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Terrorism as a Mathematical Problem

Cornell graduate student Alexander Gutfraind, organizer of a minisymposium on terrorism for SIAM's 2009 conference on dynamical systems, reports on recent advances in the area.

Terrorism has tremendous effects on both global politics and everyday life. Until recently, however, despite horrifying loss of life and vast expenditures on security, very little mathematical work was done in this area.

The dearth of research on terrorism is not entirely surprising, given the inherent difficulties of modeling adaptive socio-economic systems like terrorism. Mathematicians new to the area, for example, might be surprised to find an extensive literature on the seemingly trivial problem of defining "terrorism." What makes this taxonomic problem difficult is the similarity of terrorism to many other forms of violence. As a result, data on incidents or groups was lacking until recently, and a scholarly community was slow to emerge within the social sciences.

As to modeling, the scarcity of data—serious enough in itself—is compounded by the complexity of the systems being studied. Terrorism has psychological, sociological, and political aspects, among many others; any model can capture only a small cross-section of such a system. Fortunately, quantitative studies of well-structured elements within this chaos promise to yield new insights into terrorism.

Recent Progress

Mathematical research in counter-terrorism is a growing area, with exciting advances emerging from work on new problems. Colleagues who know of my interest in terrorism often assume that I work mainly on data mining tools for identifying terrorism suspects. Data mining is indeed an important and challenging problem [11]: The data comes in many different formats and is often incomplete, and algorithms must achieve very high rates of accuracy based on relatively small training sets of actual incidents. Data mining is only one of many research areas in which methods and tools have been successfully applied to problems in terrorism, however. In the three examples briefly presented in the remainder of this article, the underlying models are based on methods from complex systems, graph theory, and dynamical systems.

Complex systems. One important recent discovery to emerge from the complex systems community is that the patterns observed in terrorist violence are surprisingly universal across conflicts. In particular, in models of the frequency of severe terrorist events, numbers of casualties have a power-law distribution. The exponent depends on the conflict, but exponents from conflicts and insurgencies across the globe appear to be converging over time toward 2.5 [8] (Figure 1).

These findings are explained by a simple "fusion–fission" model developed by Johnson et al. in which attacks are carried out by independently operating cells [8]. In the model, 2.5 is not simply a product of judiciously chosen model parameters: Several studies have shown it to be robust to various realistic generalizations for terrorism and insurgencies [2,11]. Such a model is significant because it runs counter to the dominant view holding that most terrorist organizations organize and carry out operations in a very strategic, centralized manner. Even the existence of the power-law relation in terrorism was not previously known to the political science community.

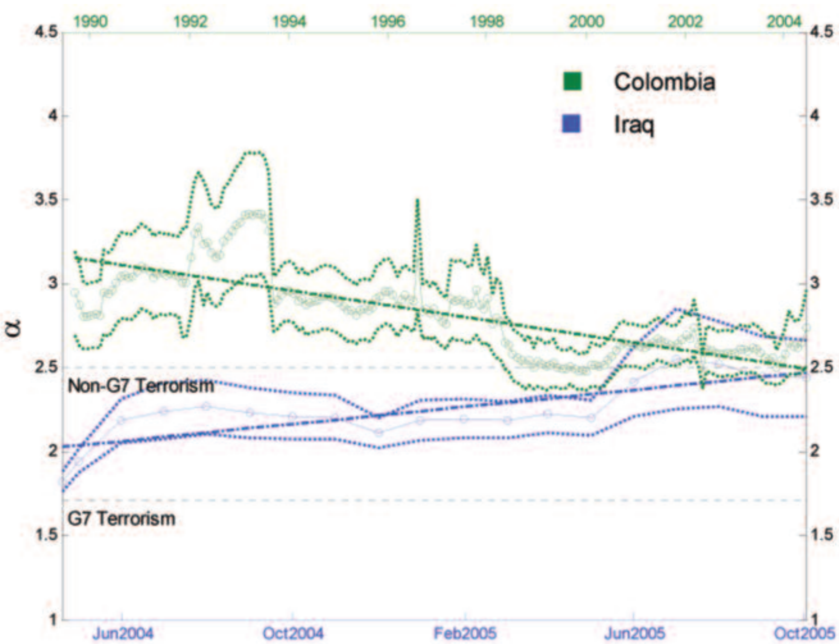

Figure 1. *Variation through time of the power-law exponent for Iraq and Colombia, shown as blue and green curves with circles, together with their corresponding 95% confidence bands. The time-windows for Iraq and Colombia are 400 and 800 days, respectively, with each time-window sliding forward one month at a time. Fits through these points (straight lines) suggest a current value of 2.5 for both wars. Clauset et al. report a refined alpha value of 2.48 for all terrorist events combined [2].*

Graph theory. Using basic results from graph theory, Woo recently explained why most terrorist plots in North America and Europe over the past decade (a total of at least 50) were intercepted [13]. The model is based on the observation that an operation like 9/11 requires the efforts of many individuals who are connected through a social network. For simplicity, the network is modeled as a connected clique, which means that the capture of any one of the conspirators would likely jeopardize the plot. (Richer models of the network structure of terrorist groups are described in [3, 5].)

In Woo's model, each link in the network can be independently detected with probability p, depending on such factors as the number of law enforcement agents tasked with tracking suspects as well as on the willingness of citizens to offer information. From percolation theory, a clique of size $N \geq 1/2p^2$ will almost surely be intercepted. Because the probability p is squared, the model indicates that increased surveillance will pay nonlinear dividends. Indeed, as counter-terrorism efforts resulted in increased p, the terrorists adapted by reducing the numbers of individuals involved in plots. If the model is correct, law enforcement agencies should also be successful in deterring plots that involve sophisticated weapons, such as weapons of mass destruction. Because the deployment of such weapons requires the participation of large numbers of people, it has become too risky.

Dynamical systems. In studies of terrorism, certain variables, such as the level of violence and properties of terrorist groups, evolve in time and space. The speakers in a minisymposium on dynamic models at this year's SIAM Conference on Applications of Dynamical Systems discussed models based on ordinary differential equations [6], hidden Markov models [4], control theory [1], and dynamic graphs [9].

One specific ODE model that has had a substantial practical impact examined the threat to the milk distribution system from botulinum toxin [12]. The researchers observe that the milk distribution system begins with the collection of milk from individual farms, after which it is placed in larger and larger tanks, processed, and then distributed to suppliers and, eventually, consumers. Because of this bow-like structure, toxin injected into a single milk tank up-stream would have the potential to poison a very large number of people. The model indicates that 10^5 casualties, mostly infants, could result from less than 1 gram of toxin, but that an inexpensive early testing protocol could mitigate the threat. The model and the paper sparked an outcry. The researchers, some said, had exposed a potentially catastrophic vulnerability that terrorists could exploit. But the eventual outcome was inspiring: By attracting media and legislative attention to this threat, the authors Wein and Liu were able to push the relevant agencies to take preventive measures.

Future

The work described here is a subjective selection from the many research efforts under way in areas that also include operations research [14] and economics [7]. As hinted by the descriptions, the models that are successful are not necessarily the most sophisticated methodologically. Indeed, social scientists and practitioners rarely adopt black-box tools—overly technical models whose underlying methods they do not understand. New models have enormous potential to improve our understanding of terrorism and help cope with its threat. Terrorism research can also benefit the broader mathematical sciences community as a source of new problems and methods.

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