \mu_i) + d_i(d_i + 1)\lambda_i \mu_i)^{1/(d_i+1)},$$

which proves our conjecture for a multiaffine generalisation of the *semiproper colouring* partition function with two proper colours. Our key technical steps for both theorems are obtained by using a custom mathematical research agent built on top of Gemini Deep Think, which can be seen as a benchmark demonstrating that the current state-of-the-art language models can, in part, assist with mathematical research.

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### CP3

#### Coloring Sparse Random Cayley Graphs

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### CP4

#### Connectivity Keeping Trees in Triangle-Free Graphs

In 2012, Mader conjectured that for any tree  $T$  of order  $m$ , every  $k$ -connected graph  $G$  with minimum degree at least  $\lfloor \frac{3k}{2} \rfloor + m - 1$  contains a subtree  $T' \cong T$  such that  $G - V(T')$  remains  $k$ -connected. In 2022, Luo, Tian, and Wu considered an analogous problem for bipartite graphs and conjectured that for any tree  $T$  with bipartition  $(X, Y)$ , every  $k$ -connected bipartite graph  $G$  with minimum degree at least  $k + \max\{|X|, |Y|\}$  contains a subtree  $T' \cong T$  such that  $G - V(T')$  remains  $k$ -connected. In this paper, we relax the bipartite assumption by considering triangle-free graphs and prove that for any tree  $T$  of order  $m$ , every  $k$ -connected triangle-free graph  $G$  with minimum degree at least  $2k + 3m - 4$  contains a subtree  $T' \cong T$  such that  $G - V(T')$  remains  $k$ -connected. Furthermore, we establish refined results for specific subclasses such as bipartite graphs or graphs with girth at least five. This is a joint work with Shinya Fujita, Boram Park, and Homoon Ryu.

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### CP4

#### Every Connected Graph Admits a Local Antimagic Orientation and Almost Every Graph Admits An Antimagic Orientation

An undirected graph  $G$  is said to admit an antimagic orientation if there exist an orientation  $D$  and a bijection between  $E(G)$  and  $\{1, 2, \dots, |E(G)|\}$  such that any two vertices have distinct vertex sums, where the vertex sum of a vertex is the sum of the labels of the in-edges minus that of the out-edges incident to the vertex. It is conjectured by Hefetz, Mütze, and Schwartz that every connected graph admits an antimagic orientation. A weak version of this problem is to require the distinct vertex sums only for the adjacent vertices. In that case, we say the graph admits a local antimagic orientation. Chang, Jing, and Wang conjectured that every connected graph admits a local antimagic orientation. In this talk, we give an affirmative

answer to the conjecture of Chang et al., and show that almost every graph satisfies the conjecture of Hefetz et al.

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### CP4

#### On Almost König-Egervary Graphs

Let  $\alpha(G)$  denote the cardinality of a maximum independent set, while  $\mu(G)$  be the size of a maximum matching in graph  $G = (V, E)$ . If  $\alpha(G) + \mu(G) = |V|$ , then  $G$  is a König-Egervary graph, while if  $\alpha(G) + \mu(G) = |V| - 1$ ,

then  $G$  is an almost König-Egervary graph (known also as a 1-König-Egervary graph). If  $G$  is not König-Egervary, but there exists a vertex  $v \in V$  (an edge  $e \in E$ ) such that  $G - v$  ( $G - e$ ) is König-Egervary, then  $G$  is called a vertex almost König-Egervary graph (an edge almost König-Egervary graph, respectively). This presentation explores the interrelations between almost König-Egervary graphs and their vertex- and edge-based counterparts. We demonstrate that every graph  $G$  that is either an edge-almost or a vertex-almost König-Egervary graph is necessarily almost König-Egervary. However, we show that the converse holds only under some specific additional conditions.

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### CP5

#### A Note on Large Degenerate Induced Subgraphs in Sparse Graphs

Given a graph  $G$  and a non-negative integer  $d$  let  $\alpha_d(G)$  be the order of a largest induced  $d$ -degenerate subgraph of  $G$ . In this talk, we will prove that for any pair of non-negative integers  $k > d$ , if  $G$  is a  $k$ -degenerate graph, then  $\alpha_d(G) \geq \max\{\frac{(d+1)n}{k+d+1}, n - \alpha_{k-d-1}(G)\}$ . For  $k$ -degenerate graphs this improves a more general lower bound of Alon, Kahn, and Seymour. By modifying our argument, we obtain improved lower bounds on  $\alpha_d(G)$  for graphs of bounded genus. This extends earlier work on degenerate subgraphs of planar graphs.

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### CP5

#### On 2-Connected Graphs Avoiding Cycles of Length Divisible by 4

For two integers  $k$  and  $\ell$ , an  $(\ell \bmod k)$ -cycle means a cycle of length  $m$  such that  $m \equiv \ell \pmod{k}$ . In 1977, Bollobas

proved a conjecture of Burr and Erdős by showing that if  $\ell$  is even or  $k$  is odd, then every  $n$ -vertex graph containing no  $(\ell \bmod k)$ -cycles has at most a linear number of edges in terms of  $n$ . Since then, determining the exact extremal bounds for graphs without  $(\ell \bmod k)$ -cycles has emerged as an interesting question in extremal graph theory, though the exact values are known only for a few integers  $\ell$  and  $k$ . Recently, Györi, Li, Salia, Tompkins, Varga and Zhu proved that every  $n$ -vertex graph containing no  $(0 \bmod 4)$ -cycles has at most  $\lfloor \frac{19}{12}(n-1) \rfloor$  edges, and they provided extremal examples that reach the bound, all of which are not 2-connected. In this paper, we show that a 2-connected graph without  $(0 \bmod 4)$ -cycles has at most  $\lfloor \frac{3n-1}{2} \rfloor$  edges, and this bound is tight by presenting a method to construct infinitely many extremal examples.

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MS1

### A squishy grid

The grid graph has vertex set being the 2-dimensional integer lattice, and two vertices are adjacent if they differ by one in exactly one coordinate. We prove that there is a "squishy grid": a weighing of the edges of the grid graph such that the distance between any two vertices is within an additive constant of their Euclidean distance. Some version of this problem goes back to Erdős, and this result disproves conjectures of Cai, Chen, Du, Filtser, Pettie, and Skora.

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MS1

### A Lovsz-Kneser theorem for triangulations

In a highly influential paper from 1978, Lovsz used topological methods to determine the chromatic number of the Kneser graph of the set of  $k$ -element subsets of a set with  $n$  elements. In this talk, we will discuss the Kneser graph of the set of triangulations of a convex  $n$ -gon and a recent proof that the chromatic number of this graph is  $n-2$ . Joint work with Anton Molnar, Michael Zheng and Daniel Zhu.

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MS1

### Euclidean Noncrossing Steiner Spanners

A Euclidean noncrossing Steiner  $(1+\epsilon)$ -spanner for a point set  $P \subset \mathbb{R}^2$  is a planar straight-line graph that, for any two points  $a, b \in P$ , contains a path whose length is at most  $1+\epsilon$  times the Euclidean distance between  $a$  and  $b$ . We construct a Euclidean noncrossing Steiner  $(1+\epsilon)$ -spanner with  $O(n/\epsilon^{3/2})$  edges for any set of  $n$  points in the plane. This result improves upon the previous best upper bound of  $O(n/\epsilon^4)$  obtained nearly three decades ago. We also establish an almost matching lower bound: There exist  $n$  points in the plane for which any Euclidean noncrossing Steiner  $(1+\epsilon)$ -spanner has  $\Omega_\mu(n/\epsilon^{3/2-\mu})$  edges for any  $\mu > 0$ . Our lower bound uses recent generalizations of the Szemerédi-Trotter theorem to disk-tube incidences in geometric measure theory. (Joint work with Sujoy Bhore, Sndor Kisfaludi-Bak, Lazar Milenkovic, Karol Wegrzycki, and Sampson Wong.)

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MS1

### Tba

Planar separator theorems and  $r$ -divisions are among the most useful tools in planar graph theory, but in many geometric applications one wants to control more than just size and boundary. In this talk I will describe a mixed-cap  $r$ -division theorem in which one can simultaneously control several weight functions on vertices, edges, and faces, and a sparse-set variant that keeps designated subsets from accumulating inside the final regions. I will then show how these refinements can be used in incidence geometry. In particular, I will explain an  $r$ -division-based proof of the  $O(n^{4/3})$  bound for incidences between  $n$  points and  $n$  pseudo-segments, and discuss how the same framework fits into the broader PachSharir paradigm. I will also describe how these decomposition tools suggest ErdosHajnal type phenomena for intersection graphs of pseudo-segments. All results are joint with Andrew Suk.

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MS2

### $R(3, k)$ in Two Bites

The triangle-Ramsey number  $R(3, k)$  is the smallest  $n$  such that any  $n$ -vertex graph contains either a triangle or an independent set of size  $k$ . Through the hard work of many researchers, around 30 years ago the order of magnitude of  $R(3, k)$  was determined to be  $\frac{k^2}{\log k}$ . The correct leading constant is now of serious interest; the main result of this talk improves the best known lower bound on this constant from  $\frac{1}{3}$  (a recent result of Campos, Jenssen, Michelen, and Sahasrabudhe) to  $\frac{1}{2}$ , using a flexible random construction.

Based on joint work with Zion Hefty, Paul Horn, and Florian Pfender.

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### MS3

#### Jeu De Taquin Forests and the Inverse Infinite Rsk Correspondence

the infinite RSK correspondence maps infinite sequences of distinct numbers in  $[0,1]$  to an infinite standard Young tableau, and can be viewed as the projective limit of the Robinson-Schensted correspondence applied to permutations of finite orders. When the infinite sequence is an i.i.d. sequence of uniform random variables in  $[0,1]$ , in previous work we showed that infinite RSK establishes an isomorphism of measure-preserving dynamical systems between the shift map on i.i.d.  $U[0,1]$  sequences and the jeu de taquin map on infinite Young tableau together with the infinite Plancherel measure. The calculation of the inverse map establishing this isomorphism required successive iterations of the jeu de taquin map. After surveying these results, I will describe a new point of view showing that the entries in the original i.i.d. sequence can in fact be recovered in a more direct way through a graphical decomposition of the infinite tableau into a "jeu de taquin forest", a novel combinatorial structure encoding in a visible way useful information about the behavior of the tableau under inverse RSK and jeu de taquin. Based on joint work with Piotr Sniady.

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### MS4

#### Roth-type Theorems in Dense $K_{s,t}$ -free Sets

We prove that for all integers  $2 \leq s \leq t$  dense  $K_{s,t}$ -free subsets of  $[N]$  must always contain nontrivial solutions to any translation-invariant equation in at least five variables. This generalizes recent results of Conlon-Fox-Sudakov-Zhao and Prendiville. Joint work with Cosmin Pohoata and Max Wenqiang Xu.

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### MS4

#### Quasipolynomial Bounds for the Corners Theorem

We will present recent progress on the corners problem. More specifically, we will sketch an argument that any subset  $A$  of a finite abelian group  $G$  such that  $A$  avoids configurations of the form  $(x, y), (x + d, y), (x, y + d)$  has density about most  $\exp(-(\log |G|)^c)$ . Time permitting, we will also discuss the problems connection to communication complexity in theoretical computer science. Based on

joint work w/ Michael Jaber, Yang P. Liu, Shachar Lovett, and Mehtaab Sawhney.

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### MS4

#### Intersecting Families in Polynomials over Finite Fields

Recently, Adriaensen, and Aguglia-Csajbók-Weiner studied an analogue of the celebrated Erdős-Ko-Rado theorem for one-variable polynomials over finite fields: for two polynomials  $f(x)$  and  $g(x)$ , we naturally say that they intersect if their graphs intersect. In this talk, I will discuss the same problem for multivariate polynomials over finite fields and highlight several interesting phenomena that are unique to multivariate polynomials. This is joint work with Shamil Asgarli and Bence Csajbók.

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### MS4

#### Chowla's Cosine Problem Through Spectral Graph Theory

Cosine polynomials of the form  $f(x) = \cos(a_1x) + \cos(a_2x) + \dots + \cos(a_nx)$  appear extensively in number theory and combinatorics. An old problem of Ankeny and Chowla asks: if  $a_1 \dots a_n$  are distinct positive integers, how small must the minimum value of  $f(x)$  on  $[0, 2p]$  be? Concurrently with Benjamin Bedert, we show that the minimum value of  $f(x)$  must be polynomial in  $n$ . Our proof is based on a new theorem in spectral graph theory: if  $G$  is a graph with average  $d$  and adjacency least eigenvalue at least  $-d^2$ , then  $G$  must contain a clique of size  $d^{1-O(?)}$ . Joint work with Zhihan Jin, Aleksa Milojevic, and István Tomon, with thanks to Ilya Shkredov.

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### MS5

#### Range of Random Lipschitz Functions from the Discrete Hypercube and Related Structures

We study the typical behavior of integer-valued graph homomorphisms and Lipschitz functions on the Hamming cube. We show that a uniformly random centered  $Z$ -homomorphism on the two middle layers of  $Q_{2d-1}$  takes, with high probability, at most five distinct values. More generally, for  $M^2 \log M \ll d$ , a random  $M$ -Lipschitz function on  $Q_d$  concentrates on an interval of length  $M+1$  on all but a vanishing fraction of vertices.

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### MS5

#### Small Eigenvalues, Large Cuts, and Chowla's Cosine Problem

What can be said about graphs which do not have very negative eigenvalues? We prove that every graph with average degree  $d$  and smallest eigenvalue of the adjacency matrix  $|\lambda_n| \leq d^\gamma$  contains a clique of size  $d^{1-O(\gamma)}$ . This simple statement had several very interesting consequences - it yields the first polynomial bound for Chowla's cosine problem (1965): for every finite set  $A \subseteq \mathbb{Z}_{>0}$ , the minimum of the cosine polynomial satisfies

$$\min_{x \in [0, 2\pi]} \sum_{a \in A} \cos(ax) \leq -|A|^{1/10-o(1)}.$$

Also, the same methods show that any  $m$  edge  $H$ -free graph  $G$  has a maximum cut of size at least  $m/2 + \Omega(m^{0.51})$ . This makes progress on an old conjecture of Alon, Bollobas, Krivelevich and Sudakov which posits that  $\text{MaxCut}(G) \geq m/2 + \Omega(m^{3/4})$ . Based on joint work with Zhihan Jin, István Tomon and Shengtong Zhang.

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### MS5

#### The Turn Density of 4-Uniform Tight Cycles

I will discuss several recent results on the Turn density of long cycle-like hypergraphs. These results (due to KamcevLetzterPokrovskiy, BaloghLuo, and myself) all follow a similar framework, and I will outline a general strategy to prove Turn-type results for tight cycles in larger uniformities or for other "cycle-like" hypergraphs. One key ingredient in this framework, which I hope to prove in full, is a hypergraph analogue of the statement that a graph has no odd closed walks if and only if it is bipartite. More precisely, for various classes  $\mathcal{C}$  of "cycle-like"  $r$ -uniform hypergraphs including, for any  $k$ , the family of tight cycles of length  $k$  modulo  $r$  we equivalently characterize  $\mathcal{C}$ -hom-free hypergraphs as those admitting a certain type of coloring of  $(r-1)$ -tuples of vertices. This provides a common generalization of results due to KamcevLetzterPokrovskiy and BaloghLuo.

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### MS5

#### Spin Models on Random Regular Graphs

Spin models are a source of many interesting problems in statistical physics but can often be viewed as purely combinatorial objects; for example, the Ising model, introduced as a model for magnetization, is a parametrized distribution over 2-vertex-colorings of a graph. The behavior of the Ising model on the random regular graph is well-understood, in part due to the limiting behavior of the local graph structure. We study the fixed-magnetization Ising

model (where the number of, say, blue vertices is fixed) in the so-called non-reconstruction regime. We characterize the local structure of the distribution and use this to prove that the positive-temperature Zdeborov–Boettcher conjecture on max-cut and min-bisection holds up to the reconstruction threshold: on the random  $d$ -regular graph, the expected fraction of bichromatic edges in the anti-ferromagnetic Ising model plus the expected fraction of bichromatic edges in the zero-magnetization ferromagnetic Ising model equals  $1+o(1)$ .

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### MS6

#### Binary matroids excluding an even circuit

We study the maximum size of a simple binary matroid of rank  $n$  which has no circuit of length exactly  $2k$ . We prove an upper bound of the form  $O(2^{n/k})$ , which matches a lower bound construction coming from coding theory. That is, for every  $k$  at least 2, there exists a constant  $C$  such that every set  $A$  of at least  $C2^{n/k}$  vectors in  $GF(2)^n$  contains  $2k-1$  linearly independent vectors whose sum is in  $A$ . Joint work with Liana Yepremyan.

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### MS6

#### Induced rational exponents near two

Given a bipartite graph  $H$  and a natural number  $s$ , let  $*(n, H, s)$  denote the maximum number of edges in an  $n$ -vertex graph that contains neither  $K_{s,s}$  nor an induced copy of  $H$ . Hunter, Milojević, Sudakov, and Tomon conjectured that  $*(n, H, s) = O_{H,s}(n)$  whenever  $H$  is connected. Motivated by this conjecture and the rational exponents conjecture, Dong, Gao, Li, and Liu [?] conjectured that for every rational  $r \in (1, 2)$  there is a bipartite graph  $H$  and an  $s_0$  such that  $*(n, H, s) = \Theta(n^r)$  for all  $s \geq s_0$ . We prove that the latter conjecture holds for all rationals  $r = 2 - a/b$ , where  $a, b \in \mathbb{N}$  satisfy  $b \geq \max\{a, (a-1)^2\}$ . Our result extends a well-known result of Conlon and Janzer to the induced setting and adds more evidence to support the former conjecture. This is joint work with Sean Longbrake.

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## MS6

### Multicolour size Ramsey number of a path

The  $r$ -color size Ramsey number of a path  $P_k$  is the smallest number of edges in a graph with the property that every  $r$ -coloring of the edges of  $G$  contains a monochromatic copy of  $P_k$ . The two-color case, first posed by Erdos, was resolved by Beck in 1983, but the question of the optimal dependence on the number of colors  $r$  remained open. The best known upper bound is  $O(r^2 \log rk)$  with the best constant due to Dudek and Pralat, while the lower bound  $\Omega(r^2 k)$  is due to Krivelevich was generally believed to be the truth. In this talk, I will discuss recent work with Csongor Beke and Julian Sahasrabudhe that closes this  $\log r$  gap and determines the  $r$ -color size Ramsey number of the path up to constant factors. Somewhat surprisingly, we do this by improving the lower bound via a new randomized edge-coloring strategy.

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## MS6

### Inequalities for Walks in Graphs

Let  $G$  be a graph and  $w_k(G)$  denote the number of walks in  $G$  of length  $k$ . There are a number of known inequalities of form  $w_{a_1}(G) \cdots w_{a_n}(G) \geq w_{b_1} \cdots w_{b_n}$  for certain  $a_1, \dots, a_n, b_1, \dots, b_n$  with  $a_1 + \dots + a_n = b_1 + \dots + b_n$ . We ask "What are all the true inequalities of this form?" and give a simple answer.

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## MS7

### Some New Constructions in Additive Combinatorics

We prove super exponential lower bounds for the three color van der Waerden numbers. This is the first super-polynomial improvement over the  $c3^k/k$  lower bound established by Erdos and Lovasz in 1975. Joint with Jacob Fox.

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## MS7

### Three applications of coverings to difference patterns

We show that a conceptually simple covering technique has surprisingly rich applications to density theorems and conjectures on patterns in sets involving set differences. These applications fall into three categories: (i) analogues of these

statements to distance 2 versions of the pattern, (ii) reduction of these statements to relative versions, and (iii) reductions of these statements to a quasirandom case with respect to some quantities that affect the number of realisations of the pattern.

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## MS7

### On Exponential Freiman Dimension

The exponential Freiman dimension of a finite set  $A \subset \mathbb{R}^m$ , introduced by Green and Tao in 2006, represents the largest positive integer  $d$  for which  $A$  contains the vertices of a non-degenerate  $d$ -dimensional parallelepiped. For every  $d \geq 1$ , we precisely determine the largest constant  $C_d > 0$  (exponential in  $d$ ) for which

$$|A + A| \geq C_d |A| - O_d(1)$$

holds for all sets  $A$  with exponential Freiman dimension  $d$ . Joint work with Akshat Mudgal, Cosmin Pohoata, and Xuancheng Shao

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## MS7

### Convexity and Sum Sets

This talk considers a rather vague question that generalises the sum-product principle: to what extent are convex functions guaranteed to disrupt additive structure? We study some more precise formulations of this question, including presenting an example of a convex set containing many non-trivial arithmetic progressions.

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## MS8

### Hypergraphs Accumulate

We show that the set of Turán densities of  $k$ -uniform hypergraphs has infinitely many accumulation points in  $[0, 1)$  for every  $k \geq 3$ .

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## MS8

### Asymptotically Half of Binary Words are Shuffle

**Squares**

A binary shuffle square is a binary word of even length that can be partitioned into two disjoint, identical subwords. We prove that asymptotically half of all binary words of length  $2n$  are shuffle squares, verifying a conjecture of He, Huang, Nam, and Thaper. We reframe the problem using a discrete stochastic process and define an algorithm for partitioning words. Joint work with Xiaoyu He.

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**MS8****Separators for Geometric Intersection Graphs**

A balanced separator in an  $n$ -vertex graph is a set of vertices whose removal results in a graph whose connected components each contain at most  $2n/3$  vertices. A famous result of Lipton and Tarjan states that planar graphs have balanced separators of size  $O(\sqrt{n})$ . In a more geometric setting, string graphs (intersection graphs of curves in the plane) with  $m$  edges are known to have balanced separators of size  $O(\sqrt{m})$ . Interestingly, this result can be viewed as an extension of the Lipton–Tarjan separator theorem. In this talk, I will discuss some ways in which these results do and do not generalize to intersection graphs of higher-dimensional geometric objects. Joint work with Jacob Fox.

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**MS8****Even-Degeneracy of a Random Graph**

A graph is even-degenerate if one can iteratively remove a vertex of even degree at each step until at most one edge remains. Recently, Janzer and Yip showed that the Erdős–Rényi random graph  $G(n, 1/2)$  is even-degenerate with high probability, and asked whether an analogous result holds for any general  $G(n, p)$ . We answer this question for any constant  $p \in (0, 1)$  in affirmation by proving that  $G(n, p)$  is even-degenerate with high probability.

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**MS9****New Bounds for Linear Arboricity and Related Problems**

In 1980, Akiyama, Exoo, and Harary conjectured that the edges of every graph with maximum degree  $\Delta$  can be decomposed into at most  $\lceil (\Delta + 1)/2 \rceil$  linear forests, where a linear forest is a collection of vertex-disjoint paths. This conjecture was confirmed asymptotically by Alon in 1988 who showed that  $(1 + o(1)) \cdot \Delta/2$  linear forests suffice. The current best bound is due to Lang and Postle who showed that there is a decomposition into at most  $\Delta/2 + O(\sqrt{\Delta} \log^4 \Delta)$  linear forests. We show that any graph on vertices can be decomposed into at most  $\Delta/2 + O(\log n)$  linear forests. This improves the previous upper bounds for  $\Delta = \Omega(\log^2 n)$ . Along the way, we show that any  $d$ -regular graph on  $n$  vertices has a spanning linear forest with at most  $2 \cdot n/(d + 1)$  paths. This resolves a conjecture of Feige and Fuchs and confirms a well-known conjecture of Magnant and Martin up to a factor of 2.

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**MS9****The (fractional)  $K_r$ -decomposition threshold for  $r$ -partite graphs**

A central topic in extremal design theory is determining the minimum degree threshold for a graph  $G$  (satisfying some simple necessary conditions) to have a  $K_r$ -decomposition. Especially the partite case, where we additionally require  $G$  to be balanced and  $r$ -partite, remains elusive. By the works of Montgomery and Barber–Khn–Lo–Osthus–Taylor, it is known that having at least  $(1 - 10^{-6} r^{-3} + o(1))v(G)/r$  neighbors to each part is sufficient. We improve this by providing better bounds for the fractional case of the problem. Joint work with Sean Longbrake, Michael Simkin, and

Liana Yepremyan.

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### MS9

#### Folkman graphs and finite geometry

Folkman's theorem asserts the existence of graphs  $G$  which are  $K_{s+1}$ -free, but which have the property that every two-coloring of  $E(G)$  contains a monochromatic copy of  $K_s$ . The quantitative aspects of  $f(s)$ , the least  $n$  such that there exists an  $n$ -vertex graph with both properties above, are notoriously difficult; a series of improvements to  $f(3)$ , the smallest nontrivial Folkman number, over the span of two decades witnessed the solution to two \$100 Erdős problems, and the current record due to Lange, Radziszowski, and Xu now stands at  $f(3) \leq 786$ , the proof of which is computer-assisted. More generally, the bound  $f(s) \leq c^{s^3}$  of Balogh and Samotij (proved using random graphs and hypergraph containers), where  $c > 0$  is an absolute constant, is the state of the art for larger cliques. In this talk, I will discuss Folkman-like properties of some pseudorandom graphs coming from finite geometry. These constructions present new potential paths to two fundamental questions in the area of restricted Ramsey theory: first, yet another \$100 problem of Ron Graham to show that  $f(3) \leq 100$ ; and secondly, a longstanding folklore conjecture that  $f(s) \leq C^s$  for some fixed  $C > 0$ . This is based on joint work with Steven van Overberghe and with Patrick Morris, Maya Sankar, and Liana Yepremyan.

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### MS10

#### Finite Matrix Multiplication Algorithms from Infinite Groups

In 2003, Cohn and Umans introduced a group-theoretic approach to constructing algorithms for multiplying matrices. The approach involves finding three subsets of a group  $G$  that satisfy a six-fold "additive combinatorial" condition (or rather, its non-Abelian multiplicative analogue). The quality of the construction trades off the size of those sets against the dimensions of the irreducible representations of  $G$  - bigger sets and smaller irreps are better. In this talk we discuss recent work with Blasiak, Cohn, Pratt, and Umans where we generalize this to allow using only a subset of a group's irreps instead of all of them. Ultimately this allows us to use algebraic groups (even infinite ones) to construct the additive-combinatorial sets needed for matrix multiplication algorithms.

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### MS10

#### Littlewood-Offord Theorems for Quadratic Forms: Going Beyond $N^{-1/2}$

Consider a degree- $d$  polynomial  $f(x_1, \dots, x_n)$  of independent Rademacher random variables  $x_1, \dots, x_n$ . The polynomial Littlewood-Offord problem studies to what extent  $f(x_1, \dots, x_n)$  can concentrate on a single value. Meka, Nguyen and Vu provided a nearly optimal answer: the probability that  $f(x_1, \dots, x_n) = 0$  is at most  $n^{-1/2+o(1)}$  unless  $f$  is close to a zero polynomial. In this paper, we prove several results supporting the general philosophy that this  $n^{-1/2+o(1)}$  bound can be significantly improved unless  $f$  is 'close to a polynomial with special algebraic structure'. For quadratic polynomials, we make progress towards a conjecture of Costello. We show that the probability that  $f(x_1, \dots, x_n) = 0$  is at most  $n^{-1/2-c}$  (for some constant  $c > 0$ ) unless  $f$  is close to a reducible polynomial and that this probability is at most  $n^{-1+o(1)}$  unless  $f$  is close to a polynomial with constant rank. Joint work with Matthew Kwan, Lisa Sauermaun and Yiting Wang.

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### MS10

#### A Nearly Tight Littlewood-Offord Theorem for Multilinear Forms

Given a  $d$ -degree polynomial  $f(x_1, \dots, x_n)$  of independent Rademacher random variables  $x_1, \dots, x_n$ , the (polynomial) Littlewood-Offord problem studies to what extent can  $f(x_1, \dots, x_n)$  concentrate on a single point. In this talk we focus on when  $f$  is a multilinear form. For  $d = 1$  and  $d=2$ , it is known that the answer is  $O(n^{-d+o(1)})$  unless  $f$  is close to a reducible polynomial (in terms of number of entries needed to be changed). In general, for  $d$ -multilinear forms, Costello conjectured the answer is  $O(n^{-d/2+o(1)})$ . This was disproved by Kwan, Sah and Sawhney. Nevertheless, we establish a bound of  $O(n^{-1+o(1)})$ , which is nearly tight up to the  $o(1)$  on the exponent. Part of the proof is a local-to-global lemma for reducibility of tensors. It has applications to property testing for tensor ranks. Joint work with Zhihan Jin, Matthew Kwan and Lisa Sauermaun.

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### MS10

#### A Weak Regularity Lemma for Polynomials

A regularity lemma for polynomials provides a decomposition in terms of a bounded number of approximately independent polynomials. Such regularity lemmas play an important role in numerous results, yet suffer from the familiar shortcoming of having tower-type bounds or worse. We design a new, weaker regularity lemma with strong bounds. The new regularity lemma in particular provides means to quantitatively study the curves contained in the

image of a polynomial map, which is beyond the reach of standard methods. Applications include a significantly strengthened bound for a problem of Karam, which relates the generalized rank of a polynomial with its univariate degree. This is joint work with Guy Moshkovitz.

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MS11

### Entropy Method and Mixture Bound

The entropy method has been used in many recent works in extremal combinatorics. With the help of Shannon entropy, significant progress has been made on several classical problems, such as the union-closed conjecture and the Sidorenko conjecture. In our recent work, we use the entropy method to give new proofs of the KruskalKatona theorem and Turns theorem, as well as some of their generalizations. The new ingredient in our approach is a method for upper bounding the sum of  $2^{H(X_i)}$  for random variables  $X_1, \dots, X_k$  whose supports do not overlap too much. We call this method the mixture bound, and it can be viewed as an entropic version of double counting. In this talk, I will introduce the mixture bound and show some examples of how it can be applied.

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MS11

### Ramsey Numbers in Kneser Graphs

The Kneser graph  $KG(n, r)$  is the graph whose vertices are the  $r$ -subsets of  $[n]$  in which an edge is present between two vertices if their corresponding  $r$ -sets are disjoint. We introduce the  $r$ -Kneser Ramsey number  $R_r^{KG}(s, t)$  as the minimum integer  $n$  such that every red/blue edge-coloring of  $KG(n, r)$  contains a red  $s$ -clique or a blue  $t$ -clique. In this talk, we will discuss general bounds on the numbers  $R_r^{KG}(s, t)$  as well as progress on a related Ramsey-type problem raised by Holmsen, Hrusak, and Roldan-Pensado.

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MS11

### Sampling Colorings with Fixed Color Class Sizes

In 1970 Hajnal and Szemerédi proved a conjecture of Erdős that for a graph with maximum degree  $\Delta$ , there exists an equitable  $\Delta + 1$  coloring; that is a coloring where color class sizes differ by at most 1. In 2007 Kierstead and Kostochka reproved their result and provided a polynomial-time algorithm which produces such a coloring. In this paper we study the problem of approximately sampling uniformly random equitable colorings. A series of work gives polynomial-time sampling algorithms for colorings without the color class constraint, latest improvement being by Carlson and Vigoda for  $q \geq 1.809\Delta$ . In this paper we give a polynomial-time sampling algorithm for equitable colorings when  $q > 2\Delta$ . Moreover, our results extend to colorings with small deviations from equitable (as a corollary establishing their existence). The proof uses the framework of the geometry of polynomials for multivariate polynomials, and as a consequence establishes a multivariate local Central Limit Theorem for color class sizes of uniform random colorings.

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MS11

### Local Permutation Removal

The permutation removal lemma was first obtained by Klímašová and Král, and later reproved by Fox and Wei in the context of permutation property testing. In this talk, we study a local version of the permutation removal problem. We show that for any permutation  $\sigma \notin \{12, 21, 132, 231, 213, 312\}$ , there exists  $\varepsilon(\sigma) > 0$  such that any sufficiently large integer  $N$ , there is a permutation  $\pi$  of length  $N$  that  $\varepsilon$ -far from being  $\sigma$ -free with respect to the  $\rho_\infty$  distance, yet  $\pi$  contains only a single copy of  $\sigma$ . We define the  $\rho_\infty$  distance as a  $L^\infty$ -variant of the earth mover distance between two permutations  $\pi_1$  and  $\pi_2$  of length  $n$  as

$$\rho_\infty(\pi_1, \pi_2) = \frac{1}{n-1} \min_{\theta \in S_n} \left( \max_{1 \leq i \leq n} \max (|i - \theta(i)|, |\pi_1(i) - \pi_2(\theta(i))|) \right).$$

We will also discuss our result on the local induced graph removal problem. This is a joint work with Fan Wei.

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## MS12

### Survey of Random Discrete Structures

One of the most interesting features of Erdős-Rényi random graphs is the percolation phase transition, where the global structure intuitively changes from only small components to a single giant component plus small ones. In this talk we discuss the percolation phase transition in the random  $d$ -process, which is a time-evolving random graph model with bounded degrees: starting with an empty graph on  $n$  vertices, new random edges are added step-by-step so that the maximum degree remains at most  $d$ . For fixed  $d \geq 3$ , we (i) show that the  $d$ -process undergoes a giant component phase transition, and (ii) determine the asymptotic size of the giant component just after the phase transition. For  $d = 2$ , we also show that the giant component has a non-trivial distribution at the end of the 2-process. These results verify a conjecture of Balinska and Quintas from 1990, and solve a problem of Wormald from 1997. The proofs are based on an interplay between discrete and continuous methods, with connections to ideas and heuristics arising in percolation theory as well as aggregation and coagulation theory. For example, our arguments track a large system of  $O(d^4)$  many random variables via the differential equation method: these variables are used as input to suitable branching process approximation arguments, which require an asymptotic analysis of the associated unusually large system of  $O(d^4)$  many differential equations. Based on joint works with Nick Wormald and Laura Eslava.

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## MS13

### Isotropy and Completeness Indices of Multilinear Maps

We introduce two new invariants for multilinear maps: the isotropy index and the completeness index. These invariants quantify the structure of the kernel of the multilinear maps. We establish fundamental bounds linking these indices to well-known tensor invariants such as the partition rank, geometric rank, analytic rank, and the height of an ideal. The main applications of these results are three-fold: (1) By using the completeness index as an interpolator, we prove the optimal polynomial relations between the subrank and other tensor ranks (partition, geometric, analytic), resolving open problems raised by KoppartyMoshkovitzZuiddam and DerksenMakamZuiddam. (2) A Ramsey-type Theorem: We prove that any multilinear map on a sufficiently large space must have either a large isotropy index or a large completeness index. This generalizes a recent result and confirms a conjecture of Qiao, providing explicit bounds for the associated Ramsey numbers. (3) A Probabilistic Algorithm: We develop an efficient probabilistic algorithm to estimate the completeness index, which consequently provides a polynomial-time probabilistic algorithm for estimating the height of a poly-

nomial ideal. This is a joint work with Ke Ye.

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## MS13

### Schur Complements for Tensors and Multilinear Commutative Rank

We show that three notions of rank for matrices of multilinear forms are equivalent. This result generalizes a classical result of Flanders, corrects a minor hole in work of Fortin and Reutenauer, answers a question of Lampert on the relation between the analytic and slice ranks of trilinear forms, and establishes a special case of the conjecture that the analytic and partition ranks of a tensor are equivalent.

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## MS14

### Rooted $C_5$ -Minors

Let  $G$  be a graph and  $x_1, x_2, \dots, x_k$  be distinct vertices of  $G$ . We say  $(G, x_1x_2 \dots x_k)$  has a  $C_k$ -minor or  $G$  has a  $C_k$ -minor rooted at  $x_1x_2 \dots x_k$ , if there exist pairwise disjoint sets  $X_1, X_2, \dots, X_k \subseteq V(G)$ , such that for all  $i \in [k]$ ,  $G[X_i]$  is connected,  $x_i \in X_i$ , and  $G$  has an edge between  $X_i$  and  $X_{i+1}$ , where  $X_{k+1} = X_1$ . When  $k = 3$  it is easy to determine when  $(G, x_1x_2x_3)$  contains a  $C_3$ -minor. For  $k = 4$ , Robertson, Seymour and Thomas gave a characterization of  $(G, x_1x_2x_3x_4)$  with no  $C_4$ -minor, which, in particular, implies that such  $G$  has connectivity at most 5. In this talk, I will discuss a characterization of  $(G, x_1x_2x_3x_4x_5)$  with no  $C_5$ -minor under a weak connectivity condition. As a consequence, if  $G$  is 10-connected then, for all distinct vertices  $x_1, x_2, x_3, x_4, x_5$  of  $G$ ,  $(G, x_1x_2x_3x_4x_5)$  has a  $C_5$ -minor.

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## MS14

### Weak coarse Menger property of minor-closed families

A graph class  $\mathcal{F}$  has the weak coarse Menger property if there exist functions  $f$  and  $g$  such that for any subsets  $X$  and  $Y$  of vertices in a graph  $G$  in  $\mathcal{F}$  and integers  $k$  and  $r$ , either there exist  $k$  paths in  $G$  from  $X$  to  $Y$  with pairwise at distance at least  $r$ , or there exists a union of  $f(k, r)$  balls of radius  $g(k, r)$  hitting all paths from  $X$  to  $Y$ . Nguyen, Scott and Seymour proved that the class of all graphs does not have the weak coarse Menger property and asked whether

minor-closed families have it. We answer this question affirmatively in a stronger form by showing that rooted fat  $K_2$ -minors have the coarse Erdos-Posa property in minor-closed families. Our result extends to every length space quasi-isometric to a locally finite infinite graph with an excluded finite minor, such as complete Riemannian surfaces of finite Euler genus, metric graphs with an excluded finite minor, and string graphs.

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#### MS14

##### Near Optimal Bounds for the Unique Linkage Function

In Graph Minors XXI, Robertson and Seymour proved that there exists a function  $f$ , such that for every instance of the  $k$ -Disjoint Paths problem with a unique solution, the treewidth of the input graph is at most  $f(k)$ . This function gave rise to the celebrated Irrelevant Vertex Technique and is central to all known FPT algorithms for  $k$ -Disjoint Paths and  $H$ -minor testing. Originally, no constructive bound for  $f$  was known. Huynh and Geelen gave a bound in form of an iterated power tower and Kawarabayashi and Wollan provided a shorter proof, estimating their bound to still be quadruple exponential. In this talk we explain how recent breakthroughs in constructive graph minor theory may lead to bounds of the form  $2^{\text{poly}(k)}$  which matches known lower bounds for  $f$  up to the degree of the polynomial.

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#### MS15

##### On 2-factors of Hamiltonian Graphs

Let  $k \geq 2$ . We show that, for a sufficiently small  $\varepsilon > 0$ , any sufficiently large  $n$ -vertex Hamiltonian graph of minimum degree at least  $n^{1-\varepsilon}$  contains a 2-factor consisting of exactly  $k$  cycles. This is the first minimum-degree condition which is polynomially smaller than linear. Our methods yield an analogous result when the host graph is not required to contain a Hamilton cycle, but only a 2-factor consisting of at most  $k$  cycles; this answers a question of Bucić, Jahn,

Pokrovskiy and Sudakov.

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#### MS15

##### Spread Distributions on Spanning Subgraphs of Dirac Graphs

A graph  $H$  embeds in a graph  $G$  if  $G$  contains a subgraph isomorphic to  $H$ . Questions about when a graph embeds in another are central in combinatorics. Dirac-type embedding results address minimum-degree conditions on a graph  $G$  to ensure an embedding of some graph  $H$ , typically when  $H$  and  $G$  have the same number of vertices. In this talk, we will discuss robustness of Dirac-type embedding results. The archetypal example of this type of result is Krivelevich, Lee, and Sudakov's robust version of Dirac's Theorem, which states that if  $p = \omega(n^{-1} \log n)$  and  $G$  is an  $n$ -vertex graph with minimum degree at least  $n/2$ , then asymptotically almost surely the random subgraph of  $G$  in which edges are included independently with probability  $p$  contains a Hamilton cycle. We will discuss recent extensions of this result utilizing the ParkPham Theorem or one of its variants. A crucial notion for this is that of the *spreadness* of a certain type of probability distribution.

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#### MS15

##### Covering Random Digraphs with Hamilton Cycles

A covering of a digraph  $D$  by Hamilton cycles is a collection of directed Hamilton cycles (not necessarily edge-disjoint) that together cover all the edges of  $D$ . In this talk we show that for  $1/2 \geq p \geq \frac{\log^{20} n}{n}$ , the random digraph  $D_{n,p}$  typically admits an optimal Hamilton cycle covering. Specifically, the edges of  $D_{n,p}$  can be covered by a family of  $t$  Hamilton cycles, where  $t$  is the maximum of the in-degree and out-degree of the vertices in  $D_{n,p}$ . Notably,  $t$  is the best possible bound, and our assumption on  $p$  is optimal up to a polylogarithmic factor. This is joint work with Asaf Ferber and Mason Shurman.

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### MS15

#### Efficient and Optimal High-Dimensional Planar Assignments

The (2-dimensional) assignment problem is to find, in an edge weighted bipartite graph, a perfect matching of minimum total weight. Efficient algorithms for this problem have been known since the advent of modern algorithmic analysis. Moreover, if the edge weights are i.i.d.  $\text{Exp}(1)$  random variables and the host graph is complete bipartite, seminal results of Aldous state that the expected weight of the optimal assignment tends to  $\zeta(2)$ . We consider high-dimensional versions of the problem. Here, we are given a cost array  $M$ , indexed by  $[n]^k$ , and with i.i.d.  $\text{Exp}(1)$  entries. The objective is to find a 0, 1-matrix  $A$  minimizing  $\sum_{x \in [n]^k} A_x M_x$ , subject to the constraint that every axis-parallel line in  $A$  sums to 1. This is the random planar assignment problem. We prove that the expected cost of an optimal assignment is  $\Theta(n^{k-2})$ . Moreover, we describe a randomized algorithm that finds such an assignment with high probability. The main tool is iterative absorption, as developed by Glock, Khn, Lo, and Osthus. The results answer questions of Frieze and Sorkin. The algorithmic result is in contrast to the axial assignment problem (in which each axis-parallel hyperplane in  $A$  sums to 1). For the latter, the best known bounds (due to Frankston, Kahn, Narayanan, and Park) exploit the connection between ‘spread’ distributions and optimal assignments and are not algorithmic. Joint work with Ashwin Sah and Mehtaab Sawhney.

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### MS16

#### Equilibrium-Based Learning Dynamics in Spiking Architectures

While research in designing brain-inspired algorithms have attained a stage where such Artificial Intelligence platforms are being able to outperform humans at several cognitive tasks, an often-unnoticed cost is the huge computational expenses required for running these algorithms in hardware. Recent explorations have also revealed several algorithmic vulnerabilities of deep learning systems like adversarial susceptibility, lack of explainability, catastrophic forgetting, to name a few. This talk reviews recent developments in the domain of neuromorphic computing paradigms from an overarching system science perspective with an end-to-end co-design focus from computational neuroscience and machine learning to hardware and applications. We will delve into methodologies that treat spiking architectures as continuously evolving dynamical systems, revealing intriguing parallels with the learning dynamics in the brain. We will discuss methods like Equilibrium Propagation, Implicit Differentiation, among others that address multiple challenges of training spiking architectures and highlight the necessity for bio-plausible local learning and increasing model scalability in spiking architectures. The methodologies discussed enable spiking ar-

chitectures to transition beyond simple vision-related tasks to complex sequence learning problems and large language model (LLM) architectures.

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### MS16

#### Neural-Inspired Probabilistic Graph Simulation of Non-Homogeneous Turbulent Flow

Simulating turbulent flows exactly requires solving a system of partial differential equations (PDEs) at such small scales that the computation is beyond the capabilities of most computers. Low-pass filtering at small scales or averaging the PDEs and using heuristics that capture chaotic behavior are more manageable but provide solutions that obscure fine details. Fortunately, the solution to the averaged PDEs is described by a probability density function (PDF), which we discretize as a probabilistic graph and sample via Monte Carlo random walks, simulating chaotic behavior to arbitrary accuracy. However, whereas the simplest examples of turbulent flows are described by static PDFs, more complex, non-homogeneous turbulent flows have PDFs that evolve over time. We demonstrate how a random walk through a probabilistic graph approximating a dynamic PDF can be implemented with event-driven, neural-inspired hardware. Each walker is a discrete spike transmitted between nodes in a probabilistic graph comprised of custom neural circuits or neuron models. We further compare to CPU and GPU implementations to demonstrate the orders-of-magnitude improved energy efficiency of our approach. SNL is managed and operated by NTESS under DOE NNSA contract DE-NA0003525.

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### MS16

#### Decomposing Neural Networks with Gnats: Graphical Neural Activity Threads

To understand the computational capacities of the brain and how we might develop brain-inspired AI algorithms, we need powerful abstractions for neural computation. Ideally, these abstractions should be naturally adapted to the spiking and synaptic dynamics of real brains. We present an alternative approach to analyzing spiking neural networks that avoids many of the implicit assumptions in current approaches for spiking network analysis and offers a route to new computational abstractions. Current approaches for building computational abstractions for spiking dynamics begin by sorting spikes into time bins and constructing population activity vectors that trace the dynamics of neural activity in a high dimensional space over time. While fruitful, these approaches necessarily smear out intrinsic relations between spikes and may obscure computationally-relevant features of neural dynamics. Our approach begins by constructing a directed acyclic graph directly from the

synaptic relations between individual spikes. These synaptic relations must support the computations in the spiking network. The analysis combines spiking activity and the connectome into a unified combinatorial object, without time bins. We show how this directed graph naturally decomposes into weakly connected subgraphs we call Graphical Neural Activity Threads (GNATs). These GNATs are well-defined and provide a picture of information flow through a spiking network. Furthermore, GNATs are defined by the relative timings between spikes and are thus robust to spike timing variations. I will then describe an algorithm that can efficiently find isomorphic GNATs in large spiking neural datasets. By identifying isomorphic GNATs, we identify putatively isomorphic computations. I will show how GNATs arising in the dynamics of spiking network models are constructed out of other GNATs, analogous to sampling in music production. Thus, GNATs exhibit compositionality. Because of their naturalness, robustness, and compositionality, GNATs provide a powerful basis for computational abstraction in spiking neural networks.

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### MS16

#### Neural codes and stimulus space structure

One major problem in neuroscience is to understand how the brain uses neural activity to form representations of the external world. It is known that combinatorial information in the firing patterns of neurons often reflects important features of the stimuli that generated them. How can we efficiently extract such information? This talk will introduce some of the algebraic and topological methods currently in use for encoding and extracting combinatorial structure from neural codes, and also discuss how this structure can be used to infer features of the underlying stimulus space.

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### MS17

#### Controlling Major Apices in the Polynomial Graph Minor Structure Theorem

We discuss a version of the Graph Minor Structure Theorem due to Robertson and Seymour with not only polynomial bounds, but also an optimal bound on the number of 'major apices' required. This version was first proven without explicit bounds by Dvorak and Thomas and has proven useful for several colouring results, e.g. the Clustered Hadwiger Conjecture that was recently resolved by Dujmovic, Esperet, Morin, and Wood. To clarify the above, the Graph Minor Structure Theorem asserts that there exist functions  $f, g$  such that for every graph  $H$  with  $t := |V(H)|$ , every  $H$ -minor-free graph can be obtained via the clique-sum operation from graphs which embed into surfaces where  $H$  does not embed after deleting at most  $f(t)$

many vertices with at most  $t^2 - 1$  many 'vortices' which are of 'depth' at most  $g(t)$ . We show that the above can be proven in such a way that  $f$  and  $g$  are polynomial functions, all but at most  $\text{apex}(H)$  of the deleted vertices only have neighbours in vortices (where  $\text{apex}(H)$  is the number of vertices that need to be deleted to make  $H$  planar), and the number of vortices in the resulting structure remains polynomial (though no longer quadratic in  $t$ ).

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### MS17

#### Directed Tree-Cutwidth

The first major step towards the graph minor structure theorem by Robertson and Seymour was the *grid theorem*, a result describing that every graph of large treewidth contains a grid as a minor. In 2014, Wollan provided a definition for a tree-like decomposition and a width parameter, *tree-cutwidth*, with respect to immersions, a different graph containment relation. He provided results linking this parameter to immersions of large walls. This talk presents a version of this parameter for directed graphs, the *directed tree-cutwidth*. The main result is a grid theorem for directed tree-cutwidth, establishing that it is linked to directed immersions of large cylindrical walls.

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### MS17

#### An approximate Tutte-decomposition for arbitrary connectivity

Tutte's seminal theorem states that every 2-connected graph can be decomposed along 2-separations into pieces that are 3-connected, cycles, or  $K_2$ 's. This result was extended to 3-connected graphs by Carmesin and Kurkofka, and recently to 4-connected graphs by Kurkofka and Planken. A key feature of these decompositions is that

they display all  $k$ -separations of the graph for the corresponding  $k$ . We extend these results to arbitrary connectivity in an approximate sense. We show that every almost- $k$ -connected graph  $G$  admits a tree-decomposition along  $k$ -separations whose torsos are either almost- $(k+1)$ -connected or have the cyclic structure of a *flower*. Moreover, every  $k$ -separation of  $G$  is displayed, up to bounded error, either by the tree-decomposition or by a flower. Thus, our result can be viewed as a concise structure that approximately encodes all  $k$ -separations. Flowers, originating in matroid theory, capture highly structured families of crossing separations and thereby generalize cycles to higher connectivity. A key new insight is that, under mild assumptions, three crossing  $k$ -separations always cross two-dimensionally. In particular, this may explain why the classical two-dimensional corner-diagram method is so effective. Our approach further uses covering spaces from topology.

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## MS17

### Tree-Width of a Graph Excluding an Apex-Forest or a Wheel as a Minor

The Grid Minor Theorem states that for every planar graph  $H$ , there exists a smallest integer  $f(H)$  such that every graph with tree-width at least  $f(H)$  contains  $H$  as a minor. The only known lower bounds on  $f(H)$  beyond the trivial bound  $f(H) \geq |V(H)| - 1$  come from the maximum number of disjoint cycles in  $H$ . In this paper, we study  $f(H)$  for planar graphs  $H$  with no two disjoint cycles. We prove that  $f(H) = |V(H)| - 1$  for every apex-forest  $H$ . This result improves a bound of Leaf and Seymour and contains all known large graphs  $H$  meeting the trivial lower bound to our knowledge. We also prove that  $f(H) \leq \max\{\frac{3}{2}|V(H)| - \frac{9}{2}, |V(H)| - 1\}$  for every wheel  $H$ .

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## MS18

### A Codegree Version of Baranyai's Theorem

Let  $2 \leq k < n$  be integers such that  $k$  divides  $n$ . The classical Baranyai's theorem states that every complete  $k$ -uniform hypergraph can be decomposed into perfect matchings. We prove a codegree condition of Baranyai's theorem as follows: For fixed  $k$  and  $\varepsilon > 0$ , the following holds for all sufficiently large  $n$ . An  $n$ -vertex  $k$ -graph  $G$  with  $\delta_{k-1}(G) \geq (1/2 + \varepsilon)n$  can be decomposed into perfect matchings if and only if  $k \mid n$  and  $G$  is vertex-regular. This generalizes Baranyai's theorem when  $n$  is sufficiently large and solves a conjecture proposed by Glock, Kuhn and Osthus. The talk is based on joint work with Stefan Glock and Andrew Lane.

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## MS18

### On Reflecting n-Queens Configurations

An  $n$ -queens configuration is a placement of  $n$  queens on an  $n \times n$  chessboard, such that no two queens are in the same row, column, or diagonal. In 1967, Klarner proposed a variation of this problem concerning the existence of reflecting  $n$ -queens configurations. The problem considers the feasibility of placing  $n$  mutually non-attacking queens on the reflecting chessboard, an  $n \times n$  chessboard with  $1 \times n$  'reflecting strip' of squares added along one side of the board. A queen placed on the reflecting chessboard can attack the squares in the same row, column, and diagonal, with the additional feature that its diagonal path can be reflected via the reflecting strip. Klarner noted the equivalence of this problem to a number theory problem proposed by Slater, which asks: for which  $n$  is it possible to pair up the integers  $1$  through  $n$  with the integers  $n+1$  through  $2n$  such that no two of the sums or differences of the  $n$  pairs of integers are the same. In this talk, I will introduce the  $n$ -queens problem and its state of the art, and discuss the existence and enumeration of the reflecting  $n$ -queens configurations for sufficiently large  $n$ . This is joint work with Alex Divoux and Tom Kelly.

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## MS18

### Thresholds for Decompositions into Long Odd Cy-

cles

An  $F$ -decomposition of a graph  $G$  is a set of subgraphs of  $G$ , each isomorphic to  $F$ , whose edge sets partition the edge set of  $G$ . I will speak about a result showing that, for each odd  $k \geq 5$ , any graph  $G$  of sufficiently large order  $n$  with minimum degree at least  $(\frac{1}{2} + \frac{1}{2k-4} + o(1))n$  has a  $C_k$ -decomposition if and only if  $k$  divides  $|E(G)|$  and all vertex degrees in  $G$  are even. Our methodology also leads to results on  $F$ -decompositions for other 3-partite graphs  $F$ . This is joint work with Darryn Bryant, Daniel Horsley, Barbara Maenhaut and Richard Montgomery.

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### MS19

#### Making a Case for Neuromorphic Combinatorial Optimization Based on Quantum Tunneling

Combinatorial optimization has been proposed as a domain where neuromorphic architectures could outperform both classical and quantum architectures. In this talk, we discuss the synaptic underpinnings of neuromorphic architectures that can facilitate the discovery of new or difficult-to-find solutions for ISING-like combinatorial optimization problems. We show that synapses operating based on the physics of Fowler-Nordheim quantum tunneling can naturally interpolate between an  $\mathcal{O}(1/t)$  temporal regime (supporting fast short-term dynamics such as attractor-like convergence and rapid time-to-solution) and an  $\mathcal{O}(1/\log t)$  regime (supporting long-term discovery with asymptotic global-optimality guarantees). As a result, FN-quantum tunneling synapses can naturally address the neuromorphic stability/plasticity dilemma that also appears in simulated-annealing-based (SA) optimization. In SA, provable convergence to global optima requires an annealing schedule that scales as  $\mathcal{O}(1/\log t)$  which we show induces an effective rare-event acceptance (memory overwrite) rate that decays as  $\mathcal{O}(1/t)$  over long time-scales corresponding to an optimal memory consolidation. We demonstrate the benefits of the multi-time-scale FN dynamics for solving ISING problems. However, the proposed neuromorphic architecture is general enough to be applied to other combinatorial optimization and stochastic simulation frameworks.

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### MS19

#### Towards a General Geometry of Spike Train Representation

Through correspondence between internal neural activity and statically defined external variables, the neuroscientific study of representation has yielded insights into what information is represented where in the brain. Yet this approach cannot explain how representation is instantiated as a dynamic process by the brain. Here we consider how a changing stimulus space might be adaptively differentiated

through coincident spiking patterns across a population of neurons, its combinatorial code. We expose how the constraints of Dale's Law organize a population's code into a conical geometry through the global overlap of all RFs. This geometry links spiking sparsity to the resolution at which stimuli can be differentiated. We exploit this relationship, hypothesizing that the conical geometry functions as a self-consistent model of stimulus space which enables precise and efficient representation of stimulus relationships across changing environments. We test our hypothesis in vivo, quantifying how neurons in auditory regions of the songbird brain adapt their RFs to the statistics of bird-song stimuli, revealing that the conical geometry shifts to optimally resolve stimulus relationships. We demonstrate how coincident spiking patterns are adaptively repurposed to align emergent internal models to external variables, reframing representation from a static set of correspondences into a dynamic process where the meaning of a spike is interpreted through the activity of a population.

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### MS19

#### Optimization on Neuromorphic Spiking Neural Networks

Biologically inspired spiking neural networks can now be simulated better than ever using a new class of spiking neuromorphic processors. Pairing these emerging platforms with graphs of biological connectomes can lead to new understanding from both the mapping of and behaviors found in complex spiking neural networks graphs. This presentation will discuss recent and on-going efforts around neuromorphic simulations of *Drosophila melanogaster*. By mapping this complex graph to neuromorphic hardware, we are able to better understand and contextualize hardware restrictions and potential optimizations. Additionally, we will discuss using optimization methods to help identify and understand behaviors found in this connectome. Ultimately, we hope these findings will inform and improve the design of future neuromorphic algorithms.

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### MS20

#### Vertex Contraction-Deletion Minors

There are a number of containment relations for graphs, including subgraph, induced subgraph, minor, topological minor, and immersion. These can be defined in terms of reduction relations. We discuss a containment relation

that does not seem to have been previously studied, but which has some natural motivation. A graph  $H$  is a *vertex contraction-deletion minor*, or *VCD minor*, of a graph  $G$  if  $H$  can be obtained from  $G$  by vertex contractions and vertex deletions. Here a *vertex contraction* contracts all edges incident with a given vertex. The VCD minor relation is a weakening of the  $t$ -minor relation introduced by Gerards and Shepherd in 1998, and for bipartite graphs is equivalent to the  $t$ -minor relation. We discuss some basic properties of this concept. We show that although planar graphs are closed under VCD minors, they cannot be described by a finite set of excluded VCD minors. On the other hand, we show that trees are well-quasi-ordered under VCD minors. We characterize  $C_4$ -VCD-minor-free graphs, which are a generalization of chordal graphs, in terms of a simple infinite family of forbidden induced subgraphs. We discuss some preliminary work on characterizing  $K_{1,3}$ -VCD-minor-free graphs, which are a subset of claw-free graphs

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## MS20

### Graph Product Structure in Hereditary Setting

We prove that the celebrated Planar Graph Product Structure Theorem by Dujmovic et al can be formulated with the *induced subgraph* containment relation. Precisely, we prove that if a graph  $G$  is a subgraph of the strong product of a graph  $Q$  of bounded maximum degree (such as a path) and a graph  $M$  of bounded tree-width, then  $G$  is isomorphic to an induced subgraph of the strong product of  $Q$  and a graph  $M'$  whose tree-width is at most exponential in the maximum degree of  $Q$  and the tree-width of  $M$ . In particular, every planar graph is isomorphic to an induced subgraph of the strong product of a path and a graph of tree-width 39. All these results are underlined by a dense variant of the graph product structure concept (informally, considering now a path times a graph of bounded clique-width), which is formalized in the definition of  $\mathcal{H}$ -clique-width where  $\mathcal{H}$  is any class of bounded degree graphs (such as the class of all paths). The original motivation behind  $\mathcal{H}$ -clique-width is in looking at graph structure theory through the lens of first-order logic, and the new results have been used, e.g., to prove that the class of the 3D grids, as well as a class of certain natural modifications of 2D grids, are not first-order transducible from classes admitting a product structure, and in particular not from the class of planar graphs.

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## MS20

### Graphs of Linear Neighborhood Complexity

Mader proved that graphs with a forbidden minor have at most a linear number of edges in their number of vertices. Recently, there has been a lot of progress on proving 'dense analogs' of theorems about sparse graphs; these graphs should still somehow be 'structurally simple', even though they can contain many edges. In this talk, we will discuss several dense analogs of Mader's Theorem. Instead of having a linear number of edges, these graphs will have linear neighborhood complexity. This talk may touch on matroids, vertex-minors, the first-order logic of graphs, and/or applications to graph coloring.

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## MS20

### 2-Factors in $3/2$ -Tough Maximal Planar Graphs

The toughness of a graph  $G$  is defined as the minimum value of  $|S|/c(G - S)$  over all cutsets  $S$  of  $G$  if  $G$  is non-complete, and is defined to be  $\infty$  if  $G$  is complete. For a real number  $t$ , we say that  $G$  is  $t$ -tough if its toughness is at least  $t$ . Following a classic 1956 result of Tutte, every more than  $\frac{3}{2}$ -tough planar graph on at least three vertices has a 2-factor. In 1999, Owens constructed a sequence of maximal planar graphs with toughness  $\frac{3}{2} - \varepsilon$  for any  $\varepsilon > 0$ , but the graphs do not contain any 2-factor. He then posed the question of whether there exists a maximal planar graph with toughness exactly  $\frac{3}{2}$  and with no 2-factor. We answer this question affirmatively by constructing such an example. In our construction, many pairs of degree-3 vertices share a common neighbor. Motivated by this, we investigate under what conditions a  $\frac{3}{2}$ -tough maximal planar graph contains a 2-factor. We show that if  $G$  is a  $\frac{3}{2}$ -tough maximal planar graph of order at least three, then  $G$  has a 2-factor provided that the distance between any two distinct degree-3 vertices is at least three.

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## MS21

### Hypergraph Nash-Williams

The study of combinatorial designs includes some of the oldest questions at the heart of combinatorics. In a breakthrough result of 2014, Keevash proved the existence of  $(n, q, r)$ -Steiner systems (equivalently  $K_q^r$ -decompositions of  $K_n^r$ ) for all large enough  $n$  satisfying the necessary divisibility conditions. In 2021, Glock, Khn, and Osthus proposed a generalization of this result by conjecturing a hypergraph version of the celebrated Nash-Williams' Conjecture. Namely, they posited that if a large enough  $K_q^r$ -divisible  $r$ -graph  $G$  on  $n$  vertices has minimum  $(r - 1)$ -degree (denoted  $\delta_{r-1}(G)$ ) at least  $(1 - \Theta_r(1/q^{r-1}))n$ , then  $G$  admits a  $K_q^r$ -decomposition, where this bound is

motivated by hypergraph Turán Theory. We prove that if  $G$  is a large enough  $K_q^r$ -divisible  $r$ -graph with  $\delta_{r-1}(G) \geq \left(1 - O_r\left(\frac{1}{\binom{q}{r-1}}\right)\right)n$ , then the existence of a fractional decomposition implies that  $G$  admits a  $K_q^r$ -decomposition. Our proof uses the newly developed method of refined absorption and establishes a non-uniform Turán theory. Combined with the best-known fractional decomposition threshold from Delcourt, Lesgourgues, and Postle, this proves that  $(1 - O_r(1/q^{r-1+\epsilon}))n$  suffices for the Hypergraph Nash-Williams' Conjecture, dramatically closing the gap between what was known and the above conjecture. Joint work with Luke Postle.

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## MS21

### Towards a High-Dimensional Dirac's Theorem

Dirac's theorem determines the sharp minimum degree threshold for graphs to contain perfect matchings and Hamiltonian cycles. There have been various attempts to generalize this theorem to hypergraphs with larger uniformity by considering hypergraph matchings and Hamiltonian cycles. In this talk, we consider another natural generalization of perfect matchings, Steiner triple systems. As a Steiner triple system can be viewed as a partition of pairs of vertices, it is a natural high-dimensional analogue of a perfect matching in graphs. We prove that for sufficiently large integer  $n$  with  $n \equiv 1$  or  $3 \pmod{6}$ , any  $n$ -vertex 3-uniform hypergraph  $H$  with minimum codegree at least  $\left(\frac{3+\sqrt{57}}{12} + o(1)\right)n = (0.879\dots + o(1))n$  contains a Steiner triple system. We conjecture that the number  $\frac{3+\sqrt{57}}{12}$  can be replaced with  $\frac{3}{4}$ , which would provide an asymptotically tight high-dimensional generalization of Dirac's theorem. We will also discuss several applications of our method, which was used to find an approximate Steiner triple system.

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## MS21

### Forbidding Alternating Cycles Using Forbidden Submatchings with Reserves

In this talk, we present variety of problems pertaining to (not necessarily properly) edge-coloring graphs while forbidding alternating cycles. We utilize the "forbidden submatchings with reserves method" recently developed by Delcourt and Postle. This is joint work with Michelle Delcourt and Reaz Huq.

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## MS21

### On Nash-Williams' Conjecture and Its Generaliza-

## tions

In 1847, Kirkman proved that there exists a Steiner triple system on  $n$  vertices (equivalently a triangle decomposition of the edges of  $K_n$ ) whenever  $n$  satisfies the necessary divisibility conditions (namely  $n$  is congruent to 1 or 3 mod 6). In 1970, Nash-Williams conjectured that every graph  $G$  on  $n$  vertices with minimum degree at least  $3n/4$  (for  $n$  large enough and  $G$  satisfying the necessary divisibility conditions) has a triangle decomposition. Nash-Williams' Conjecture is a central open question in extremal design theory. Here we discuss recent progress on the conjecture and its generalizations. This is joint work with subsets of Michelle Delcourt, Cicely Henderson, and Thomas Lesgourgues.

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## MS22

### Settling the Complexity of Recognizing Leaf Powers

Leaf powers are a graph class that can be viewed as encoding pairwise distances within a tree in a way that is robust to noise and errors. Edges of leaf powers correspond to "close" taxa, while non-edges correspond to taxa that are "far apart" according to some threshold  $k$ . More precisely, a graph  $G$  is a  $k$ -leaf power if there exists a tree  $T$  whose leaves are the vertices of  $G$ , such that  $uv$  is an edge of  $G$  if and only if the distance between  $u$  and  $v$  in  $T$  is at most  $k$ . Then, a graph is a leaf power if it is a  $k$ -leaf power for some  $k$ . Recognizing leaf powers would provide a useful tool for reconstructing phylogenies from imperfect distance data, yet many of the central questions about this graph class have remained open since its introduction in 2002. In this talk, I will present answers to several major open problems in the area. In particular, I will discuss whether leaf powers and  $k$ -leaf powers can be recognized in polynomial time, and whether leaf powers admit a meaningful graph-theoretic characterization in terms of forbidden induced subgraphs.

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## MS22

**What are unrooted tree-child networks? [Note that the title is slightly different from what is currently published on the SIAM DM website. No problem, in case it cannot be changed.]**

TRooted tree-child networks are a popular class of phylogenetic networks that have favorable properties from a mathematical and algorithmic perspective without being overly simplistic. In this talk, we explore two classes of unrooted phylogenetic networks with the goal of expanding the notion of tree-child networks to the unrooted setting. First, we consider unrooted phylogenetic networks that are tree-child orientable and establish that they are computationally hard to recognize even in the binary case. Second, we introduce unrooted  $q$ -cuttable networks and show that

they have several desirable properties that are similar to those of rooted tree-child networks. Joint work with Leo van Iersel, Mark Jones, and Norbert Zeh.

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## MS22

### The Space of Equidistant Phylogenetic Cactuses

TexAn equidistant X-cactus is a type of rooted, arc-weighted, directed acyclic graph with leaf set X, that is used in biology to represent the evolutionary history of a set of species. In this talk, I will introduce the space of equidistant X-cactuses and some combinatorial properties of these phylogenetic networks. These properties can be used to show that this space is a CAT(0)-orthant space, for which there are existing algorithms for computing geodesic and Frchet means and performing other statistical analyses. This is joint work with Katharina T. Huber, Vincent Moulton, Andreas Spillner, and Katherine St. John.

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## MS22

### A sharp lower bound for the number of phylogenetic trees displayed by a tree-child network

Phylogenetic networks generalize evolutionary trees by allowing for reticulation events such as hybridization or horizontal gene transfer. A classical result states that a normal phylogenetic network with  $k$  reticulations displays exactly  $2^k$  phylogenetic trees. In this talk, I will present an analogous result for a broader class of networks: tree-child networks that contain no underlying 3-cycles. Specifically, we prove that such a network with  $k = 2$  reticulations displays at least  $2^{\lfloor k/2 \rfloor}$  phylogenetic trees if  $k$  is even and at least  $(3/2)v2 \cdot 2^{\lfloor k/2 \rfloor}$  if  $k$  is odd. Moreover, we show that these bounds are sharp and characterize the tree-child networks that attain these bounds.

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## MS23

### The Small Parsimony Problem (SPP) for Gene Content Evolution

Understanding how a gene family, a set of genomic regions, or a set of related genomes has evolved from a common ancestor is an important step towards elucidating many biological questions. If a rooted phylogenetic tree is already available for the considered biological units represented at the leaves of the tree, the problem reduces to the Small Parsimony Problem (SPP), which consists of inferring the content of ancestral nodes, in a way maximizing a certain parsimony or probabilistic criterion. In this presentation, I will review the recent work on the SPP for inferring ances-

tral gene contents based on different evolutionary models, from the simplest one restricted to optimizing a scenario of gains and losses, to more comprehensive ones also involving duplications and transfers.

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## MS23

### k-Nearest Common Leaves algorithm for phylogenetic tree completion

Phylogenetic trees represent the evolutionary histories of taxa and support tasks such as clustering and Tree of Life reconstruction. Many established comparison methods, including the Robinson-Foulds (RF) distance, assume identical taxon sets. A methodological gap remains for trees with distinct but overlapping taxa. Existing approaches either prune non-common leaves, which can discard information, or complete both trees such that they share the same taxa. Completion is more comprehensive, but current methods typically ignore branch lengths, which are essential for identifying evolutionary patterns. In this talk, I will present k-Nearest Common Leaves (k-NCL), an algorithm for completing rooted phylogenetic trees defined on different but overlapping taxa. The method uses branch lengths and topological characteristics and does not rely on a specific distance measure. The k-NCL algorithm is designed to preserve evolutionary relationships in the trees under comparison. The running time is  $O(n^2)$ , where  $n$  is the size of the union of the two leaf sets. Additional properties include preservation of original distances and topology, symmetry, and uniqueness of the completion. k-NCL is evaluated on biological datasets of amphibians, birds, mammals, and sharks. Experimental results show that RF combined with k-NCL improves phylogenetic tree clustering performance compared to the RF(+) tree completion approach.

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